HW 5: State Estimation in Oil Well Drilling

Due: Wednesday, May 1, at 11:59pm CST

Total Points: 10 pts

This assignment will provide hands-on practice for state estimation with application to oil and gas energy systems. The assignment is organized in a tutorial fashion, thereby allowing you to practice state estimation on a relevant real-world energy system example.

Movie-goers will recognize this problem from the 1998 science fiction thriller "Armageddon." In this movie, Bruce Willis, an experienced and gritty offshore oil-driller, must drill a hole into an asteroid plummeting towards Earth to destroy it. If only he knew about state estimation, then maybe he could prevent the insufferable Ben Affleck from marrying Liv Tyler... and save Earth.

Reading: The following four articles describe state estimation in various energy applications: batteries [2], buildings [3], traffic [4], and geophysical systems [5]. Please peruse for your own self-interest. There are no questions regarding this reading.

Background: The process of oil well drilling involves creating a borehole several hundred meters deep in the ground until an oil reservoir is reached. The drill string, consisting of the assembly of drill pipes, drill collars, and the rock-cutting tool referred to as drill bit (see Figure 1) rotates around its vertical axis, penetrating through the rock. At the top of the drillstring, the rotary table provides the necessary torque to put the system into a rotary motion.

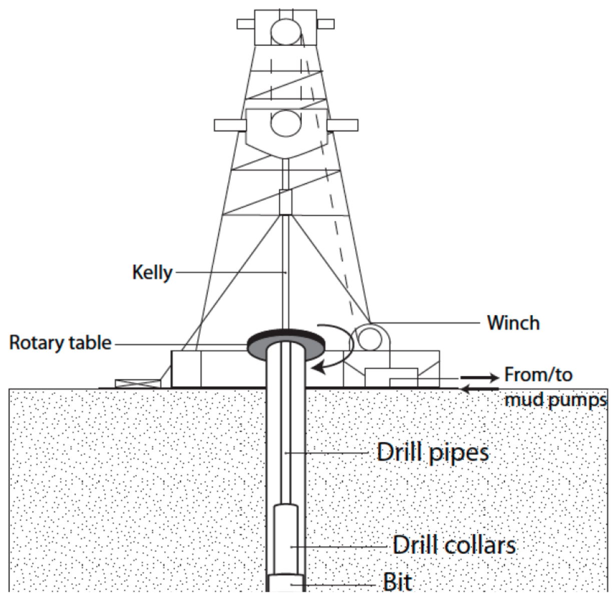

Figure 1: Schematic view of a drilling system [1].

During this process, the drill bit will stick and slip, causing oscillations in the drill string and creating mechanical fatigue to the bit itself. For this reason, it is crucial to monitor the drill bit speed. However, it's nearly impossible to place sensors hundreds of meters into the ground, especially in the perilous environment near the bit which contains flows of rock cuttings, oil, gas, water, and mud. For this reason, we seek to estimate the bit velocity by fusing a model of the drill string dynamics, and measurements of the rotary table speed.

Problem 1: Dynamic System Modeling [3.5 pts]

We model the drill-string as two rigid rotating bodies: the table/top portion and the bit/bottom portion, connected by a rotational spring, as shown in Figure 2. The top and bottom have rotational inertia J_T and J_B, respectively. A motor provides the table torque $T(t)$. The rock provides some unmeasurable frictional torque $T_f(t)$ on the drill bit at the bottom. The top and bottom also experience torques from viscous drag and the rotational spring.

(a) Define the modeling objective. What are the inputs and outputs? Which inputs are controllable and

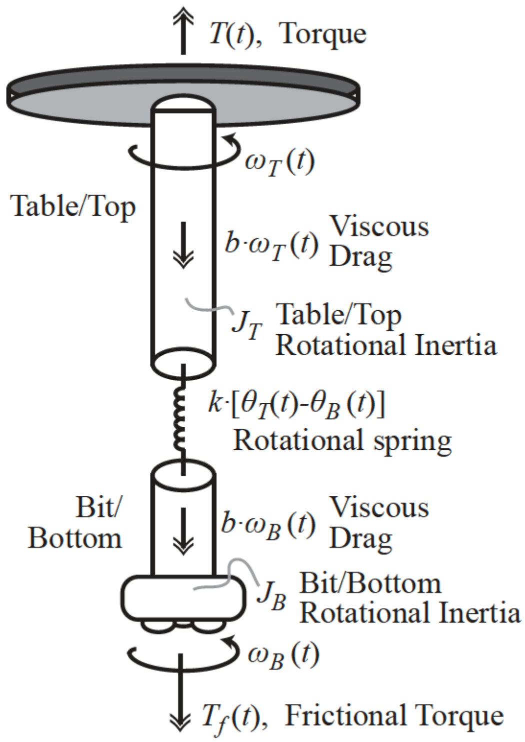

Figure 2: Free-body diagram of drill string.

uncontrollable? List the parameters. [0.5 pts]

(b) Use Newton's second law in rotational coordinates to derive the equations of motion for the top/table and bottom/bit portions of the drill string. [2 pts]

<u>Hint 1:</u> Newton's second law for rotation is given by

$$\sum_i T_i = J\alpha, \tag{1}$$

where J denotes rotational inertia, α stands for angular acceleration. The left hand side of equation (1) represents the summation of all applied torques. We will use the convention that the counterclockwise angular acceleration is positive.

<u>Hint 2:</u> The relations between angular position θ, angular velocity ω, and angular acceleration α are given by $\omega = \dot{\theta}$ and $\alpha = \ddot{\theta}$ (thus $\alpha = \dot{\omega}$).

(c) Write all the dynamical equations into matrix state-space form. What are the A, B, C matrices? Hint: $A \in \mathbb{R}^{4\times 4}$. [1 pt]

Problem 2: Observability Analysis [2 pts]

In this problem, we consider IF it's possible to estimate the bit velocity from table velocity measurements. Assume the following parameter values: $J_T = 100$, $J_B = 25$, $k = 2$, $b = 5$.

(a) Compute the Observability Matrix $\mathcal{O}$ described in Section 2 of Chapter 5. Are all the states observable from measurements of table velocity? Justify why or why not. [0.5 pts]

(b) Define a new state $\theta(t) = \theta_T(t) - \theta_B(t)$ as the relative angular displacement between the top and bottom. Re-derive the state-space model replacing θ_T, θ_B with θ only. Provide the new A, B, C matrices. [1 pt]

(c) Compute the Observability Matrix $\mathcal{O}$ for this modified model. Is the bit velocity observable from measurements of the table velocity? Justify why or why not. [0.5 pts]

Problem 3: Measurement Data [1 pt]

Download `HW5_Skeleton.m` and `HW5_Data.csv` from Canvas. Import `HW5_Data.csv`. In one figure, with two subplots, plot the table torque $T(t)$ and measured table velocity $\omega_T(t)$ versus time. Use legends, x- and y-labels, appropriate font sizes, and line widths. Provide the plot in your report.

Problem 4: Luenberger Observer [3.5 pts]

In this problem we design a Luenberger observer to estimate bit speed from the 3-state model and measurements. You are encouraged to experiment with the eigenvalue placement to obtain a good balance between estimation speed and robustness to noise.

(a) Compute the eigenvalues of the open loop system, A. Write down the Luenberger observer equations in your report. [0.5 pts]

(b) Design the output error injection gain, L, as described in Chapter 5, Section 3. Select some desired eigenvalues for the error system matrix $(A - LC)$. Remember, the only requirements are that (i) the real parts must be strictly negative, and (ii) complex eigenvalues must be in complex conjugate pairs. Provide your selected eigenvalues and observer gain L in the report. HINT: You need the `place` command in Matlab. This requires the Matlab Control Systems Toolbox. [1 pt]

(c) Write down the state-space matrices for your Luenberger observer, denoted $A_{\text{lobs}}, B_{\text{lobs}}, C_{\text{lobs}}$, such that the Luenberger observer equations in Problem 4(a) can be re-expressed as

$$\dot{\hat{x}}(t) = A_{\text{lobs}}\hat{x}(t) + B_{\text{lobs}}\overline{u}(t), \tag{2}$$

$$\hat{y}(t) = C_{\text{lobs}}\hat{x}(t). \tag{3}$$

Note, (i) $\overline{u}(t)$ in equation (2) is different than $u(t)$ in Problem 4(a) (think about why), and (ii) the frictional torque is unknown and unmeasured and therefore not included in the Luenberger observer. [1 pt]

(d) In one figure, create two subplots. On top, plot the estimated bit speed $\hat{\omega}_B(t)$ versus the true value `omega_B_true` from `HW5_Data.csv`. On bottom, plot the estimation error $\omega_B(t) - \hat{\omega}_B(t)$. Use these plots to tune the eigenvalues of $(A - LC)$. Provide this plot in your report, and document your eigenvalue placement for $(A - LC)$. HINT: There is no single best answer. Tune to your preference. [1 pt]

Problem 5 and Problem 6 are bonus questions. We did not cover these two topics in the classroom. However, you are encouraged to self-study Section 4 and Section 5 from the Chapter 5 course note and make an attempt on these two problems.

Problem 5: Kalman Filter (KF) Design [1.5 pts]

In this problem we design a Kalman Filter, using the canonical model format

$$\dot{x}(t) = Ax(t) + Bu(t) + w(t), \tag{4}$$

$$y_m(t) = Cx(t) + n(t), \tag{5}$$

where $x(t) = [\theta(t), \omega_T(t), \omega_B(t)]^\top$, $u(t) = T(t)$, $w(t) = T_f(t)$, and $y_m(t) = \omega_T(t) + n(t)$. The process noise $w(t)$ encapsulates unknown frictional torque on the drill bit. The measurement noise $n(t)$ represents noise in the table velocity sensor $\omega_T(t)$.

(a) Implement the Kalman filter equations in your code. Use the skeleton code `ode_kf.m` on Canvas. Assume the measurement noise covariance is $N = 0.02$, the mean initial condition is $\overline{x}_0 = 0_{3\times 1}$, and the initial condition covariance is $\Sigma_0 = I_{3\times 3}$. Tune the process noise covariance $W \in \mathbb{R}^{3\times 3}$, and provide the value in your report. Explain your selection. [1 pt]

(b) In one figure, create two subplots. On top, plot the estimated bit speed $\hat{\omega}_B(t)$ versus the true value `omega_B_true` from `HW5_Data.csv`. Also plot the $\pm$ one-standard deviation bound, i.e. $\hat{\omega}_B(t) \pm \sqrt{\Sigma_{3,3}(t)}$. On bottom, plot the estimation error $\omega_B(t) - \hat{\omega}(t)$. Use these plots to tune the process covariance W. Provide this plot in your report. [0.5 pts]

Problem 6: Extended Kalman Filter (EKF) Design [1 pt]

Now we consider a nonlinear rotational spring law. In Figure 2 and Problem 1, we relate spring displacement $\theta(t)$ and spring torque by Hooke's law: $k\theta(t)$. This is a linear relationship. In reality, spring torque exhibits a nonlinear relationship. In this problem, let us replace Hooke's law with the more realistic nonlinear spring torque relationship: $k_1\theta(t) + k_2\theta^3(t)$. Consider values of $k_1 = 2$ and $k_2 = 0.25$.

Design an Extended Kalman Filter (EKF) from Chapter 5, Section 5. In your report:

(a) Derive expressions for the Jacobians: $F(t) = \dfrac{\partial f}{\partial x}(\hat{x}(t), u(t))$, $H(t) = \dfrac{\partial h}{\partial x}(\hat{x}(t), u(t))$. [0.25 pts]

(b) Create a new MATLAB script `ode_ekf.m` and implement EKF by copying, pasting, and modifying your KF code. Use the covariance W you used from Problem 5. In one figure, create two subplots. On top, plot the estimated bit speed $\hat{\omega}_B(t)$ versus the true value `omega_B_true` from `HW5_Data.csv`. Also plot the $\pm$ one-standard deviation bound, i.e. $\hat{\omega}_B(t) \pm \sqrt{\Sigma_{3,3}(t)}$. On the bottom, plot the estimation error $\omega_B(t) - \hat{\omega}(t)$. [0.75 pts]

Deliverables

Submit the following on Gradescope. Be sure that all files are named exactly as specified (including spelling and case), and make sure the MATLAB function declaration is exactly as specified.

`LASTNAME_FIRSTNAME_HW5.PDF`
`LASTNAME_FIRSTNAME_HW5.m`
`ode_kf.m` (optional)
`ode_ekf.m` (optional)

References

[1] Conrad Sagert, Florent Di Meglio, Miroslav Krstic, and Pierre Rouchon. Backstepping and flatness approaches for stabilization of the stick-slip phenomenon for drilling. *IFAC Proceedings Volumes*, 46(2):779–784, 2013.

[2] Scott J Moura. Estimation and control of battery electrochemistry models: A tutorial. In *2015 54th IEEE Conference on Decision and Control (CDC)*, pages 3906–3912. IEEE, 2015.

[3] Peter Radecki and Brandon Hencey. Online building thermal parameter estimation via unscented kalman filtering. In *2012 american control conference (acc)*, pages 3056–3062. IEEE, 2012.

[4] Daniel B Work, Olli-Pekka Tossavainen, Sébastien Blandin, Alexandre M Bayen, Tochukwu Iwuchukwu, and Kenneth Tracton. An ensemble kalman filtering approach to highway traffic estimation using gps enabled mobile devices. In *2008 47th IEEE Conference on Decision and Control*, pages 5062–5068. IEEE, 2008.

[5] Jeffrey L Anderson. Ensemble kalman filters for large geophysical applications. *IEEE Control Systems Magazine*, 29(3):66–82, 2009.