First exploration to a system with a strong interaction to an enviroment

Eduardo Villaseñor

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1 Introduction

We are interested in studying a central spin 1/2 system that has a strong interaction to an environment. To accomplish this, we simulate the environment E using a kicked spinchain and have our central system C having some strong interaction with some parts of this spinchain.

As a first exploration, two similar models were used as fig. 1 shows. Having in the first model the central system interaction with only one of the parts of the chain, in contrast with the second one in which the central system interacts with three parts of the chain.

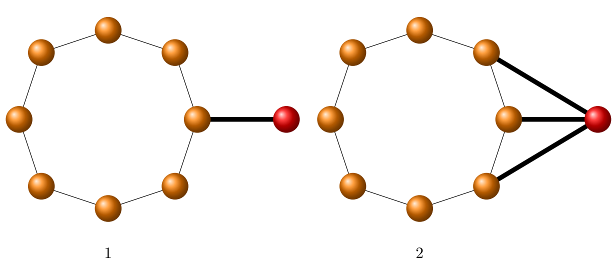

Figure 1: Visual representation of the two models used. The orange dots represent the environment E and the red one the central system C.

To describe the model we introduce the Hamiltonians for H_{CE} the interaction between C and E, and H_E for the internal interaction between the particles in the environment.

$$H_{CE} = J_c \sum_{i,j} \sigma_i^z \sigma_j^z; \quad H_E = \sum_{k \in E} J_k \sigma_k^z \sigma_{k+1}^z. \tag{1}$$

The Ising interaction between each of the the spins on the chain is randomized, such that $J_k \in [J_s - \Delta J_s, J_s + \Delta J_s]$. Thus the Ising Hamiltonian that describes the complete model

$$H_I = H_{CE} + H_E. \tag{2}$$

Additionally, the magnetic field Hamiltonian described as usual for the entire model.

$$H_{II} = \sum_i \vec{\sigma}_i \cdot \vec{b}, \tag{3}$$

where we can always chose a reference frame were the field is only b_x and b_z.

We see that the system is characterized by the set of parameters $[J_c, J_s, \Delta J_s, bx, by]$. Also we must mention that over this full work the initial state of the system $|\psi\rangle = |\psi^E_{\text{rand}}\rangle \otimes |\psi^C_{\text{rand}}\rangle$ was always used.

2 Finding the correct parameters

We are interested in finding the correct parameters in which the system has an chaotic behaviour. To accomplish this we analyse the systems spectral density and the nearest neighbour distribution. For both models with the parameters $[1.4, 1.4, 0.2, 1.4, 1.4]$ we found such behaviour as fig 2 and fig. 3 show for models 1 and 2 respectively.

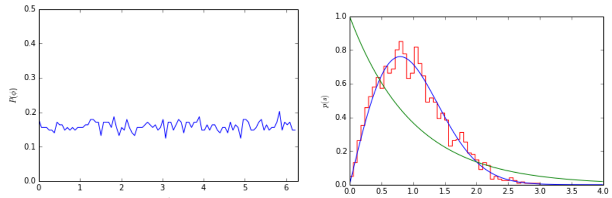

Figure 2: (left)Spectral density and (right)the nearest neighbour distribution for model 1.

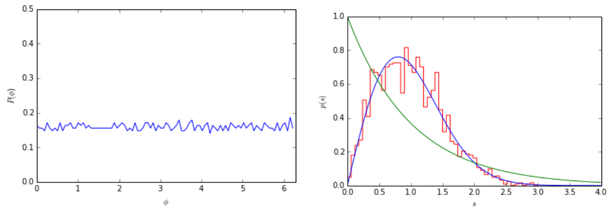

Figure 3: (left)Spectral density and (right)the nearest neighbour distribution for model 2.

It's important to note that different values of J_c were used, fixing the other parameters, and a very similar behaviour was found. And so the set of parameters to be used is $P = [J_c, 1.4, 0.2, 1.4, 1.4]$.

3 Purity decay of C

First we are interested in knowing how much the randomness of the system (in the parameters J_k) affects the Purity of C. We calculate the Purity over times as $\gamma(t) = \mathrm{Tr}(\rho(t)^2)$, where $\rho(t)$ corresponds to the reduced density matrix of $|\psi(t)\rangle$ over C. The results in fig. 4 show how for small values of J_c there is a considerable variation for different *runs* of the system, however for larger values of J_c there is no considerable variation.

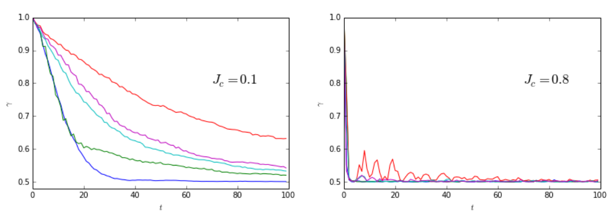

Figure 4: Different *runs* using model 1, were different random variables J_k were created using parameters P and fixed $dim(E) = 10$ The left panel corresponds with $J_c = 0.1$, and the right panel to $J_c = 0.8$.

3.1 Varying the size of E

Now we explore the purity when we vary the size of E. The results for purity decay in each of the models are shown of fig. 5.

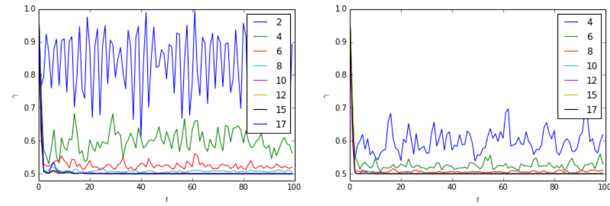

Figure 5: Purity decay fixing the value $J_c = 0.8$ and using P for the other parameters. The legend shows the total dimension of the system, that is $dim(E) + 1$. The left panel shows for model 1 and the right one shows model 2.

3.2 Varying the parameter J_c

To complete the exploration for these two models, we now fix all the variables and vary the parameter J_c. Our interests now is in finding the behaviour of the purity γ for long times. To do so we define the average over two points in times as

$$< \gamma >_{T_1}^{T_2} = \frac{1}{T_2 - T_1 + 1} \sum_{t=T_1}^{T_2} \gamma(t). \tag{4}$$

Fig. 6 show the calculations for $< \gamma >_{80}^{100}$ obtained for both models.

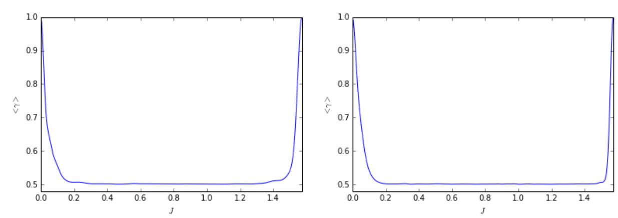

Figure 6: Values $< \gamma >_{80}^{100}$ varying J_c, with parameters P and a total of 10 qubits. For both models, 1 and 2, left and right respectively.

We see how for both models the behaviour of γ, in long times, basically goes to its lowest value possible with the exception where the system becomes trivial near $J_c = 0$ and $J_c = \frac{\pi}{2}$ being J_c $\frac{\pi}{2}$-periodic.