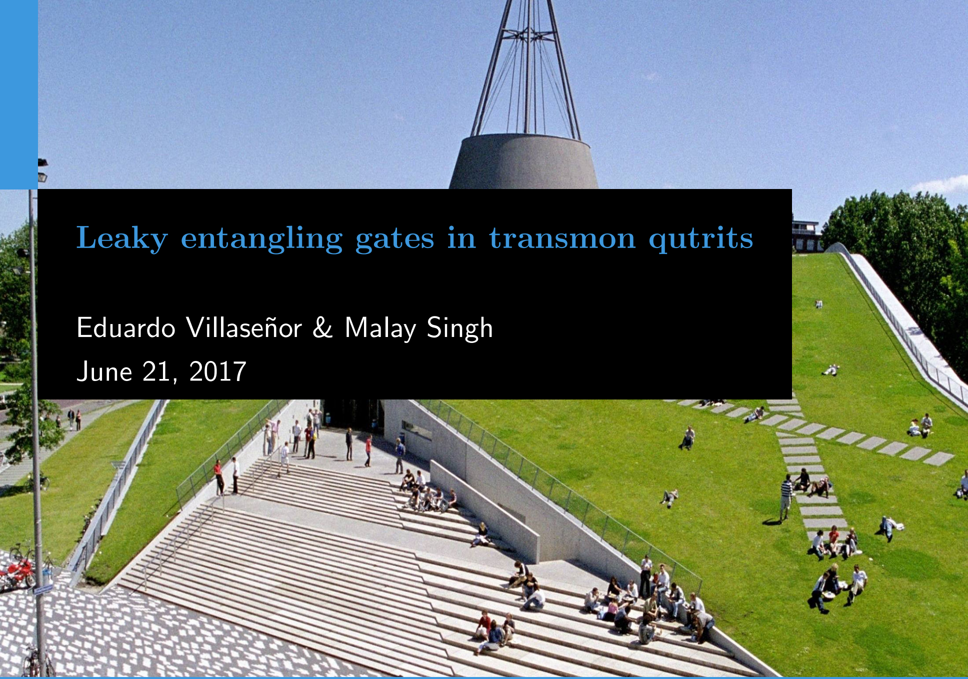


Leaky entangling gates in transmon qutrits

Eduardo Villaseñor & Malay Singh

June 21, 2017

Introduction

- Superconducting qubits are a promising platform for quantum computing.
- Transmon qubit is an anharmonic oscillator, higher states are not taken into account when not being used(iSWAP).
- High fidelity entangling two qubit-gates are needed for every application that requires creation of entanglement.
- Try to replicate and expand the results of the paper "Two-qubit gate operations in superconducting circuits with strong coupling and weak anharmonicity" New Journal of Physics 14 (2012) 073041

The system

The system consists in 2 superconducting circuits, based in Joshephson Johnson junctions coupled via a resonator or two capacitively coupled qubits. The Hamiltonian for two capacitively coupled qubits:

$$H^{\text{direct}} = \sum_{n=1}^{N-1} \left[\left(n\omega_A - \epsilon_n^A\right) |n\rangle_A\langle n| + \left(n\omega_B - \epsilon_n^B\right) |n\rangle_B\langle n| \right] + g J_A^x \otimes J_B^x,$$

Using the rotating wave approximation

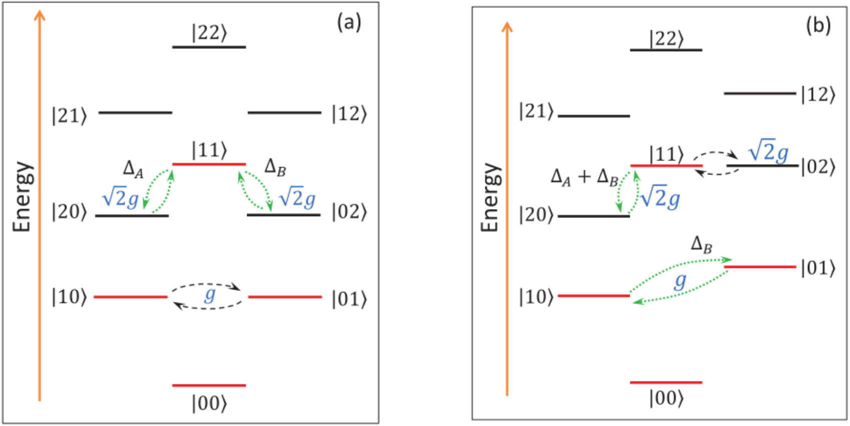
$$H_I^{\text{direct}} = \sum_{j=A,B} \left[\omega_j |1\rangle_j\langle 1| + \left(2\omega_j - \Delta_j\right) |2\rangle_j\langle 2|\right]$$

$$+g[|01\rangle\langle 10| + \sqrt{2}|02\rangle\langle 11| + \sqrt{2}|20\rangle\langle 11| + 2|12\rangle\langle 21| + \text{h.c.}],$$

N is the number of levels. Time independent evolution for time $t_g = \frac{\pi}{2g}$ and $t_g = \frac{\pi}{\sqrt{2}g}$ generates an iSWAP gate and C-Z gate respectively.

2 Coupled qutrits

Product states of the hamiltonian for $\omega_A = \omega_B$ and $\omega_B = \omega_A + \Delta_B$.



$iSWAP, \omega_A = \omega_B$ $\quad$ $CPHASE, \omega_B = \omega_A + \Delta_B$

The model

We use the *qutrit* basis states

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} \quad |1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad |2\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

With the two qubit operators in this base

$$\begin{aligned} CPHASE &= |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| - |11\rangle\langle 11| \\ iSWAP &= |00\rangle\langle 00| - i\,|01\rangle\langle 10| - i\,|10\rangle\langle 01| + |11\rangle\langle 11| \end{aligned}$$

Master equation

Using the master equation

$$\dot{\rho}(t) = \mathcal{L}[\rho] = -i[\mathcal{H}, \rho(t)] + \sum_{k=1}^{M} \left(L_k \rho(t) L_k^\dagger - \frac{1}{2} L_k^\dagger L_k \rho(t) - \frac{1}{2} \rho(t) L_k^\dagger L_k \right)$$

Aditionally we would like to implement the other types of decoherence, that is dephasing and relaxation.

$$\sigma_Z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix} \qquad \sigma_{1\to 0} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$\sigma_{2\to 1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

TU Delft

Master equation

Making the tranformation $\rho = |\psi\rangle \langle\psi| \rightarrow |\psi\rangle \otimes |\psi*\rangle$. We can obtain a flattnend representation of the Master equation, where

$$\rho(t) = \exp[(\mathcal{H} + \mathcal{G})t]\rho(0)$$

With each term:

- Unitary evolution

$$\mathbf{H} = -i(\mathcal{H} \otimes \mathbb{1} - \mathbb{1} \otimes \mathcal{H})$$

- Linbladian

$$\mathcal{G} = \sum_{m=0}^{M} \bar{L}_m \otimes L_m - \frac{1}{2}\mathbb{1} \otimes (L_m^\dagger L_m) - \frac{1}{2}(\bar{L}_m^\dagger \bar{L}_m) \otimes \mathbb{1}$$

TU Delft

Fidelity

The main goal is to test the parameter space and find the values $\Delta_A, \Delta_B, \omega$ where the target operations $CPHASE$ and $iSWAP$ cn be recovered. We use the Fidelity to measure the distance between the target gate and the actual evolution.

$$F = 1 - \frac{1}{16}||U_T - P^\dagger U(t_g)P||^2$$

With P the proyector to two-qubit space

$$P = |00\rangle\langle 00| + |01\rangle\langle 01| + |10\rangle\langle 10| + |11\rangle\langle 11|$$

Fidelity for Master equation

For the master equation we used a initial random state ρ_0. To whom we evolved $\rho(t_g) = U(t_g)\rho_0 = \exp[(\mathcal{H} + \mathcal{G})t_g]\rho_0$.
And then compared to $\rho_T = U_T \rho U_T^\dagger$. Using the trace distance as a measure

$$F = 1 - \mathsf{Tr}(\rho_T, \rho(t_g)) = 1 - \sum_i \frac{1}{2}|\lambda_i|,$$

with λ_i the eigenvalues of $\rho_T - \rho(t_g)$

Optimization

After evolution time t_g gates are still off by single qubit unitaries. A transformation using single qubit gates is required.

$$U'(t_g) = U_I(\theta_1)U_Z^B(\theta_2)U_Z^A(\theta_3)U(t_g),$$

with

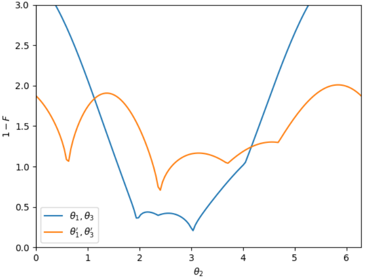
- $U_I(\theta_1) = e^{i\theta_1 \mathbb{1}}$,
- $U_Z^B(\theta_2) = e^{i\theta_2 \sigma_Z^B}$,
- $U_Z^A(\theta_3) = e^{i\theta_3 \sigma_Z^A}$.

Were $(\theta_1, \theta_2, \theta_3)$ must maximize F (minimize Trace distance in the case of Master eq.)

Optimization

The problem has many global minimua. Basin-hopping algorithm was required to find consistently the global minima. It's a stochastic optimization algorithm like simulated annealing:

1. Random perturbation of coordinates.
2. Iterate once using a local optimization algorithm (Nelder-Mead) to find a minimum.
3. Of all the iterations choose the smallest as the global minima .



TU Delft

Finding the Fidelity

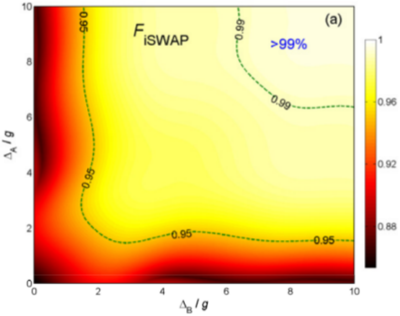
The steps of our routine are as follows:

1. Select the system parameters $\omega_A, \omega_B, \Delta_A, \cdots$
2. Find the evolution operator U usign the Hamiltonian (Master eq.)
3. Maximize Fidelity by optimizing the parameters $(\theta_1, \theta_2, \theta_3)$ using BH algorithm.

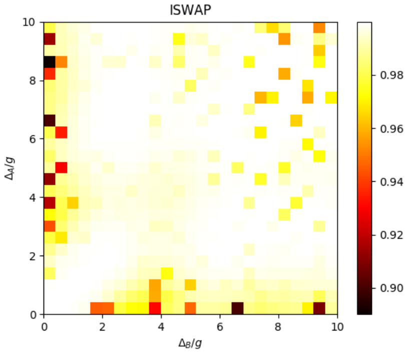
TUDelft

Results

Without the Master eq. For $iSWAP$



Reference

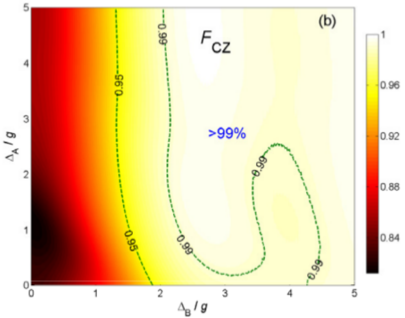


Results

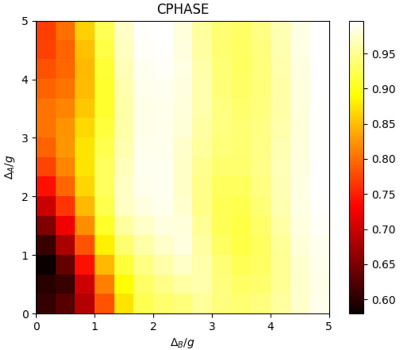
† *Two-qubit gate operations in superconducting circuits with strong coupling and weak anharmonicity*, Xin-You Lü, S Ashhab, Wei Cui,

Comparing of results with paper

For the $CPHASE$ gate.



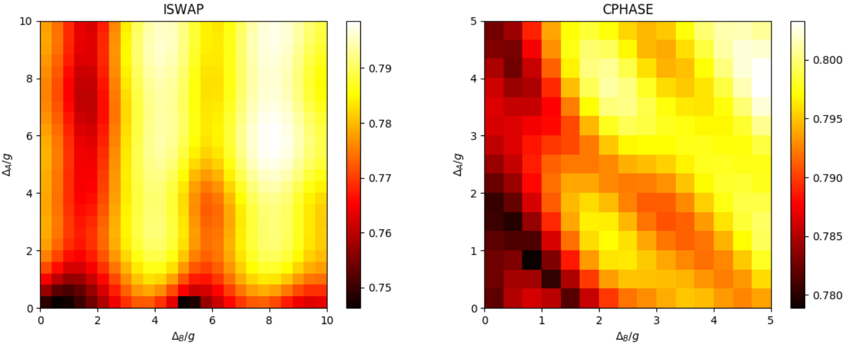
Reference



Results

Using the master equation

Using the parameters for the master eq. : $\Gamma_{1\to 0} = 3.1 \times 10^5$, $\Gamma_{2\to 1} = 1.55 * 10^5$ and $\Gamma_{dephasing} = 4.188 \times 10^5$.Ref. : Coherence and Decay of Higher Energy Levels of a Superconducting Transmon Qubit;Michael J. Peterer. MIT group.



Conclusions and outlook

- We were able to simulate the system and obtain the $CPHASE$ and $iSWAP$ operations which consistent results as in the literature
- Using the master eq. we expanded the model to include dephasing and relaxation in the operations.
- Our program allows us to calculate given the system parameters, the maximum fidelity that can be achieved in the two qubit operations.

TUDelft

Ooutlook

- We want to move one qubit with to the other and bring the two qubits in resonance using a flux pulse. We need to do a time dependent master equation simulation for that. Target is to optimize the shape of the flux pulse keeping in mind all the experimental constraints using with fidelity as a cost function.
- Include even higher levels.
- Use two optimized iSWAPS with 5 unitaries to generate a CNOT gate. Explore the possibility of adding decoupling sequences to combat dephasing(Low frequency noise).