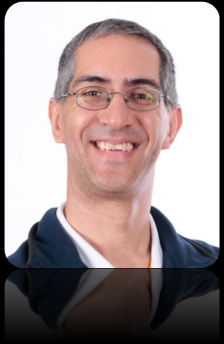
CS61C News

- Errata on lecture 1: The cumulative section of the final will clobber the midterm



UC Berkeley
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CS61C

Great Ideas in Computer Architecture

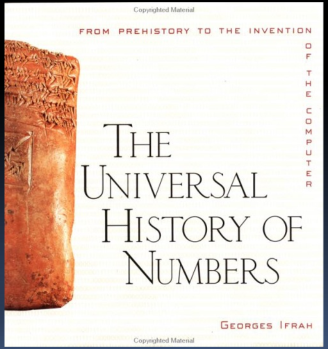
(a.k.a. Machine Structures)

UC Berkeley
Lecturer
Justin Yokota

Number Representation

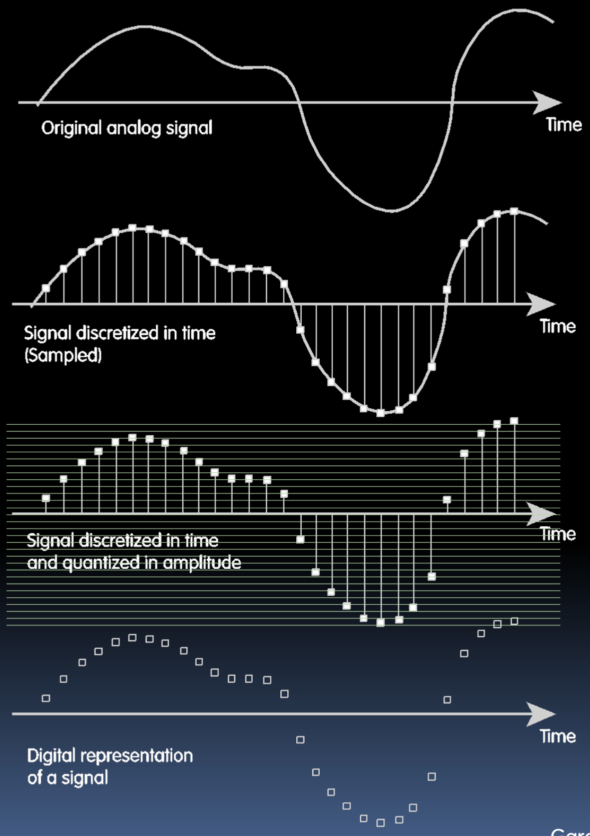
Great book ⇒
The Universal History of Numbers

by Georges Ifrah



Data input: Analog → Digital

- Real world is analog!
- To import analog information, we must do two things
  - Sample
    - E.g., for a CD, every 44,100ths of a second, we ask a music signal how loud it is.
  - Quantize
    - For every one of these samples, we figure out where, on a 16-bit (65,536 tic-mark) "yardstick", it lies.



Digital data not necessarily born Analog…

hof.povray.org

BIG IDEA: Bits can represent anything!!

- Characters?
  - 26 letters ⇒ 5 bits ($2^5 = 32$)
  - upper/lower case + punctuation
    ⇒ 7 bits (in 8) ("ASCII")
  - standard code to cover all the world's languages ⇒ 8,16,32 bits ("Unicode")
    www.unicode.com
- Logical values?
  - 0 → False, 1 → True
- colors ? Ex: ***Red (00)*** ***Green (01)*** ***Blue (11)***
- locations / addresses? commands?
- MEMORIZE: N bits ⇔ at most 2^N things

☑

Respond at **pollev.com/ddg**

Text **DDG** to **22333** once to join, then **A, B, C, D, or E**

L02a Number Representation: How many bits to represent π (Pi)?

1	A
9 (π = 3.14, so that's 011 "." 001 100)	B
64 (Since Macs are 64-bit machines)	C
Every bit the machine has!	D
∞	E

Binary
Decimal
Hex

Number vs Numeral

Numeral

A symbol or name that stands for a number
e.g., 4 , four , quatro , IV , IIII , …
…and Digits are symbols that make numerals

Above the abstraction line

Abstraction Line

Below the abstraction line

Number

The "idea" in our minds…there is only ONE of these
e.g., the concept of "4"

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Base 10 (Ten) Numerals, Decimals

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

Example:

$3275 = 3275_{10} = (3 \times 10^3) + (2 \times 10^2) + (7 \times 10^1) + (5 \times 10^0)$

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Base 2 (Two) Numerals, Binary (to Decimal)

Digits: 0, 1 (<u>bi</u>nary digi<u>ts</u> → bits)

Example: "1101" in binary? ("0b1101")

$$1101_2 = (1 \text{x} 2^3) + (1 \text{x} 2^2) + (0 \text{x} 2^1) + (1 \text{x} 2^0)$$

$$= 8 + 4 + 0 + 1$$

$$= 13$$

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Base 16 (Sixteen) #s, Hexadecimal (to Decimal)

Digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F
10,11,12,13,14,15

Example: "A5" in Hexadecimal?

$0xA5 = A5_{16} = (10 \times 16^1) + (5 \times 16^0)$

$= 160 + 5$

$= 165$

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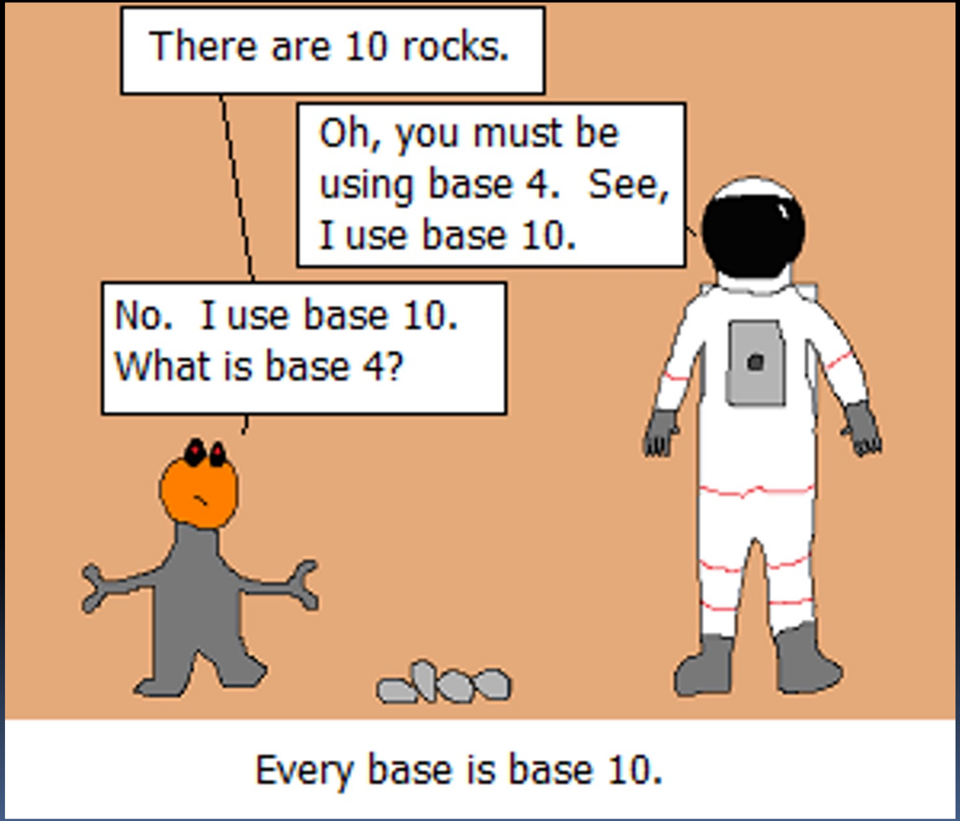
Every Base is Base 10...



Convert from Decimal to Binary

- E.g., 13 to binary?
- Start with the columns

~~13~~
~~5~~
~~1~~
0

$2^3=8$	$2^2=4$	$2^1=2$	$2^0=1$
1	1	0	1

- Left to right, is (column) ≤ number **n**?
  - If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
  - If not, put 0 and keep going. (and stop at 0)

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Convert from Decimal to Hexadecimal

- E.g., 165 to hexadecimal?
- Start with the columns

~~165~~
~~5~~
0

$16^3=4096$	$16^2=256$	$16^1=16$	$16^0=1$
0	0	(10) A	5

- Left to right, is (column) ≤ number **n**?
  - If yes, put how many of that column fit in **n**, subtract col * that many from **n**, keep going.
  - If not, put 0 and keep going. (and Stop at 0)

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Convert Binary ←→ Hexadecimal

- Binary → Hex? Easy!
  - Always left-pad with 0s to make full 4-bit values, then look up!
  - E.g., **0b11110** to Hex?
    - **0b11110 → 0b00011110**
    - Then look up: **0x1E**
- Hex → Binary? Easy!
  - Just look up, drop leading 0s
    - **0x1E → 0b00011110 → 0b11110**

D	H	B
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

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Decimal vs Hexadecimal vs Binary

- 4 Bits
  - 1 “Nibble”
  - 1 Hex Digit = 16 things
- 8 Bits
  - 1 “Byte”
  - 2 Hex Digits = 256 things
  - Color is usually
    0-255 Red,
    0-255 Green,
    0-255 Blue.
    #D0367F=

D	H	B
00	0	0000
01	1	0001
02	2	0010
03	3	0011
04	4	0100
05	5	0101
06	6	0110
07	7	0111
08	8	1000
09	9	1001
10	A	1010
11	B	1011
12	C	1100
13	D	1101
14	E	1110
15	F	1111

Which base do we use?

- Decimal: great for humans, especially when doing arithmetic
- Hex: if human looking at long strings of binary numbers, its much easier to convert to hex and see 4 bits/symbol
  - Terrible for arithmetic on paper
- Binary: what computers use;
  you will learn how computers do +, -, *, /
  - To a computer, numbers always binary
  - Regardless of how number is written:
  - $32_{ten} == 32_{10} ==$ 0x20 $== 100000_2 ==$ 0b100000
  - Use subscripts "ten", "hex", "two" in book, slides when might be confusing

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The computer knows it, too...

```c
#include <stdio.h>
int main() {
    const int N = 1234;

    printf("Decimal: %d\n",N);
    printf("Hex:     %x\n",N);
    printf("Octal:   %o\n",N);

    printf("Literals (not supported by all compilers):\n");
    printf("0x4d2         = %d (hex)\n", 0x4d2);
    printf("0b10011010010 = %d (binary)\n", 0b10011010010);
    printf("02322         = %d (octal, prefix 0 - zero)\n", 0x4d2);
    return 0;
}
```

Output

```
Decimal: 1234
Hex:     4d2
Octal:   2322
Literals (not supported by all compilers):
0x4d2         = 1234 (hex)
0b10011010010 = 1234 (binary)
02322         = 1234 (octal, prefix 0 - zero)
```

☑

Number Representations

What to do with representations of numbers?

- What to do with number representations?
  - Add them
  - Subtract them
  - Multiply them
  - Divide them
  - Compare them

```
      1  0  1  0
+     0  1  1  1
-----------------
```

- Example: 10 + 7 = 17
  - …so simple to add in binary that we can build circuits to do it!
  - Subtraction just as you would in decimal
  - Comparison: How do you tell if X > Y ?

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What if too big?

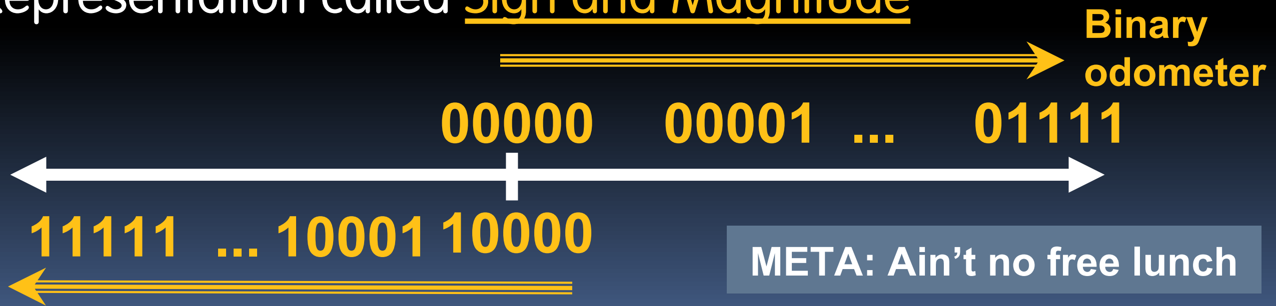
- Binary bit patterns are simply representatives of numbers. Abstraction!
  - Strictly speaking they are called “numerals”.
- Numerals really have an ∞ number of digits
  - with almost all being same (00…0 or 11…1) except for a few of the rightmost digits
  - Just don’t normally show leading digits
- If result of add (or -, *, /) cannot be represented by these rightmost HW bits, we say overflow occurred

How to Represent Negative Numbers?

(C's `unsigned int`, C18's `uintN_t`)

- So far, unsigned numbers

- Obvious solution: define leftmost bit to be sign!
  - 0 ➔ + 1 ➔ – …and rest of bits are numerical value
- Representation called Sign and Magnitude



META: Ain't no free lunch

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Shortcomings of Sign and Magnitude?

- Arithmetic circuit complicated
  - Special steps depending on if signs are the same or not
- Also, two zeros
  - 0x00000000 = $+0_{ten}$
  - 0x80000000 = $–0_{ten}$
  - What would two 0s mean for programming?
- Also, incrementing "binary odometer", sometimes increases values, and sometimes decreases!
- Therefore sign and magnitude used only in signal processors

Another try: complement the bits

- Example: $7_{10} = 00111_2$ $-7_{10} = 11000_2$
- Called One's Complement
- Note: positive numbers have leading 0s, negative numbers have leadings 1s.

- What is -00000 ? Answer: 11111
- How many positive numbers in N bits?
- How many negative numbers?

Shortcomings of One's Complement?

- Arithmetic still somewhat complicated
- Still two zeros
  - $0x00000000 = +0_{ten}$
  - $0xFFFFFFFF = -0_{ten}$
- Although used for a while on some computer products, one's complement was eventually abandoned because another solution was better.

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Two's Complement & Bias Encoding

Standard Negative # Representation

- Problem is the negative mappings "overlap" with the positive ones (the two 0s). Want to shift the negative mappings left by one.
  - Solution! For negative numbers, complement, then add 1 to the result
- As with sign and magnitude, & one's compl. leading 0s → positive, leading 1s → negative
  - 000000...xxx is ≥ 0, 111111...xxx is < 0
  - except 1…1111 is -1, not -0 (as in sign & mag.)
- This representation is Two's Complement
  - This makes the hardware simple!

(C's `int`, C18's `intN_t`, aka a "signed integer")

Two's Complement Formula

- Can represent positive and negative numbers in terms of the bit value times a power of 2:

$$d_{31} \times -(2^{31}) + d_{30} \times 2^{30} + \ldots + d_2 \times 2^2 + d_1 \times 2^1 + d_0 \times 2^0$$

- Example: 1101_{two} in a nibble?

$$= 1 \times -(2^3) + 1 \times 2^2 + 0 \times 2^1 + 1 \times 2^0$$

$$= -2^3 + 2^2 + 0 + 2^0$$

$$= -8 + 4 + 0 + 1$$

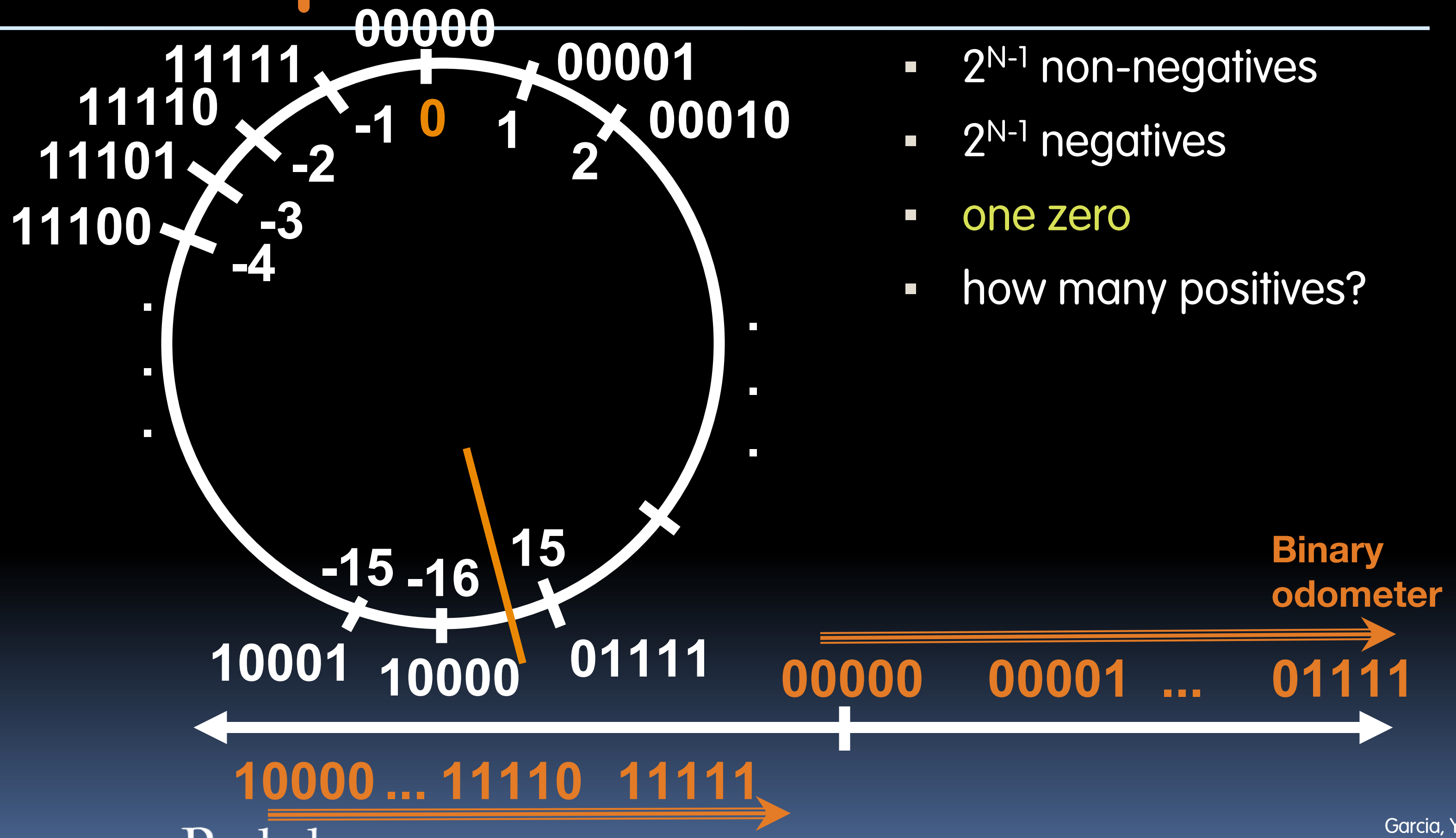
$$= -8 + 5$$

$$= -3_{ten}$$

Example: -3 to +3 to -3 (again, in a nibble):

x	:	1101_{two}	-3
x'	:	0010_{two}	
+1	:	0011_{two}	3
()'	:	1100_{two}	
+1	:	1101_{two}	-3

Two's Complement Number "line": N = 5



- 2^{N-1} non-negatives
- 2^{N-1} negatives
- one zero
- how many positives?

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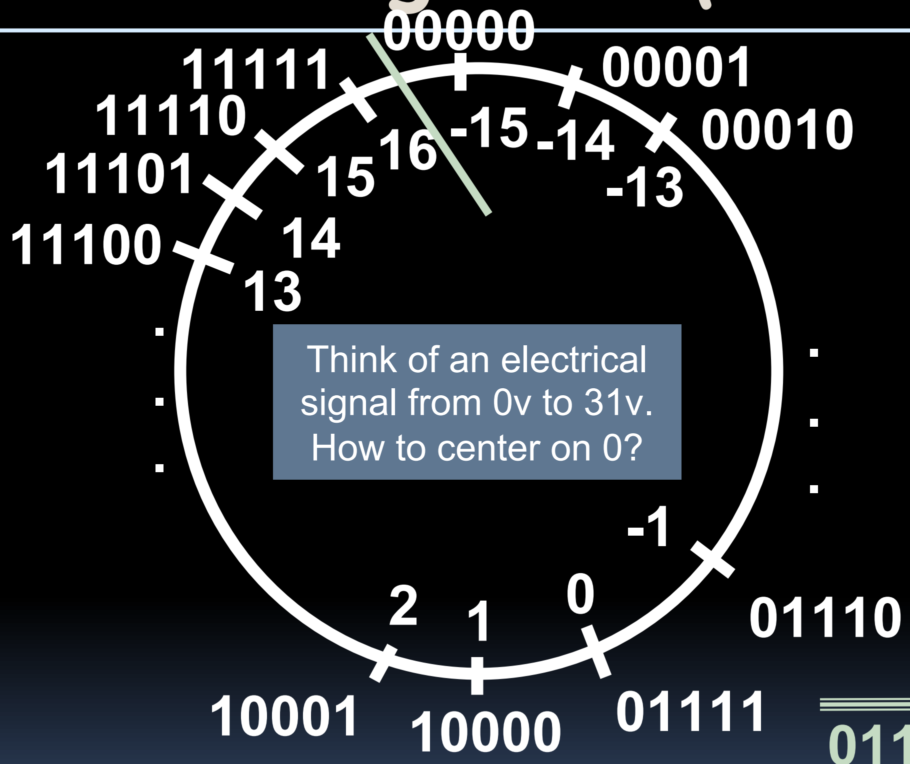
Bias Encoding: N = 5 (bias = -15)



- # = unsigned + bias
- Bias for N bits chosen as $-(2^{N-1}-1)$
- one zero
- how many positives?

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L02b How best to represent -12.75? (explain shifting binary point)

- 2s Complement (but shift binary point)
- Bias (but shift binary point)
- Combination of 2 encodings
- Combination of 3 encodings
- We can't

And in summary...

META: We often make design decisions to make HW simple

- We represent "things" in computers as particular bit patterns: N bits ⇒ 2^N things
- These 5 integer encodings have different benefits; 1s complement and sign/mag have most problems.
- unsigned (C18's **`uintN_t`**) :

00000 00001 ... 01111 10000 ... 11111

- Two's complement (C18's **`intN_t`**) universal, learn!

00000 00001 ... 01111

10000 ... 11110 11111

- Overflow: numbers ∞; computers finite, errors!

META: Ain't no free lunch

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