Tail Calls

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Announcements

Scheme Lists Practice

Dynamic Scope

Dynamic Scope

The way in which names are looked up in Scheme and Python is called lexical scope (or static scope) [You can see what names are in scope by inspecting the definition]

Lexical scope: The parent of a frame is the environment in which a procedure was *defined*

Dynamic scope: The parent of a frame is the environment in which a procedure was *called*

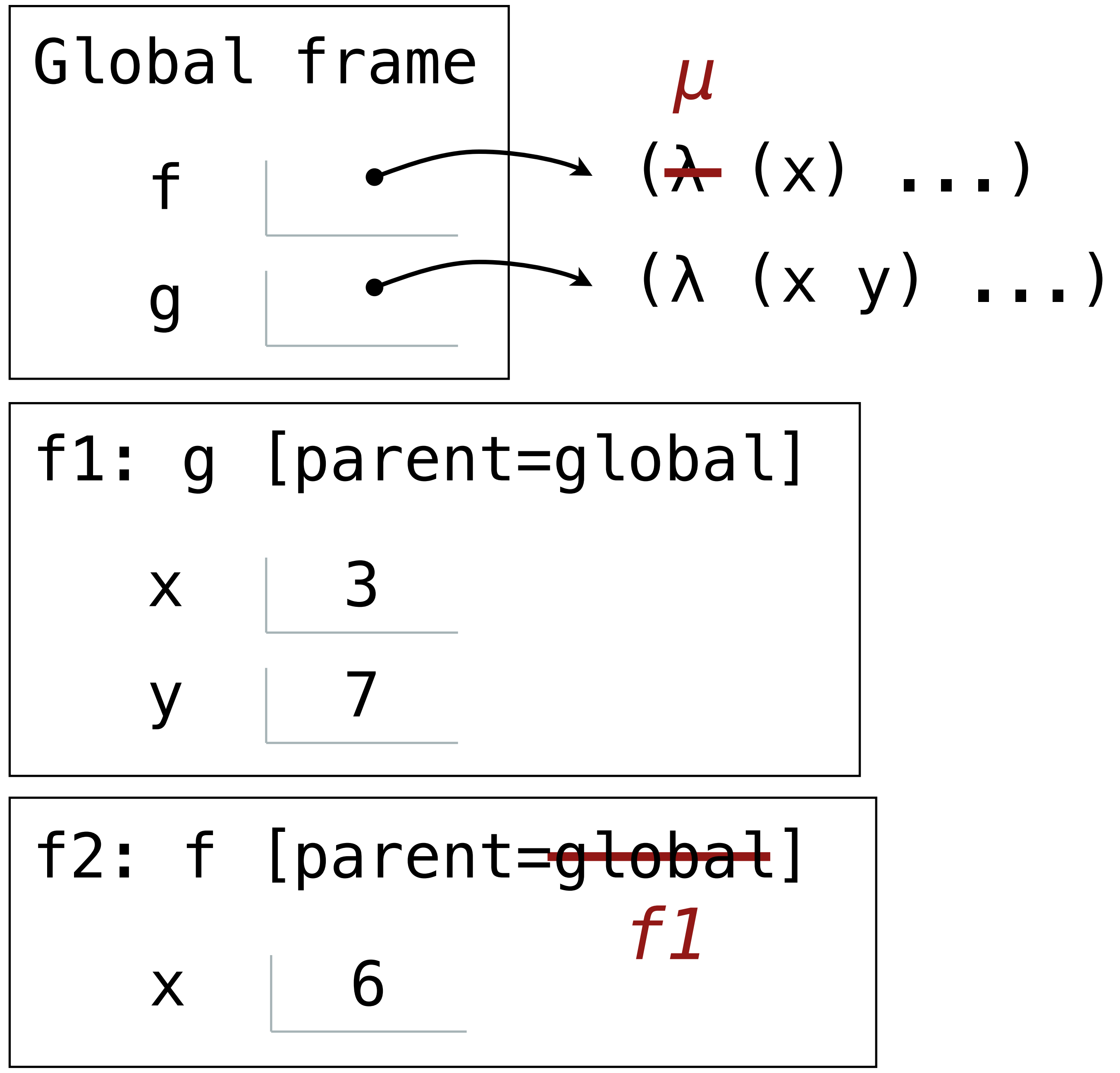
Special form to create dynamically scoped procedures (you will implement **mu** special form in Project 4 Scheme)

mu

(define f (~~lambda~~ (x) (+ x y)))

(define g (lambda (x y) (f (+ x x))))

(g 3 7)



Lexical scope: The parent for f's frame is the global frame

Error: unknown identifier: y

Dynamic scope: The parent for f's frame is g's frame

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Space

Space and Environments

Which environment frames do we need to keep during evaluation?

At any moment there is a set of active environments

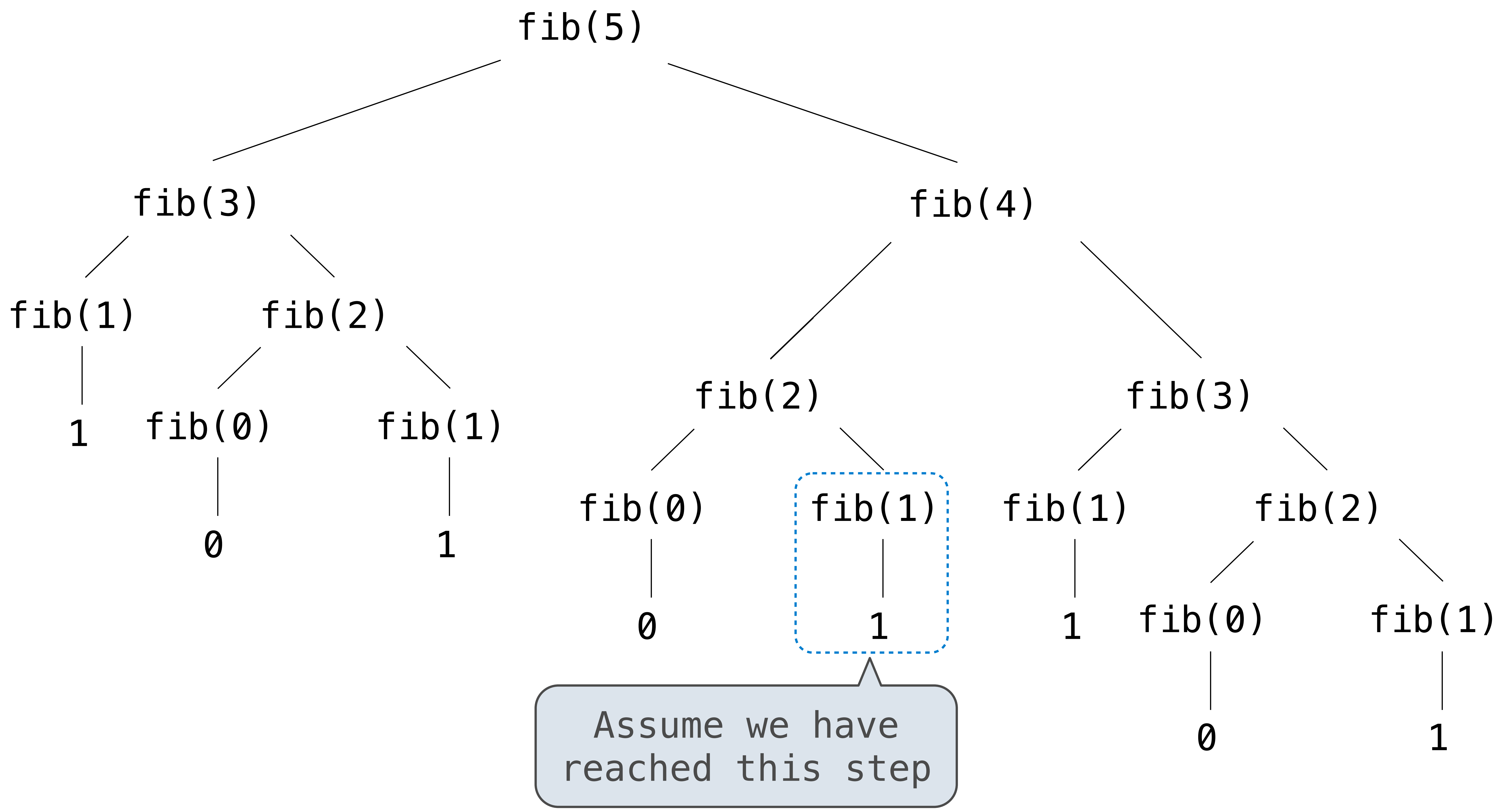
Values and frames in active environments consume memory

Memory that is used for other values and frames can be recycled

Active environments:

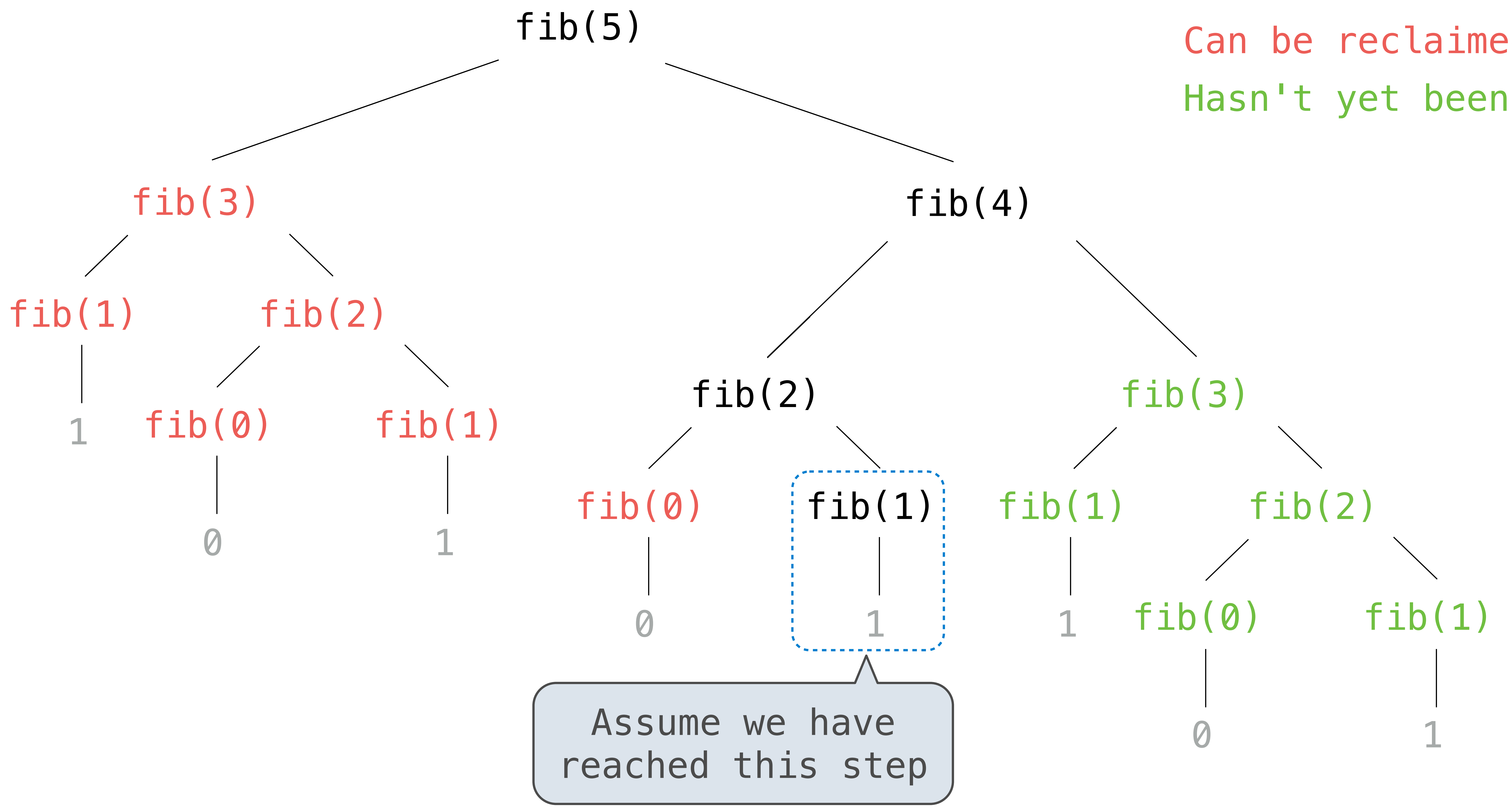
- Environments for any function calls currently being evaluated
- Parent environments of functions named in active environments

Fibonacci Space Consumption



Fibonacci Space Consumption

Has an active environment
Can be reclaimed
Hasn't yet been created



fib takes **linear space.**

(Demo)

pythontutor.com/composingprograms.html#code=def%20fib%28n%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%20or%20n%20%3D%3D%201%3A%0A%20%20%20%20%20%20%20%20return%20n%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20return%20fib%28n-2%29%20%2B%20fib%28n-1%29%0A%20%20%20%20%20%20%20%20%0Afib%286%29&mode=display&origin=composingprograms.js&cumulative=false&py=3&rawInputLstJSON=[]&curInstr=1

Tail Recursion

Functional Programming

All functions are pure functions.

No re-assignment and no mutable data types.

Name-value bindings are permanent.

Advantages of functional programming:

- The value of an expression is independent of the order in which sub-expressions are evaluated
- Sub-expressions can safely be evaluated in parallel or only on demand (lazily) (Demo)
- **Referential transparency:** The value of an expression does not change when we substitute one of its subexpression with the value of that subexpression

But... no for/while statements! Can we make recursion efficient? Yes!

Recursion and Iteration in Python

In Python, recursive calls always create new active frames.

`fact_k(n, k)` computes: `n! * k`

```
def fact_k(n, k):
    if n == 0:
        return k
    else:
        return fact_k(n - 1, n*k)
```

```
def fact_k(n, k):
    while n > 0:
        n, k = n - 1, k * n
    return k
```

Time	Space
Linear	Linear
Linear	Constant

Tail Recursion

From the Revised[7] Report on the Algorithmic Language Scheme:

> "Implementations of Scheme are required to be properly tail-recursive. This allows the execution of an iterative computation in constant space, even if the iterative computation is described by a syntactically recursive procedure."

```scheme
(define (fact_k n k)
  (if (= n 0) k
    (fact_k (- n 1)
            (* k n))))
```

Should use resources like

```python
def fact_k(n, k):
    while n > 0:
        n, k = n-1, k*n
    return k
```

How? Eliminate the middleman!

Time	**Space**
Linear	Constant

(Demo)

pythontutor.com/composingprograms.html#code=def%20factorial%28n,%20k%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%20k%0A%20%20%20%20else%3A%0A%20%20%20%20%20%20%20%20return%20factorial%28n-1,%20k*n%29%0A%20%20%20%20%0Afactorial%284,%2010%29&mode=display&origin=composingprograms.js&cumulative=true&py=3&rawInputLstJSON=[]&curInstr=0

Tail Calls

Tail Calls, Tail Contexts, Tail Recursion

A procedure call that has not yet returned is active. Some procedure calls are tail calls. A Scheme interpreter should support an unbounded number of active tail calls using only a constant amount of space.

A tail call is a call expression in a ***tail context***:

- The last body sub-expression in a **lambda** expression (or procedure definition)
- Sub-expressions 2 & 3 in a tail context **if** expression
- All non-predicate sub-expressions in a tail context **cond**
- The last sub-expression in a tail context **and**, **or**, **begin**, or **let**

A recursive procedure is tail recursive if ***all*** of its recursive calls are tail calls

```
(define (fact-k n k)
  (if (= n 0) k
    (fact-k (- n 1)
            (* k n)) ) )
```

```
(define fact-k (lambda (n k)
  (if (= n 0) k
    (fact-k (- n 1)
            (* k n)) ) ))
```

Example: Length of a List

```
(define (length s)
  (if (null? s) 0
    (+ 1 (length (cdr s))) ) )
```

Not a tail context

A call expression is not a tail call if more computation is still required in the calling procedure

Linear recursive procedures can often be re-written to use tail calls

```
(define (length-tail s)
  (define (length-iter s n)
    (if (null? s) n
      (length-iter (cdr s) (+ 1 n)) ) )
  (length-iter s 0) )
```

Recursive call is a tail call

Break

Tail Recursion Practice

Tail Recursion Examples

Which Procedures are Tail Recursive?

Which of the following procedures run in constant space?

```scheme
;; Compute the length of s.
(define (length s)
  (+ 1 (if (null? s)
           -1
           (length (cdr s))) ) )

;; Return whether s has any repeated elements.
(define (has-repeat2 s)
  (if (null? s)
      #f
      (if (contains (cdr s) (car s))
          #t
          (if (has-repeat2 (cdr s))
            #t
            #f                          )   )   ) )
```

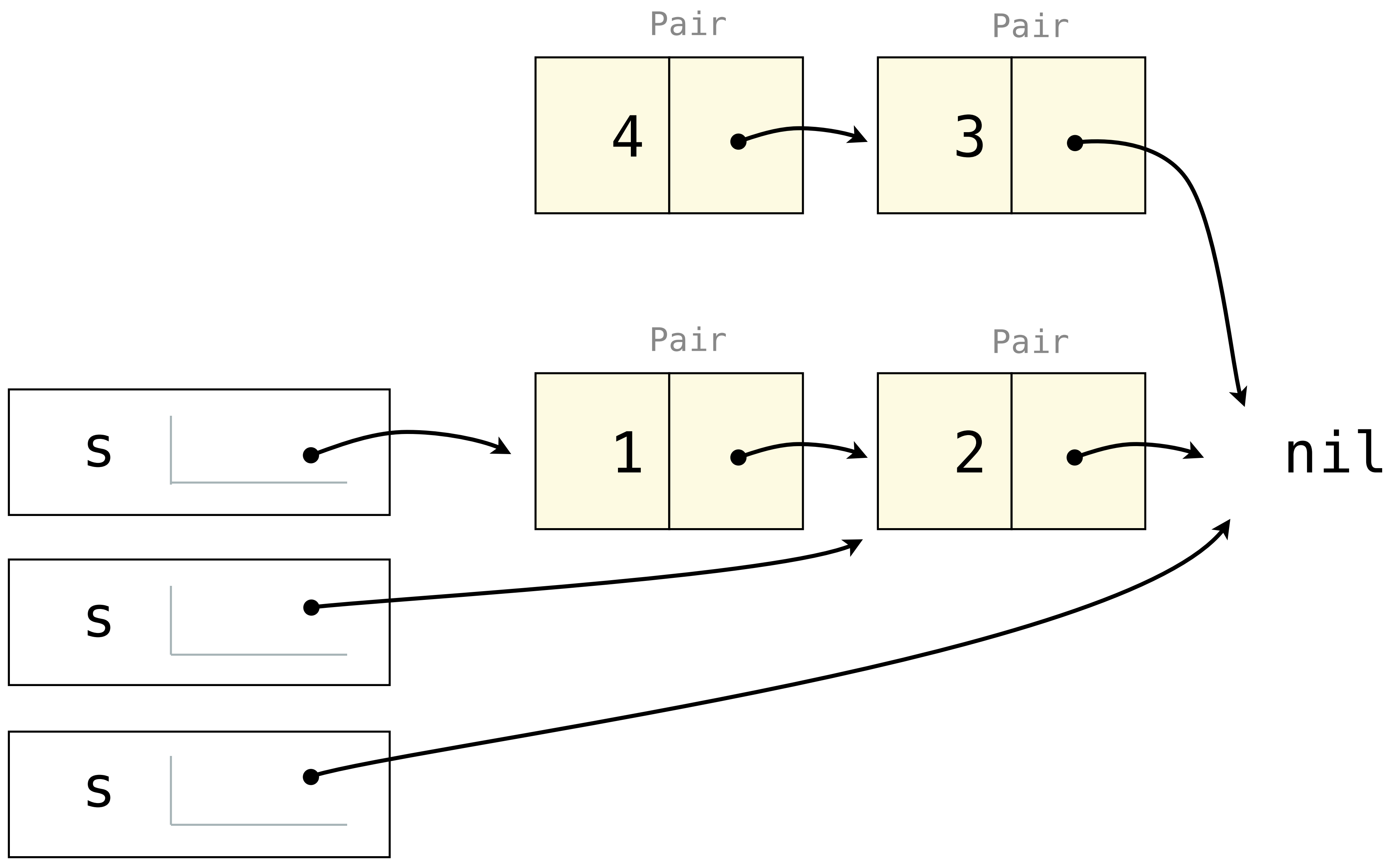
```scheme
;; Return whether s contains v.
(define (contains s v)
  (if (null? s)
      #f
      (if (= v (car s))
          #t
          (contains (cdr s) v))))

;; Return whether s has any repeated elements.
(define (has-repeat s)
  (if (null? s)
      #f
      (if (contains (cdr s) (car s))
          #t
          (has-repeat (cdr s))) ) )
```

Tail Recursion with Scheme Lists

```scheme
(define (map procedure s)
  (if (null? s)
      nil
      (cons (procedure (car s))
            (map procedure (cdr s))) ) )
```

```scheme
(map (lambda (x) (- 5 x)) (list 1 2))
```




```scheme
(define (map procedure s)
  (define (map-reverse s m)
    (if (null? s)
        m
        (map-reverse (cdr s)
                     (cons (procedure (car s))
                           m)) ) )
  (reverse (map-reverse s nil)))
```

```scheme
(define (reverse s)
  (define (reverse-iter s r)
    (if (null? s)
        r
        (reverse-iter (cdr s)
                      (cons (car s) r)) ) )
  (reverse-iter s nil))
```

Tail Recursion Techniques

Base case should return the complete answer (rather than a partial solution).

Define a helper with an extra parameter to keep track of progress so far.

Sketch an iterative solution (e.g. in Python) – names that are iteratively updated need to be tracked as function arguments in recursion.

Verify all recursive calls are tail calls.

(Demo)

Tail Call Optimization

Who'da Thunk?

Thunk: An expression wrapped in an argument-less function.

```
thunk1 = lambda: 2 * (3 + 4)

thunk2 = lambda: add(2, 4)

thunk1()

thunk2()
```

Known as **Unevaluated** objects in the Scheme project.

Trampolining

Trampoline: A loop that iteratively invokes thunk-returning functions.

```
def trampoline(f, *args):
  v = f(*args)
  while callable(v):
    v = v()
  return v
```

The function needs to be thunk-returning.

```
def fact_k_thunked(n, k):
  if n == 0:
    return k
  return lambda: fact_k_thunked(n - 1, n * k)
```

```
trampoline(fact_k_thunked, 3, 1)
```

This way of executing the factorial function uses a constant number of frames.

Trampolining can simulate tail call optimization in unoptimized languages (e.g. Python).

(Demo)

https://pythontutor.com/composingprograms.html#code=def%20trampoline%28f,%20*args%29%3A%0A%20%20%20%20v%20%3D%20f%28*args%29%0A%20%20%20%20while%20callable%28v%29%3A%0A%20%20%20%20%20%20%20%20v%20%3D%20v%28%29%0A%20%20%20%20return%20v%0A%0Adef%20fact_thunked%28n,
%20k%29%3A%0A%20%20%20%20if%20n%20%3D%3D%200%3A%0A%20%20%20%20%20%20%20%20return%20k%0A%20%20%20%20return%20lambda%3A%20fact_thunked%28n%20-%201,%20n%20*%20k%29%0A%20%20%20%20%0Atrampoline%28fact_thunked,%203,%201%29&cumulative=true&mode=edit&origin=composingprograms.js&py=3&rawInputLstJSON=%5B%5D