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INVESTMENT DEMAND: AN EMPIRICAL CONTRIBUTION TO THE AGGREGATION PROBLEM*

BY J. C. G. BOOT AND G. M. DE WIT[1]

1. INTRODUCTION AND SUMMARY

PREVIOUS ANALYSES of problems arising in the study of aggregation have been largely theoretical in character. A notable exception is Balderston's and Whitin's article [1] on aggregation in input-output models, in which are shown the kind of divergencies to which various patterns of aggregation may lead. It is hardly necessary to stress the need for empirical analysis in this field, but there are particular difficulties to be solved, as the present paper will show. It is concerned not with input-output relations, but with relationships of the more "conventional" regression type.

The organization of this article is as follows. In Section 2 we give a brief re-formulation of Professor Theil's aggregation theory, which is applied in Section 4 to the coefficients of a macroeconomic investment equation. The underlying investment theory is due to Dr. Y. Grunfeld and is explained in Section 3. A considerable problem in the empirical analysis of aggregation is the fact that micro and macroparameters are unknown. At the micro-level this is solved by proceeding under the assumption that the parameters coincide with their least squares estimates; at the macro-level, by a statistical extension of aggregation theory such that the estimated macrocoefficients are formulated as the sum of a "true value," an aggregation bias, and a sampling error. For the particular case considered here, which deals with investment of ten large American corporations in the period 1935–1954, the aggregation bias of the multiplicative coefficients appears to be relatively small. The sampling errors can be attributed in a simple manner to individual firms and to separate sample years; the large firms appear to dominate these errors. The numerical characteristics of the auxiliary regression equations are also discussed, since the coefficients of these equations play a crucial role in the determination of the macroparameters (Section 5).

* Manuscript received June 8, 1959.

[1] During the preparation of this report Professor Theil has greatly assisted the authors by his highly stimulating remarks and valuable suggestions. Our thanks are also due to Dr. Grunfeld, cf. footnote 5.

The last two sections are devoted to the nature of the macrodisturbances. These are of great importance for the estimation of macroregressions because of the conditions which are usually imposed on them. Section 6 is concerned with the individual least-squares estimated macrodisturbances; it is shown that they are each the sum of a "true value," an aggregation bias, and a sampling error (just as are the macrocoefficients). In Section 7 the sample variance of the estimated macrodisturbances is analyzed, together with the first-order autocovariance. It is shown that a statistical appraisal of the significance of various components of the variance and the autocovariance is a useful tool in the analysis of macroregressions.

2. AGGREGATION BIAS OF MACROPARAMETERS

Our starting point is Theil's aggregation theory [5], which can be briefly summarized as follows. We assume that there are certain micro-units (e.g. firms or family households), indicated by the index $i = 1, \cdots, I$, which are aggregated to one single macro-unit. For each micro-unit we have a theory which states that some variable y_i depends linearly on, let us say, two other variables, x_{1i} and x_{2i}, except for a disturbance u_i. For example, a household's demand for dwelling-space (y_i) depends on its income (x_{1i}) and the number of family members (x_{2i}). If we add a t in parentheses to indicate the period to which the variables refer, we can write such a microtheory in the form:

$$y_i(t) = \alpha_i + \beta_{1i}x_{1i}(t) + \beta_{2i}x_{2i}(t) + u_i(t) \quad (i = 1, \cdots, I) , \tag{2.1}$$

where the α_i, β_{1i}, β_{2i} are microparameters. In general, these parameters will be different for different j, implying that the reaction of micro-unit i to a unit change in x_{1i} differs from the reaction of micro-unit j to a unit change in x_{1j} and similarly for x_{2i} and x_{2j}. The u's are regarded as random with zero expectation, while the x-variables are supposed to take nonstochastic values.[2]

We assume further that the microvariables y_i, x_{1i}, x_{2i} are aggregated by summation:

$$y(t) = \sum_{i=1}^{I} y_i(t) \; ; \quad x_1(t) = \sum_{i=1}^{I} x_{1i}(t) \; ; \quad x_2(t) = \sum_{i=1}^{I} x_{2i}(t) \; ; \tag{2.2}$$

and that, on the analogy of (2.1), a linear relationship in these aggregates is introduced:

$$y(t) = \alpha + \beta_1 x_1(t) + \beta_2 x_2(t) + u(t) \, . \tag{2.3}$$

[2] No assumption is made here about the correlations of the u's, either about correlations over time, or about correlations for pairs (u_i, u_j). In Section 7, however, it will be assumed that there are no lagged correlations among the microdisturbances.

This procedure is fully in accordance with the conventional way of aggregating. If an individual household's demand for dwelling-space is assumed to depend on this household's family income and the number of its family members, then it is usually postulated that total demand depends on total personal income and the size of the population.

It will be clear that certain difficulties are associated with this procedure, because summation of all microequations (2.1) for $i = 1, \cdots, I$ does not express the aggregate y in terms of the aggregate x_1 and x_2, but in terms of all individual microvariables x_{1i}, x_{2i}. This problem is solved as follows. We assume that observations $y(t)$, $x_1(t)$, $x_2(t)$ on the aggregates y, x_1, x_2 are available for a series of consecutive periods $t = 1, \cdots, T$. Then we consider the least-squares regression of y on x_1 and x_2 for this sample of observations, after which the macroparameters α, β_1, β_2 of (2.3) are defined as the expectations of the least-squares regression coefficients; the macrodisturbances $u(t)$ for $t = 1, \cdots, T$ are then implicitly defined.[3] Essentially, this approach implies that we determine the expectations of the macroeconomic least-squares coefficients, given the microtheory as postulated in (2.1).

The next problem is: What is the relationship between the macroparameters of (2.3), thus interpreted, and the microparameters of (2.1)? The answer (which will not be proved here because we need a generalized result in the next sections) appears to be as follows. First, we consider the so-called *auxiliary regression equations*, which are the least-squares regressions of each of the explanatory microvariables on the two explanatory macrovariables during the period $t = 1, \cdots, T$:

$$(2.4) \qquad \begin{cases} x_{1i}(t) = A_{1i} + B_{1.1i}x_1(t) + B_{2.1i}x_2(t) + V_{1i}(t) \\ x_{2i}(t) = A_{2i} + B_{1.2i}x_1(t) + B_{2.2i}x_2(t) + V_{2i}(t) \, , \end{cases}$$

the A's and B's being regression coefficients and the V's residuals.[4] No special economic meaning is to be attached to the relations (2.4); they are introduced simply because their coefficients facilitate the way in which the macroparameters are expressed in terms of microparameters. These expressions are as follows:

[3] A similar procedure is available when we use not least-squares but any other linear unbiased estimation method (see Theil [**5**, pp. 116 seq.]), but this is not pursued here. It will also be clear that the method expounded here can easily be generalized for larger numbers of explanatory macrovariables. We confine ourselves here to the case of two macro-x's in view of the empirical case to be discussed below.

[4] Note that the A's and B's as well as the V's are nonstochastic because of our assumption that all x-variables take nonstochastic values.

$$(2.5)\qquad \begin{cases} \alpha = \sum_i \alpha_i + \sum_i A_{1i}\beta_{1i} + \sum_i A_{2i}\beta_{2i} \\ \beta_1 = \sum_i B_{1,1i}\beta_{1i} + \sum_i B_{1,2i}\beta_{2i} \\ \beta_2 = \sum_i B_{2,1i}\beta_{1i} + \sum_i B_{2,2i}\beta_{2i} \, . \end{cases}$$

It follows from (2.5) that it is *not* true that the constant term of the macrorelation (α) depends only on the constant terms of the microrelations, nor that β_1 depends only on the β_{1i} and that β_2 depends only on the β_{2i}. In other words, for each of the three macroparameters we find that they depend on *corresponding* as well as on *noncorresponding* parameters. This is a rather negative result, but it is fortunately possible to be somewhat more specific with respect to the weights of the microparameters in (2.5) (i.e., the coefficients of the auxiliary regressions). It can be proved that

$$(2.6)\qquad \begin{cases} \sum_i A_{1i} = \sum_i A_{2i} = 0 \\ \sum_i B_{1,1i} = \sum_i B_{2,2i} = 1 \\ \sum_i B_{1,2i} = \sum_i B_{2,1i} = 0 \, . \end{cases}$$

Comparing (2.5) and (2.6), we find that the weights of the noncorresponding microparameters always add up to zero, while the corresponding microparameters for β_1 and β_2 have weights that add up to one. In view of this result, it is worth-while to re-write the last two equations of (2.5) as follows:

$$(2.7)\qquad \begin{cases} \beta_1 = \dfrac{1}{I}\sum_i \beta_{1i} + \sum_i \Big(B_{1,1i} - \dfrac{1}{I}\Big)\beta_{1i} + \sum_i B_{1,2i}\beta_{2i} \\ \beta_2 = \dfrac{1}{I}\sum_i \beta_{2i} + \sum_i B_{2,1i}\beta_{1i} + \sum_i \Big(B_{2,2i} - \dfrac{1}{I}\Big)\beta_{2i} \, . \end{cases}$$

Taking the first equation of (2.5), together with (2.7), we can conclude that the constant term α of the macrorelation is equal to the sum of the constant terms of all microrelations, and that the multiplicative macroparameters β_1 and β_2 are equal to the (unweighted) means of the corresponding microparameters, apart from certain deviations which are called the *aggregation biases* of these macroparameters. In the case of α this bias is

$$\sum_i A_{1i}\beta_{1i} + \sum_i A_{2i}\beta_{2i} \, ,$$

the first term representing the aggregation bias due to the β_{1i}, the second that due to the β_{2i}. Hence, for α the aggregation bias is wholly due to noncorresponding microparameters. For β_1 the bias is

$$\sum_i \left(B_{1,1i} - \frac{1}{I}\right)\beta_{1i} + \sum_i B_{1,2i}\beta_{2i} ,$$

of which the first term represents the aggregation bias due to the corresponding microparameters β_{1i}. For β_2 the results are analogous.

3. GRUNFELD'S INVESTMENT THEORY[5]

The procedure described in the preceding section will be applied to $I = 10$ large American corporations, for which gross investment (including maintenance and repairs) is taken as dependent microvariable. A list of these firms, arranged in order of decreasing average investment during the period 1935–1954, is presented in Table 1; this table also contains the amounts of average investment and the abbreviations of the names of these firms that will be used in later tables. The aggregate amount of investment of this group of firms was about 7 per cent of total private investment in the United States in 1952.

Grunfeld rejects (observed) profits as an explanatory variable for investment; instead, he prefers expected profits, for which he takes as a measurable substitute the "market value of the firm," viz., the total value of outstanding stock at end-of-year stock market quotations[6]. He then introduces the "desired capital stock" at time t, $C^*(t)$, which is supposed to be a linear (increasing) function of the market value at t, $F(t)$:

$$C^*(t) = c_1 + c_2F(t) . \tag{3.1}$$

Let $C(t)$ be the existing stock, then $c_1 + c_2F(t) - C(t)$ is desired net

[5] This is the correct place to describe our indebtedness to Dr. Y. Grunfeld and also to Professor Z. Griliches of the University of Chicago. Although it will be clear that we consider Grunfeld's theory and data [**3**] primarily as a convenient starting point of our aggregation analysis, it will be equally clear that we cannot claim credit for any conceivable merit which his theory may have, and that we are open to discredit for having followed him if his theory proves to have defects. We should also mention that Grunfeld and Griliches wrote a joint paper [**4**] on the aggregation side of Grunfeld's investment analysis as well as of Griliches' analysis of the demand for fertilizer, which was published in another paper [**2**]. The main difference between the present aggregation analysis and that of Grunfeld and Griliches is that the latter is confined to multiple correlation coefficients of micro and macroregressions. The present paper deals with micro and macroparameters as well as residual variances (and hence implicitly with multiple correlations). Another difference is that this paper includes some large corporations which were considered by Grunfeld only after the bulk of his computations was completed, and which were therefore excluded from his aggregation analysis.

[6] For the firms 2, 4, 6 and 8 of Table 1 the value was measured by Grunfeld at average December 31 and January 31 stock market quotations, which gave somewhat better results.

TABLE 1

LIST OF AMERICAN CORPORATIONS ANALYZED

No.	Firm	Abbreviation	Average amount of investment*
1	General Motors Corporation	G.M.	608.0
2	United States Steel Corporation	U.S. Steel	410.5
3	General Electric	G. Elec.	102.3
4	Chrysler Corporation	Chrys.	86.1
5	Atlantic Refining	Atl. Ref.	61.8
6	International Business Machines	I.B.M.	55.4
7	Union Oil	U. Oil	47.6
8	Westinghouse	West.	42.9
9	Goodyear Tire and Rubber	G. Year	41.9
10	Diamond Match	D. Match	3.1

* Measured in millions of dollars of 1947 purchasing power per year.

investment. Assuming that a constant fraction q_1 of desired net investment is carried out in the year between t and $t+1$, we get for actual net investment of that year

$$q_1\{C^*(t) - C(t)\} = q_1c_1 + q_1c_2F(t) - q_1C(t) \,. \tag{3.2}$$

Remembering that it is gross investment including maintenance and repairs that we want to describe, and assuming that replacement investment plus maintenance and repairs equals a constant fraction q_2 of the existing capital stock $C(t)$, we obtain

$$\begin{aligned} I(t+1) &= q_1\{C^*(t) - C(t)\} + q_2C(t) \\ &= q_1c_1 + q_1c_2F(t) + (q_2 - q_1)C(t) \,, \end{aligned} \tag{3.3}$$

where $I(t+1)$ is gross investment including maintenance and repairs in the year following time t. Empirical testing of this relation shows that it out-performs current profits as well as lagged profits; moreover, it offers predictions superior to those of conventional naive models in nearly all cases.[7] As for the numerical values of the coefficients, it seems reasonable to assume that the q's and c_2 are between 0 and 1; in addition, Grunfeld offers the suggestion that q_2 will usually be below 0.3. It will be noted that the coefficient of capital stock in (3.3) may be positive. This may seem strange for an investment equation, but it must be remembered that the dependent variable is not net investment but gross investment including maintenance and

[7] Either stating: "same investment as last year" [$I(t+1) = I(t)$], or "same change in investment as last year" [$\Delta I(t+1) = \Delta I(t)$, where Δ is the operator of first backward differences].

TABLE 2

MICROREGRESSIONS DESCRIBING INVESTMENT OF 10 AMERICAN CORPORATIONS*

	G.M.	U.S. Steel	G. Elec.	Chrys.	Atl. Ref.
$\bar{R}^2$	.907	.377	.653	.898	.624
Coeff. of F_{-1}	.119(.026)	.175(.074)	.027(.016)	.078(.020)	.162(.057)
Coeff. of C_{-1}	.371(.037)	.390(.045)	.152(.026)	.316(.029)	.003(.022)
Constant term	−149.78	−49.20	−9.96	−6.19	22.71
Est. var. disturbances	7537.15	8320.70	695.61	157.76	73.52
von Neumann ratio	1.09	1.09	1.08	2.02	2.39
	I.B.M.	**U. Oil**	**West.**	**G. Year**	**D. Match**
$\bar{R}^2$	.944	.722	.699	.606	.580
Coeff. of F_{-1}	.131(.031)	.088(.066)	.053(.016)	.075(.034)	.005(.027)
Coeff. of C_{-1}	.085(.100)	.123(.017)	.093(.056)	.082(.028)	.437(.080)
Constant term	−8.69	−4.50	−.48	−7.72	.16
Est. var. disturbances	58.45	79.34	93.55	74.07	1.05
von Neumann ratio	1.77	1.91	1.58	1.64	1.22

* Figures in parentheses are least-squares standard errors.

repairs.

The least-squares method applied to each of the 10 corporations gives rise to a number of estimates which are summarized in Table 2.[8] The squared coefficients of multiple correlation $\bar{R}^2$ (adjusted for loss of degrees of freedom) are in general satisfactorily high, though it should be remembered that a simple rising trend alone accounts for a rather large part of the variance of the dependent variable in most of the cases. The estimated coefficients are all positive, which is what one would reasonably hope for; some of the C-coefficients are above 0.3, which is rather large in terms of the discussion in the preceding paragraph. The standard errors of the coefficients are of the usual order of magnitude, but we should add that the large majority of the von Neumann ratios below the level 2 (indicating a positive first autocorrelation of the disturbances) suggests that these standard errors are on the optimistic side.[9]

The constant terms have only limited meaning, because capital stock has been measured as a deviation from the stock in 1933. There is also the obvious difficulty of the war years, which were characterized by much government intervention. We cannot exclude the

[8] The figures of the tables are generally specified in three or four decimal places The actual computations have all been carried out in seven decimal places or more.

[9] The ratios are of the mean square successive difference to the variance of the least squares estimated microdisturbances.

possibility that this affects our results. It will, however, be clear that our interest is largely confined to the methodology of aggregation rather than the mechanism of investment decisions, so that we feel justified in neglecting this feature.

Comparing these results with the theoretical set-up of Section 2, we note that we now have a microtheory to which the aggregation procedure can be applied—viz., by interpreting y_i, x_{1i}, x_{2i} as I, F_{-1}, C_{-1} respectively for each of the corporations—except that we do not have a fully specified numerical microtheory. We do not have the microparameters $\alpha_i, \beta_{1i}, \beta_{2i}$; we have only estimates of these parameters. The only way to make any progress at all is to make certain assumptions about the values of the microparameters. We shall make the simplest assumption possible, viz. that the microparameters coincide with their least-squares estimates. Needless to say this is a restrictive assumption and it may not be fulfilled. In the mimeographed version of this article we introduced an alternative assumption, which, however, showed generally small differences.

4. AGGREGATION BIAS AND IMPLIED SAMPLING ERRORS OF THE LEAST-SQUARES MACROCOEFFICIENTS

At the macro-level we have precisely the same parameter-estimate problem as at the micro-level; since we do not know the macroparameters, we can, at most, estimate them. A straightforward least-squares calculation gives for the macrorelation

$$I = -332.3 + \underset{(0.025)}{0.099F_{-1}} + \underset{(0.020)}{0.260C_{-1}}\,, \tag{4.1}$$

the von Neumann ratio of the estimated disturbances being 1.40.

This problem can, however, be solved in an elegant manner, without resort to additional assumptions. We shall do so in a general way, which can also be used for cases other than our investment problem. Let us write y for the column vector of values $y(t)$ taken by the dependent macrovariable (in the present case, the values of aggregate investment in each of the twenty years), and X for the matrix of values taken by the explanatory macrovariables. Here, X is of the order 20×3, the first column consisting of twenty elements which are all equal to 1 (corresponding with the constant term of the equation), while the second and third contain the values taken by the aggregate value of all firms and aggregate capital stock respectively. The vector of least-squares coefficients of the regression of y on X is then

$$\begin{bmatrix} a \\ b_1 \\ b_2 \end{bmatrix} = (X'X)^{-1}X'y \;, \tag{4.2}$$

where a, b_1, b_2 are the least-squares estimates of α, β_1, β_2 respectively. It will prove useful to introduce the matrix

$$\Xi = (X'X)^{-1}X' \;, \tag{4.3}$$

which is of order 3×20 in the present case. Let us write ξ_{0t} for a typical element of the first row of Ξ, and similarly ξ_{1t} and ξ_{2t} for the elements in the second and third row respectively. Then, comparing (4.2) and (4.3), we observe that

$$a = \sum_t \xi_{0t} y(t) \;; \quad b_1 = \sum_t \xi_{1t} y(t) \;; \quad b_2 = \sum_t \xi_{2t} y(t) \;. \tag{4.4}$$

This simple result has an important generalization, which we shall use frequently in what follows. Consider an arbitrary series of real numbers $z(1), \cdots, z(T)$ (T being 20 in our case); then

$$\sum_t \xi_{0t} z(t) \;, \quad \sum_t \xi_{1t} z(t) \;, \quad \sum_t \xi_{2t} z(t) \tag{4.5}$$

are the constant term and the two multiplicative coefficients of the least-squares regression of the z-variable on X. This follows immediately from the fact that the three expressions (4.5) are the components of $\Xi z = (X'X)^{-1}X'z$, z being the vector of values $z(t)$ taken by the z-variable.

Consider first the constant term, and combine (4.4), (2.2) and (2.1):

$$\begin{aligned} a &= \sum_t \xi_{0t} y(t) = \sum_t \xi_{0t} \sum_i y_i(t) \\ &= \sum_t \xi_{0t} \sum_i \{\alpha_i + \beta_{1i} x_{1i}(t) + \beta_{2i} x_{2i}(t) + u_i(t)\} \\ &= \sum_i \alpha_i \sum_t \xi_{0t} + \sum_i \beta_{1i} \sum_t \xi_{0t} x_{1i}(t) + \sum_i \beta_{2i} \sum_t \xi_{0t} x_{2i}(t) \\ &\qquad + \sum_t \xi_{0t} \sum_i u_i(t) \;. \end{aligned} \tag{4.6}$$

This expression is simplified as follows. First, we have $\sum \xi_{0t} = 1$;[10] second, $\sum \xi_{0t} x_{1i}(t)$ and $\sum \xi_{0t} x_{2i}(t)$ are the constant terms in the least-squares regressions of x_{1i} and x_{2i}, respectively, on the explanatory macrovariables x_1 and x_2. This follows from the remark made at the end of the preceding paragraph, and it implies that $\sum \xi_{0t} x_{1i}(t) = A_{1i}$,

[10] The proof is as follows. We can consider $\sum \xi_{0t}$ as the first component of the vector $\Xi\iota$, where ι is a column vector of unit elements of appropriate order. But ι is also the first column of X. Hence $\sum \xi_{0t}$ is the leading element of $\Xi X = (X'X)^{-1}X'X = I$, that is, it is equal to 1.

$\sum \xi_{0t} x_{2t}(t) = A_{2t}$, where A_{1t} and A_{2t} are the constant terms of the auxiliary regression (2.4). Hence we can write for (4.6)

$$\begin{aligned} a &= \sum_{\iota} \alpha_{\iota} + \sum_{\iota} A_{1\iota}\beta_{1\iota} + \sum_{\iota} A_{2\iota}\beta_{2\iota} + \sum_{t} \xi_{0t} \sum_{\iota} u_{\iota}(t) \\ &= \alpha + \sum_{t} \xi_{0t} \sum_{\iota} u_{\iota}(t) \, , \end{aligned} \tag{4.7}$$

α being given by (2.5). It is easily seen that α is the expectation of a, provided that the x-values (and hence the ξ's) are nonstochastic and that the microdisturbances have all zero expectation. We shall call $\sum \xi_{0t} \sum u_{\iota}(t)$ the *implied sampling error* of the constant-term estimate; this double sum can be interpreted as the constant term of the least-squares regression of the sum of the microdisturbances ($\sum_{\iota} u_{\iota}$) on the explanatory macrovariables x_1 and x_2.

The derivation of the multiplicative coefficients b_1 and b_2 is entirely similar. Let us take b_1 as defined in (4.4), and apply (2.2) and (2.1):

$$\begin{aligned} b_1 &= \sum_{t} \xi_{1t} y(t) = \sum_{t} \xi_{1t} \sum_{\iota} y_{\iota}(t) \\ &= \sum_{\iota} \alpha_{\iota} \sum_{t} \xi_{1t} + \sum_{\iota} \beta_{1\iota} \sum_{t} \xi_{1t} x_{1\iota}(t) + \sum_{\iota} \beta_{2\iota} \sum_{t} \xi_{1t} x_{2\iota}(t) \\ &\qquad + \sum_{t} \xi_{1t} \sum_{\iota} u_{\iota}(t) \, . \end{aligned}$$

It is easily seen that $\sum \xi_{1t} = 0$,[11] that $\sum \xi_{1t} x_{1\iota}(t) = B_{1.1\iota}$ and that $\sum \xi_{1t} x_{2\iota}(t) = B_{1.2\iota}$ as defined in (2.4), and that $\sum \xi_{1t} \sum u_{\iota}(t)$ is the coefficient of x_1 in the least-squares regression of the sum of the microdisturbances on the explanatory macrovariables x_1 and x_2. Hence

$$\begin{aligned} b_1 &= \sum_{\iota} B_{1.1\iota}\beta_{1\iota} + \sum_{\iota} B_{1.2\iota}\beta_{2\iota} + \sum_{t} \xi_{1t} \sum_{\iota} u_{\iota}(t) \\ &= \beta_1 + \sum_{t} \xi_{1t} \sum_{\iota} u_{\iota}(t) \, , \end{aligned} \tag{4.8}$$

β_1 (the expectation of b_1 under the conditions formulated in the preceding paragraph) being given by (2.5).[12] Similar results are obtained for b_2 by interchanging the indices 1 and 2.

Our analysis shows that the least-squares estimates a, b_1, b_2 are equal to their macroparameters as specified in (2.5), plus certain implied sampling errors. The macroparameters can be split up in terms of a "true value" and an aggregation bias. For the "true value" of α we take the sum of the contsant terms of the microequations ($\sum \alpha_{\iota}$),

[11] Proof: $\sum \xi_{1t}$ can be regarded as a nondiagonal element of the unit matrix of the preceding footnote.

[12] The sum-of-weight rules (2.6) are also easily verified. For example, $\sum A_{1\iota} = \sum \xi_{0t} x_1(t)$ is the element in the first row and the second column of the unit matrix of the preceding footnotes, and hence equal to zero.

for that of β_1 and β_2 the average of the corresponding microparameters $[(1/I)\sum\beta_{1i}$ and $(1/I)\sum\beta_{2i}$ respectively]. The aggregation bias has already been discussed in Section 2.

Table 3 specifies these results for our investment problem numerically. It is seen that, as far as the multiplicative coefficients are concerned, the aggregation bias due to noncorresponding microparameters is of moderate size: for β_1 it is of the order of 10 per cent of the "true value," for β_2 only 1 per cent. This is an encouraging result, but it is of course rash to accept this as a general result. The bias due to corresponding microparameters is generally larger, in particular for β_2, but in our opinion this is less serious; we shall come back to this point at the end of Section 5. For the constant term the aggregation bias is much larger, and this is largely due to the fact that all of our variables take positive values, implying that the constant term measures the value of the dependent variable outside the range of observations. If the constant term were measured closer to the center of gravity of the observations, both the aggregation bias and the implied sampling error would be reduced.[13]

It will be observed that the implied sampling errors can be ascribed

TABLE 3

AGGREGATION BIAS AND IMPLIED SAMPLING ERRORS OF THE LEAST-SQUARES MACROCOEFFICIENTS

Constant Term		
Sum of corresponding microparameters		−213.6
Aggregation bias		−126.4
due to β_{1i}	59.3	
due to β_{2i}	−185.6	
Implied sampling error		7.7
Total: estimated coefficient		−332.3
Coefficient of F_{-1}		
Average of corresponding microparameters		.0913
Aggregation bias		.0147
due to corresponding microparameters	0075	
due to noncorresponding microparameters	0072	
Implied sampling error		−.0068
Total: estimated coefficient		.0993
Coefficient of C_{-1}		
Average of corresponding microparameters		.2053
Aggregation bias		.0311
due to corresponding microparameters	.0289	
due to noncorresponding microparameters	.0022	
Implied sampling error		.0238
Total: estimated coefficient		.2602

13 Bias and sampling error vanish entirely when the constant term is measured at the center of gravity.

in a unique way to separate firms and years. For example, if we take the sampling error of the constant term, we find that it consists of terms $\xi_{0t}u_i(t)$, each of which can be allocated to some firm i in some year t. This allocation is shown in tabular form in the Appendix to this paper (Tables 11–13); here, we confine ourselves to the marginal totals, i.e., to

$$\sum_t \xi_{0t}u_i(t) ,$$

which is the total contribution of firm i to the sampling error of the constant term, and to

$$\xi_{0t}\sum_i u_i(t) ,$$

which is the total contribution of year t. A summary of these contributions—for the constant term as well as for the multiplicative coefficients—is given in Table 4. The results show that the implied sampling errors are the result of numerous positive and negative contributions, but also that some of these are much more dominant than others. For example, the contributions of the smallest firms are much smaller than those of the two largest (General Motors and U.S. Steel); an inspection of the tables in the Appendix shows that this also holds for most of the individual years. Further, it is interesting to observe that the years 1944–46, which mark the end of the war and the transitional period thereafter, contribute very little.

5. DISCUSSION OF THE AUXILIARY REGRESSION EQUATIONS

The analysis of the preceding section was to a large extent based on the coefficients of the auxiliary equations (2.4). It is worth-while to analyze these in somewhat more detail, first because very little is known about them at the empirical level, second because an analysis of these equations is not hampered by our lack of complete knowledge of the microtheory. We take all x-values as they are observed, which implies a complete knowledge of the auxiliary regression equations.

The coefficients $A_{1i}, A_{2i}, B_{1.1i}, \cdots$ of the auxiliary regressions are presented in Table 5. As such, these coefficients have relatively little meaning from an economic point of view, since the variables of the auxiliary equations are tied together in a way which is economically irrelevant. We could, however, perhaps say that in the "ideal" case the explanatory microvariables move proportionally to the corresponding aggregate. This case is indeed ideal in the sense that all macroparameters are then independent of noncorresponding microparameters. This follows directly from the fact that addition of the microequations,

TABLE 4

CONTRIBUTIONS OF SEPARATE FIRMS AND SEPARATE YEARS TO THE SAMPLING ERRORS OF THE ESTIMATED MACROPARAMETERS

Contributions of separate firms

Firm	Constant Term	Coeff. of F_{-1}	Coeff. of C_{-1}
G.M.	30.8	−.0039	.0040
U.S. Steel	7.9	−.0062	.0213
G. Elec.	−10.8	.0012	−.0006
Chrys.	10.0	−.0012	.0010
Atl. Ref.	−14.6	.0016	−.0010
I.B.M.	−6.0	.0006	−.0000
U. Oil	−11.0	.0011	−.0005
West.	−1.8	.0002	.0001
G. Year	2.0	.0000	−.0007
D. Match	1.2	−.0002	.0002
Total	7.7	−.0068	.0238

Contributions of separate years

Year	Constant Term	Coeff. of F_{-1}	Coeff. of C_{-1}
1935	5.2	−.0003	−.0009
36	−76.8	.0065	.0055
37	−11.9	.0006	.0034
38	−2.5	.0002	.0008
39	−34.6	.0041	−.0057
40	−53.3	.0060	−.0069
41	54.1	−.0058	.0057
42	38.2	−.0038	.0031
43	−36.2	.0057	−.0054
44	4.1	−.0010	.0012
45	0.5	−.0000	−.0001
46	−4.2	.0002	.0002
47	38.6	−.0031	.0004
48	−1.0	.0014	−.0029
49	11.5	−.0028	.0047
50	2.6	−.0051	.0123
51	−46 3	.0034	.0013
52	61.7	−.0089	.0100
53	−1.1	.0005	−.0013
54	59.1	−.0046	−.0016
Total	7.7	−.0068	.0238

combined with an elimination of the explanatory microvariables by means of the proportionalities just-mentioned, gives a simple equation —without aggregation bias—in macrovariables only. Now this ideal case can be handled conveniently as follows. Consider the auxiliary equations (2.4) and average them over time, after which the results are divided by the time average of x_{1i} and x_{2i} respectively. Taking account of the fact that the residuals have zero mean, we obtain

$$1 = \frac{A_{1i}}{\bar{x}_{1i}} + B_{1,1i}\frac{\bar{x}_1}{\bar{x}_{1i}} + B_{2,1i}\frac{\bar{x}_2}{\bar{x}_{1i}} \tag{5.1}$$

$$1 = \frac{A_{2i}}{\bar{x}_{2i}} + B_{1,2i}\frac{\bar{x}_1}{\bar{x}_{2i}} + B_{2,2i}\frac{\bar{x}_2}{\bar{x}_{2i}}\ , \tag{5.2}$$

where $\bar{x}_{1i}, \bar{x}_{2i}, \bar{x}_1, \bar{x}_2$ are the means of x_{1i}, x_{2i}, x_1, x_2 respectively. It is easy to see that if all microvariables move proportionally to the corresponding aggregate, $B_{1,1i}$ must be equal to $\bar{x}_{1i}/\bar{x}_1$; in other words, the second term on the right of (5.1) must be unity. It is also easily verified that the first and the third term on the right must then vanish; similarly, the third term on the right of (5.2) must be unity, the two others being zero.

The coefficients $A_{1i}/\bar{x}_{1i}, A_{2i}/\bar{x}_{2i}, \cdots$ are also specified in Table 5. It appears that their average values over all 10 firms are close to the value of the ideal case, but that there are some sizable discrepancies for individual firms, leading to standard deviations of the order of 0.3 or 0.4. In none of the six cases do we find a significant difference between the computed average and the ideal value (significance being judged by dividing the difference by $1/\sqrt{10}$ times the standard deviation); but it must be remembered, of course, that such an appraisal of significance is hampered by the fact that all quantities entering the auxiliary equations are taken as nonstochastic.

It is also of some interest to observe that the "ideal case" enables us to formulate the "true value" of a multiplicative macroparameter in a slightly different manner. In Section 2 we took the unweighted mean for this purpose, but we may as well take a weighted mean, the weights being $\bar{x}_{1i}/\bar{x}_1$ for β_{1i} and $\bar{x}_{2i}/\bar{x}_2$ for β_{2i}. This means that the first equation of (2.7) is replaced by

$$\beta_1 = \sum_i \frac{\bar{x}_{1i}}{\bar{x}_1}\beta_{1i} + \sum_i \left(B_{1,1i} - \frac{\bar{x}_{1i}}{\bar{x}_1}\right)\beta_{1i} + \sum_i B_{1,2i}\beta_{2i}\ , \tag{5.3}$$

the second equation of (2.7) being changed similarly. It follows from (5.3) that the aggregation bias due to noncorresponding microparameters remains as it is, but the true value and the aggregation bias due

TABLE 5

COEFFICIENTS OF THE AUXILIARY EQUATIONS

Firm	A_{1i}	$B_{1,1i}$	$B_{2,1i}$	$\frac{A_{1i}}{\bar{x}_{1i}}$	$B_{1,1i}\frac{\bar{x}_1}{\bar{x}_{1i}}$	$B_{2,1i}\frac{\bar{x}_2}{\bar{x}_{1i}}$
G.M.	−486.65	.4615	−.0622	−.112	1.152	−.040
U.S. Steel	756.89	.1309	−.0729	.384	.718	−.102
G. Elec.	−93.36	.2077	−.0770	−.048	1.158	−.109
Chrys.	−38.06	.0720	−.0172	−.055	1.123	−.069
Atl. Ref.	20.58	.0107	.0345	.089	.500	.411
I.B.M.	−45.39	.0122	.1208	−.108	.314	.794
U. Oil	27.14	.0121	−.0029	.181	.873	−.054
West.	−236.19	.0671	.0657	−.352	1.082	.270
G. Year	38.04	.0236	.0146	.114	.765	.121
D. Match	57.00	.0021	−.0033	.804	.327	−.130
Average				.090	.801	.109
Standard deviation				.320	.333	.301

Firm	A_{2i}	$B_{1,2i}$	$B_{2,2i}$	$\frac{A_{2i}}{\bar{x}_{2i}}$	$B_{1,2i}\frac{\bar{x}_1}{\bar{x}_{2i}}$	$B_{2,2i}\frac{\bar{x}_2}{\bar{x}_{2i}}$
G.M.	−510.19	.0127	.3698	−.787	.213	1.574
U.S. Steel	15.15	.0047	.0828	.051	.173	.776
G. Elec.	80.96	−.0108	.1581	.202	−.293	1.090
Chrys.	−155.89	.0108	.0580	−1.286	.965	1.321
Atl. Ref.	231.69	−.0062	.1167	.476	−.138	.662
I.B.M.	−3.91	−.0004	.0409	−.037	−.046	1.084
U. Oil	157.59	−.0048	.0759	.500	−.165	.665
West.	18.44	−.0037	.0389	.215	−.469	1.253
G. Year	167.13	−.0025	.0572	.561	−.091	.530
D. Match	−.96	.0002	.0016	−.162	.440	.723
Average				−.027	.059	.968
Standard deviation				.594	.412	.346

to corresponding microparameters are affected. Applying this to β_1, we find that the true value and the "corresponding" bias, which originally were 0.0913 and 0.0075, become 0.1048 and −0.0060, respectively. We observe that the sign of the aggregation bias due to corresponding microparameters is changed if we interpret this bias in the weighted manner of (5.3). For β_2 the changes are relatively smaller: True value and "corresponding" bias change from 0.2053 and 0.0289 to 0.1953 and 0.0389 respectively.

6. THE LEAST-SQUARES ESTIMATED MACRODISTURBANCES; (i) THE INDIVIDUAL ERROR TERMS

So far we have confined our attention to the macroparameters α, β_1, β_2, and to their least-squares estimates. But it is also interesting to analyze the discrepancies between the observed dependent macrovalues and the corresponding values which are "predicted" by the macroequation. This leads us to an analysis of the estimated macrodisturbances, which is largely similar to the analysis of the estimated macrocoefficients.

Let us write $v(t)$ for the least-squares estimated macrodisturbance in t, and v for the vector of these estimates. Then

$$v = y - X(X'X)^{-1}X'y = My \,, \tag{6.1}$$

where M is a square matrix of order $T(= 20$ in our case) with typical element $m_{tt'}$:

$$M = [m_{tt'}] = I - X(X'X)^{-1}X' \,, \tag{6.2}$$

I being the unit matrix of order T. It is important to note that M is analogous to Ξ in the following sense: if we postmultiply M by any column vector z of T real components, then the resulting column Mz can be interpreted as the residual vector of the least-squares regression of z on X.

Consider then an individual error term $v(t)$, and combine (6.1) with (2.2) and (2.1):

$$\begin{aligned} v(t) &= \sum_{t'} m_{tt'} y(t') = \sum_{t'} m_{tt'} \sum_{i} y_i(t') \\ &= \sum_{i} \alpha_i \sum_{t'} m_{tt'} + \sum_{i} \beta_{1i} \sum_{t'} m_{tt'} x_{1i}(t') + \sum_{i} \beta_{2i} \sum_{t'} m_{tt'} x_{2i}(t') \\ &\qquad + \sum_{t'} m_{tt'} \sum_{i} u_i(t') \,. \end{aligned}$$

It is then easily verified that $\sum_{t'} m_{tt'} = 0$,[14] that $\sum_{t'} m_{tt'} x_{1i}(t')$ and $\sum_{t'} m_{tt'} x_{2i}(t')$ are equal to the residuals $V_{1i}(t)$ and $V_{2i}(t)$, respectively, of the auxiliary regressions (2.4), and that the last term $[\sum m_{tt'} \sum u_i(t')]$ is the t-th residual of the least-squares regression of the sum of the microdisturbances on the explanatory macrovariables x_1 and x_2. Hence we can write

$$\begin{aligned} v(t) &= \sum_{i} \beta_{1i} V_{1i}(t) + \sum_{i} \beta_{2i} V_{2i}(t) + \sum_{t'} m_{tt'} \sum_{i} u_i(t') \\ &= \sum_{i} u_i(t) + \sum_{i} \beta_{1i} V_{1i}(t) + \sum_{i} \beta_{2i} V_{2i}(t) + \sum_{t'} (m_{tt'} - \delta_{tt'}) \sum_{i} u_i(t') \,, \end{aligned} \tag{6.3}$$

[14] Proof: $\sum m_{tt'}$ is the first component of $M\iota$ (cf. footnote 10) and hence the leading element of MX, which is a zero matrix.

where $\delta_{tt'}$ is the Kronecker delta ($=1$ if $t = t'$, $= 0$ if $t \neq t'$). We shall interpret the sum of the corresponding microdisturbances, $\sum u_i(t)$, as the "true" macrodisturbance (on the analogy of $\sum \alpha_i$ which is the "true value" of the macro-intercept), and

$$u(t) = \sum_i u_i(t) + \sum_i \beta_{1i} V_{1i}(t) + \sum_i \beta_{2i} V_{2i}(t) \tag{6.4}$$

as the macrodisturbance itself.[15] The aggregation bias is then the sum of the last two terms on the right of (6.4). Hence

$$v(t) = u(t) + \sum_{t'} (m_{tt'} - \delta_{tt'}) \sum_i u_i(t') \ , \tag{6.5}$$

the last term on the right being the implied sampling error of the estimated macrodisturbance. It will be observed that this sampling error can be interpreted as the inner product of two vectors of T components, the first of which is the t-th row of

$$M - I = -X(X'X)^{-1}X' \ .$$

Disregarding the negative sign on the right, we note that this matrix produces the "systematic part" of regressions on X. For whatever column z of appropriate order, $X(X'X)^{-1}X'z$ is the systematic part of the regression of z on X. So we may conclude from (6.5) that, apart from sign, the implied sampling error of the estimate of the t-th macrodisturbance is equal to the systematic part in the t-th period of the least-squares regression of the sum of the microdisturbances on the explanatory macrovariables x_1 and x_2.

The results are summarized numerically in Table 6. We observe that the aggregation bias is usually smaller in absolute value than the "true value" of the macrodisturbance, but that it is far from being negligible; also, the two components of this bias have opposite signs (and hence compensate each other) in eleven cases out of twenty.

7. THE LEAST-SQUARES ESTIMATED MACRODISTURBANCES:
(ii) VARIANCE AND FIRST-ORDER AUTOCOVARIANCE

Whereas we considered in the preceding section the components of the individual macrodisturbances, we shall now turn to parameters

[15] The result (6.4) has also been given in [5, pp. 110 seq.]; it is in accordance with the definition of the macrodisturbance as the discrepancy between the dependent macrovariable and the linear combination of the explanatory macrovariables whose weights are defined as indicated in the third paragraph of Section 2. It is further interesting to note that the sum of the $V_{1i}(t)$ and $V_{2i}(t)$ over all i is zero. For example, taking ( ) and summing over i, we obtain $\sum_{t'} m_{tt'} x_1(t')$, which is an element of the zero matrix of the preceding footnote.

TABLE 6

AGGREGATION BIAS AND IMPLIED SAMPLING ERRORS OF THE ESTIMATED MACRODISTURBANCES

Year	"True value" of macrodisturbance	Aggregation bias:			Macrodisturbance	Implied sampling error	Estimated macrodisturbance
		Total	due to β_{1i}	due to β_{2i}			
1935	107.0	62.9	5.3	57.6	169.9	25.5	195.4
36	24.7	11.1	−14.7	25.8	35.8	47.9	83.7
37	−130.2	17.7	−.6	18.3	−112.5	59.6	−52.8
38	−127.4	15.1	−46.5	61.5	−112.3	16.6	−95.8
39	−382.6	14.6	−22.9	37.4	−368.0	28.6	−339.4
40	−124.9	19.5	6.7	12.8	−105.4	31.5	−73.9
41	109.5	55.9	47.3	8.6	165.4	24.1	189.5
42	113.3	37.4	27.5	9.9	150.7	3.6	154.3
43	−26.3	−16.2	6.7	−22.9	−42.5	9.0	−33.5
44	4.9	−46.8	5.2	−51.0	−41.9	11.6	−30.2
45	−56.5	−89.3	−13.3	−75.9	−145.8	17.5	−128.3
46	214.1	−89.9	−8.5	−81.2	124.2	18.4	142.8
47	120.9	−20.8	−.9	−19.9	100.1	−18.1	82.0
48	140.3	−37.1	−24.8	−12.4	103.2	−29.9	73.3
49	−144.4	−4.8	8.3	−13.1	−149.2	−37.0	−186.2
50	−122.3	−19.5	−1.0	−18.4	−141.8	−37.7	−179.5
51	30.6	16.0	47.9	−31.8	46.6	−27.3	19.3
52	93.8	8.1	15.5	−7.4	101.9	−38.2	63.7
53	170.5	22.8	−5.1	27.9	193.3	−41.9	151.4
54	−15.1	43.1	−32.1	75.2	28.0	−63.7	−35.7

describing their statistical distribution. Their mean is not very interesting, because all terms entering equation (6.3) have zero average over time; hence the same thing must apply to the error terms themselves. The variance is more interesting, however. We shall analyze it by means of the following splitting-up:

$$v(t) = U_t + V_t + W_t \,, \tag{7.1}$$

where U_t is the "true value" of the macrodisturbance:

$$U_t = \sum_i u_i(t) \,, \tag{7.2}$$

V_t the aggregation bias:

$$V_t = \sum_i \beta_{1i} V_{1i}(t) + \sum_i \beta_{2i} V_{2i}(t) \,, \tag{7.3}$$

and W_t the implied sampling error:

$$W_t = \sum_{t'} (m_{tt'} - \delta_{tt'}) \sum_i u_i(t') \,. \tag{7.4}$$

The variance of the estimated macrodisturbance is then[16]

$$(7.5)\qquad \begin{aligned}\frac{1}{T}\sum_t v(t)^2 = \frac{1}{T}\sum_t U_t^2 + \frac{1}{T}\sum_t V_t^2 + \frac{1}{T}\sum_t W_t^2 \\ + \frac{2}{T}\sum_t U_t V_t + \frac{2}{T}\sum_t U_t W_t + \frac{2}{T}\sum_t V_t W_t ,\end{aligned}$$

i.e., this variance can be regarded as the sum of three variances and six covariances, the latter being pairwise equal. These components are shown numerically in Table 7; our comments are as follows:

(i) The first term, $(1/T)\sum U_t^2$, which is the variance of the true values of the macrodisturbances, is by far the largest. A consequence of this is that the true values of the macrodisturbances (U_t), the actual macrodisturbances including aggregation bias $(U_t + V_t)$, and the least-squares estimated macrodisturbances $(U_t + V_t + W_t)$ are highly correlated, the correlations ranging from 0.92 to 0.97. It is also interesting to note that the variance of the true values can be split up into two terms, one of which is the sum of the variances of the microdisturbances, the other being the sum of their covariances:

$$(7.6)\qquad \frac{1}{T}\sum_t U_t^2 = \sum_i \frac{1}{T}\sum_t u_i(t)^2 + \sum_{i \neq j}\sum \frac{1}{T}\sum_t u_i(t)u_j(t) .$$

This splitting-up is presented numerically in Table 7, which shows that the sum of the microcovariances represents a small but positive contribution to the variance of the true values. This is the result of a tendency towards positive correlation of the microdisturbances corresponding to separate firms, a feature which suggests that the individual firms' investments were subject to a common factor that is not introduced explicitly as an explanatory variable in the microregressions. A survey of these correlations is given in Table 8; about two thirds of them are positive.

(ii) The second term, $(1/T)\sum V_t^2$, which is the variance of the aggregation bias, is almost 10 per cent of the first term. As the aggregation bias is the sum of two terms, one of which deals with β_{1i} and $V_{1i}(t)$ and the other with β_{2i} and $V_{2i}(t)$, we can make a further splitting-up according to

$$(7.7)\qquad \frac{1}{T}\sum_t V_t^2 = \frac{1}{T}\sum_t \{\sum_i \beta_{1i} V_{1i}(t)\}^2 + \frac{1}{T}\sum_t \{\sum_i \beta_{2i} V_{2i}(t)\}^2$$

[16] In order to avoid cumbersome expressions, we shall frequently write "variance" in what follows, instead of the term "sample variance" which is, strictly speaking, more correct.

TABLE 7

COMPOSITION OF THE VARIANCE OF THE ESTIMATED MACRODISTURBANCES

$(1/T)\sum U_t^2$		19547
sum of microvariances	16237	
sum of microcovariances	3310	
$(1/T)\sum V_t^2$		1655
variance of $\sum_i \beta_{1i} V_{1i}(t)$	528	
variance of $\sum_i \beta_{2i} V_{2i}(t)$	1699	
twice the covariance	−571	
$(1/T)\sum W_t^2$		1103
$(2/T)\sum U_t V_t$		−900
$(2/T)\sum U_t W_t$		−2207
$(2/T)\sum V_t W_t$		0
Total		19198

$$+\frac{2}{T}\sum_t\{\sum_i \beta_{1i} V_{1i}(t)\}\{\sum_i \beta_{2i} V_{2i}(t)\} .$$

Table 7 shows that the second term is about three or four times as large as the first, and that the third term is negative. The latter feature implies that the two components of the aggregation bias tend to compensate each other, the former that the component referring to the capital stock of the firm dominates the other one.

(iii) The third term, $(1/T)\sum W_t^2$, which is the variance of the implied sampling errors, is about 20–30 per cent below the value of the second. It is of some interest to consider this term in more detail:

$$\begin{aligned}\sum W_t^2 &= \sum_t \{\sum_{t'}(m_{tt'} - \delta_{tt'})\sum_i u_i(t')\}^2 \\ &= \sum_t \{\sum_{t'}(m_{tt'} - \delta_{tt'})\sum_{t''}(m_{tt''} - \delta_{tt''})\sum_i\sum_j u_i(t')u_j(t'')\} .\end{aligned} \tag{7.8}$$

Assuming that there are no lagged correlations among the microdisturbances, and that the current covariances are independent of time,

$$E[u_i(t')u_j(t'')] = \begin{cases} \sigma_{ij} & \text{if } t' = t'' \\ 0 & \text{if } t' \neq t'' , \end{cases} \tag{7.9}$$

we obtain for the expectation of $\sum W_t^2$

$$\begin{aligned}E(\sum W_t^2) &= (\sum_i\sum_j \sigma_{ij})\sum_t\sum_{t'}(m_{tt'} - \delta_{tt'})^2 \\ &= (\sum\sum \sigma_{ij}) \operatorname{tr}(M - I)(M - I)' \\ &= (\sum\sum \sigma_{ij}) \operatorname{tr} X(X'X)^{-1}X'\end{aligned} \tag{7.10}$$

TABLE 8

CORRELATION MATRIX OF THE MICRODISTURBANCES

	G.M.	U.S. Steel	G. Elec.	Chrys.	Atl. Ref.	I.B.M.	U. Oil	West	G. Year	D. Match
G.M.	1	−.26	.28	−.27	−.31	.12	.51	.15	.21	−.26
U.S. Steel		1	.43	.34	.22	.36	−.27	.61	.28	.70
G. Elec.			1	−.07	−.02	.46	−.02	.73	.41	.56
Chrys.				1	.06	.20	−.12	.12	.07	.11
Atl. Ref.					1	.21	.15	.00	−.14	.12
I.B.M.						1	.13	.52	−.17	.39
U. Oil							1	.14	.19	−.22
West.								1	.52	.57
G. Year									1	.26
D. Match										1

$$E(\sum W_t^2) = (\sum\sum \sigma_{ij}) \operatorname{tr} (X'X)^{-1}X'X = \Delta \sum\sum \sigma_{ij} ,$$

where use has been made of the fact that the sum of squares of all elements of some matrix A is the trace of AA';[17] further, Δ is the number of columns of X (3 in our case). So we can conclude that, under assumption (7.9), the expectation of the third term is $(3/20)\sum\sum \sigma_{ij}$. The double sum $\sum\sum \sigma_{ij}$ is unknown, of course, but it can be estimated without bias by the estimated variance $(1/T)\sum U_t^2$ of the true values of the macrodisturbances in view of the definition (7.2). Comparing this with the figures of Table 7, we find that the estimated expectation of the third term is about 3000 and that the observed value is less than one half of this.

(iv) The fourth term, $(2/T)\sum U_t V_t$, which is twice the covariance of the true values and the aggregation bias, is negative under both assumptions. As is easily verified, the expected value of this term is zero when V_t is nonstochastic and U_t has zero expectation. It is of some interest to pursue this a little further, because if the observed value of this term were significantly different from zero, this would point to a dependence between explanatory microvariables and microdisturbances, the reason being that the residuals V_t of the auxiliary regressions are nothing else than combinations of values taken by the explanatory microvariables, while U_t is a combination of microdisturbances. There is, of course, nothing which prevents a dependence of this kind, even if the disturbances of microequations are all independent of the explanatory variables of the same equation, because V_t

[17] The trace of a matrix is the sum of the diagonal elements. In deriving (7.10), use has also been made of the fact that tr AB = tr BA (provided that both product matrices

and U_t are based on all explanatory microvariables and all microdisturbances at time t. Now the significance of the present term can be easily judged by computing the variance of $\sum U_t V_t$:

$$\text{(7.11)} \quad \begin{aligned} \operatorname{var}(\sum U_t V_t) &= \sum_t \sum_{t'} V_t V_{t'} \sum_i \sum_j E[u_i(t) u_j(t')] \\ &= (\sum \sum \sigma_{ij}) \sum_t V_t^2 , \end{aligned}$$

where use is made of assumption (7.9) (and of course also of the independence of the U's and the V's, which is our null-hypothesis here). It follows that $(2/T) \sum U_t V_t$ is distributed with zero mean and standard deviation

$$\frac{2}{T} \{(\sum \sum \sigma_{ij}) \sum_t V_t^2\}^{\frac{1}{2}} ,$$

which is estimated as about 2600. The observed values are therefore not significantly different from zero.

(v) The last two terms are very simple. The sixth, $(2/T) \sum V_t W_t$, is necessarily zero, which means that aggregation bias and implied sampling errors of least-squares estimated macrodisturbances are always uncorrelated in the sample. This is proved as follows. We write

$$\sum V_t W_t = \sum_i \sum_{t'} u_i(t') \sum_t (m_{tt'} - \delta_{tt'}) V_t ,$$

the last part of which is

$$\sum_t (m_{tt'} - \delta_{tt'}) \sum_j V_{1j}(t) \beta_{1j} + \sum_t (m_{tt'} - \delta_{tt'}) \sum_j V_{2j}(t) \beta_{2j} .$$

Both terms vanish, because

$$\sum_t (m_{tt'} - \delta_{tt'}) V_{1j}(t) = \sum_t (m_{tt'} - \delta_{tt'}) V_{2j}(t) = 0 .$$

This follows from the fact that these expressions are equal to minus the systematic parts at time t of the regressions of V_{1j} and V_{2j} on the explanatory macrovariables, which must be zero because V_{1j} and V_{2j} are least-squares residuals corresponding to these explanatory variables (cf. the end of the third papagraph).

The fifth term, $(2/T) \sum U_t W_t$, is necessarily twice the third except for sign. This means that the covariance of the true values and the implied sampling errors is always minus the variance of these errors, which is proved as follows. For T times the covariance we can write

$$\text{(7.12)} \quad \begin{aligned} \sum U_t W_t &= \sum_t \sum_i u_i(t) \sum_{t'} (m_{tt'} - \delta_{tt'}) \sum_j u_j(t') \\ &= \sum_t \sum_{t'} (m_{tt'} - \delta_{tt'}) U_t U_{t'} , \end{aligned}$$

and for T times the variance of the implied sampling errors:

$$\sum_t W_t^2 = \sum_t \sum_{t'} \sum_{t''} (m_{tt'} - \delta_{tt'})(m_{tt''} - \delta_{tt''}) \sum_i \sum_j u_i(t')u_j(t'') \tag{7.13}$$
$$= \sum_{t'} \sum_{t''} U_{t'} U_{t''} \sum_t (m_{tt'} - \delta_{tt'})(m_{tt''} - \delta_{tt''}) .$$

But for the last sum we have

$$\sum_t (m_{tt'} - \delta_{tt'})(m_{tt''} - \delta_{tt''}) = -(m_{t't''} - \delta_{t't''}) , \tag{7.14}$$

because the left-hand side is equal to the (t', t'') element of the matrix $(M - I)^2 = -(M - I)$. Combining (7.12), (7.13) and (7.14), we find that our assertion has been proved.

Finally, we turn to the first-order autocovariance, the importance of which will be clear in view of the fact that this covariance should vanish according to the classical conditions of regression analysis. We have

$$\frac{1}{T-1} \sum_t v(t)v(t-1) = \frac{1}{T-1} [\sum U_t U_{t-1} + \sum U_t V_{t-1} \tag{7.15}$$
$$+ \sum U_t W_{t-1} + \sum V_t U_{t-1} + \sum V_t V_{t-1} + \sum V_t W_{t-1}$$
$$+ \sum W_t U_{t-1} + \sum W_t V_{t-1} + \sum W_t W_{t-1}] .$$

The autocovariances, as well as the separate terms on the right of (7.15), are shown in Table 9. For the least-squares estimated macro-disturbances the autocovariance is about 0.3 times the variance, implying a first-order autocorrelation equal to that fraction. It is further seen that the first component, $\{1/(T-1)\} \sum U_t U_{t-1}$, is by far the largest under both assumptions; this is in spite of the fact that it has zero expectation under the conditions just-mentioned. To test these conditions, we consider the variance of $\sum U_t U_{t-1}$:

$$E(\sum_t U_t U_{t-1})^2 = \sum_t \sum_{t'} \sum_i \sum_j \sum_k \sum_l E[u_i(t)u_j(t-1)u_k(t')u_l(t'-1)] \tag{7.16}$$
$$= \sum_t \sum_i \sum_j \sum_k \sum_l E[u_i(t)u_k(t)]E[u_j(t-1)u_l(t-1)]$$
$$= (T-1)(\sum \sum \sigma_{ij})^2 ,$$

which implies that—under the null-hypothesis—, $\{1/(T-1)\} \sum U_t U_{t-1}$ is distributed with zero mean and standard deviation

$$\frac{1}{\sqrt{T-1}} \sum \sum \sigma_{ij} ,$$

this standard deviation being estimated as slightly more than 4500. Comparing this with the observed values of Table 9, we can conclude

TABLE 9

COMPOSITION OF THE FIRST-ORDER AUTOCOVARIANCE OF THE ESTIMATED MACRODISTURBANCES

	U_t	V_t	W_t
U_{t-1}	7820	−396	−1787
V_{t-1}	−1299	1109	271
W_{t-1}	−789	3	895
Total		5827	

that they are not significantly positive.

The other eight components of the autocovariance are much smaller, and four of them have zero expectation. The other four are $\sum V_t V_{t-1}$ (which is nonstochastic), $\sum U_t W_{t-1}$, $\sum W_t U_{t-1}$ and $\sum W_t W_{t-1}$. The expectations of the last three terms are derived as follows:

$$
\begin{aligned}
E(\sum_t U_t W_{t-1}) &= E\sum_t \{\sum_i u_i(t) \sum_{t'} (m_{t-1,t'} - \delta_{t-1,t'}) \sum_j u_j(t')\} \\
&= (\sum \sum \sigma_{ij}) \sum_t (m_{t-1,t} - \delta_{t-1,t}) = (\sum \sum \sigma_{ij}) \sum_t m_{t-1,t} \, ;
\end{aligned} \tag{7.17}
$$

$$
\begin{aligned}
E(\sum_t W_t U_{t-1}) &= E\sum_t \{\sum_i u_i(t-1) \sum_{t'} (m_{tt'} - \delta_{tt'}) \sum_j u_j(t')\} \\
&= (\sum \sum \sigma_{ij}) \sum_t m_{t,t-1} \, ;
\end{aligned} \tag{7.18}
$$

$$
\begin{aligned}
&E(\sum_t W_t W_{t-1}) \\
&= E\sum_t \{\sum_{t'} (m_{tt'} - \delta_{tt'}) \sum_{t''} (m_{t-1,t''} - \delta_{t-1,t''}) \sum_j \sum u_i(t') u_j(t'')\} \\
&= (\sum \sum \sigma_{ij}) \sum_t \sum_{t'} (m_{tt'} - \delta_{tt'})(m_{t-1,t'} - \delta_{t-1,t'}) \\
&= -(\sum \sum \sigma_{ij}) \sum_t m_{t,t-1} \, .
\end{aligned} \tag{7.19}
$$

Hence all three expectations are equal except that the last has opposite sign. Taking account of the factor $1/(T-1)$ that was applied in Table 9, we find that the expectations corresponding to (7.17) and (7.18) are about -2000, and the one corresponding to (7.19) about 2000. With only one exception, the observed values of Table 9 correspond with these expectations as far as signs are concerned.

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APPENDIX

The data underlying this study, taken from Grunfeld [3], are presented below in Table 10. The exact definitions of these variables are as follows.

I = Gross investment[1] = additions to plant and equipment plus maintenance and repairs in millions of dollars deflated by P_1.

F = Value of the firm[2] = price of common and preferred shares at December 31 (or average price of December 31 and January 31 of the following year) times number of common and preferred shares outstanding plus total book value of debt at December 31 in millions of dollars deflated by P_2.

C = The stock of plant and equipment[1] = accumulated sum of net additions to plant and equipment deflated by P_1 minus depreciation allowance deflated by P_3.

In these definitions

P_1 = Implicit price deflator of producers durable equipment (base 1947).[3]

P_2 = Implicit price deflator of G.N.P. (base 1947).[3]

P_3 = Depreciation expense deflator = ten years moving average of wholesale price index of metals and metal products (base 1947).[4]

A splitting-up of the implied sampling errors of a, b_1 and b_2 by firms and years is given in Tables 11-13.

TABLE 10

MICRODATA UNDERLYING THIS STUDY

	G.M.			U.S. Steel			G. Elec.			Chrys.		
	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}
1935	317.6	3078.5	2.8	209.9	1362.4	53.8	33.1	1170.6	97.8	40.29	417.5	10.5
36	391.8	4661.7	52.6	355.3	1807.1	50.5	45.0	2015.8	104.4	72.76	837.8	10.2
37	410.6	5387.1	156.9	469.9	2676.3	118.1	77.2	2803.3	118.0	66.26	883.9	34.7
38	257.7	2792.2	209.2	262.3	1801.9	260.2	44.6	2039.7	156.2	51.60	437.9	51.8
39	330.8	4313.2	203.4	230.4	1957.3	312.7	48.1	2256.2	172.6	52.41	679.7	64.3
40	461.2	4643.9	207.2	361.6	2202.9	254.2	74.4	2132.2	186.6	69.41	727.8	67.1
41	512.0	4551.2	255.2	472.8	2380.5	261.4	113.0	1834.1	220.9	68.35	643.6	75.2
42	448.0	3244.1	303.7	445.6	2168.6	298.7	91.9	1588.0	287.8	46.80	410.9	71.4
43	499.6	4053.7	264.1	361.6	1985.1	301.8	61.3	1749.4	319.9	47.40	588.4	67.1
44	547.5	4379.3	201.6	288.2	1813.9	279.1	56.8	1687.2	321.3	59.57	698.4	60.5
45	561.2	4840.9	265.0	258.7	1850.2	213.8	93.6	2007.7	319.6	88.78	846.4	54.6
46	688.1	4900.9	402.2	420.3	2067.7	132.6	159.9	2208.3	346.0	74.12	893.8	84.8
47	568.9	3526.5	761.5	420.5	1796.7	264.8	147.2	1656.7	456.4	62.68	579.0	96.8
48	529.2	3254.7	922.4	494.5	1625.8	306.9	146.3	1604.4	543.4	89.36	694.6	110.2
49	555.1	3700.2	1020.1	405.1	1667.0	351.1	98.3	1431.8	618.3	78.98	590.3	147.4
50	642.9	3755.6	1099.0	418.8	1677.4	357.8	93.5	1610.5	647.4	100.66	693.5	163.2
51	755.9	4833.0	1207.7	588.2	2289.5	342.1	135.2	1819.4	671.3	160.62	809.0	203.5
52	891.2	4924.9	1430.5	645.5	2159.4	444.2	157.3	2079.7	726.1	145.00	727.0	290.6
53	1304.4	6241.7	1777.3	641.0	2031.3	623.6	179.5	2371.6	800.3	174.93	1001.5	346.1
54	1486.7	5593.6	2226.3	459.3	2115.5	669.7	189.6	2759.9	888.9	172.49	703.2	414.9

[1] Source: *Moody's Industrial Manual* and Annual Reports of Corporations.

[2] Source: *Bank and Quotation Record* and *Moody's Industrial Manual.*

[3] Source: *Survey of Current Business*, July 1956 and July 1957.

[4] Source: *Historical Statistics of the U.S.* 1789-1945 and *Economic Report of the President*, January 1957, p. 161.

TABLE 10 (*concluded*)

	Atl. Ref.			I.B.M.			U. Oil		
	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}
1935	39.68	157.7	183.2	20.36	197.0	6.5	24.43	138.0	100.2
36	50.73	167.9	204.0	25.98	210.3	15.8	23.21	200.1	125.0
37	74.24	192.9	236.0	25.94	223.1	27.7	32.78	210.1	142.4
38	53.51	156.7	291.7	27.53	216.7	39.2	32.54	161.2	165.1
39	42.65	191.4	323.1	24.60	286.4	48.6	26.65	161.7	194.8
40	46.48	185.5	344.0	28.54	298.0	52.5	33.71	145.1	222.9
41	61.40	199.6	367.7	43.41	276.9	61.5	43.50	110.6	252.1
42	39.67	189.5	407.2	42.81	272.6	80.5	34.46	98.1	276.3
43	62.24	151.2	426.6	27.84	287.4	94.4	44.28	108.8	300.3
44	52.32	187.7	470.0	32.60	330.3	92.6	70.80	118.2	318.2
45	63.21	214.7	499.2	39.03	324.4	92.3	44.12	126.5	336.2
46	59.37	232.9	534.6	50.17	401.9	94.2	48.98	156.7	351.2
47	58.02	249.0	566.6	51.85	407.4	111.4	48.51	119.4	373.6
48	70.34	224.5	595.3	64.03	409.2	127.4	50.00	129.1	389.4
49	67.42	237.3	631.4	68.16	482.2	149.3	50.59	134.8	406.7
50	55.74	240.1	662.3	77.34	673.8	164.4	42.53	140.8	429.5
51	80.30	327.3	683.9	95.30	676.9	177.2	64.77	179.0	450.6
52	85.40	359.4	729.3	99.49	702.0	200.0	72.68	178.1	466.9
53	91 90	398.4	774.3	127.52	793.5	211.5	73.86	186.8	486.2
54	81.43	365.7	804.9	135.72	2927.3	238.7	89.51	192.7	511.3

	West.			G. Year			D. Match		
	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}	I	F_{-1}	C_{-1}
1935	12.93	191.5	1.8	26 63	290.6	162	2.54	70.91	4.50
36	25.90	516.0	.8	23 39	291.1	174	2 00	87.94	4.71
37	35.05	729.0	7.4	30.65	335.0	183	2.19	82.20	4.57
38	22.89	560.4	18.1	20.89	246.0	198	1.99	58.72	4.56
39	18.84	519.9	23.5	28.78	356.2	208	2.03	80.54	4.38
40	28.57	628.5	26.5	26.93	289.8	223	1.81	86.47	4.21
41	48.51	537.1	36.2	32.08	268.2	234	2.14	77.68	4.12
42	43.34	561.2	60.8	32.21	213.3	248	1.86	62.16	3.83
43	37.02	617.2	84.4	35.69	348.2	274	.93	62.24	3.58
44	37.81	626.7	91.2	62.47	374.2	282	1.18	61.82	3.41
45	39.27	737.2	92.4	52.32	387.2	316	1.36	65.85	3.31
46	53.46	760.5	86.0	56.95	347.4	302	2.24	69.54	3.23
47	55.56	581.4	111.1	54.32	291.9	333	3.81	64.97	3.90
48	49.56	662.3	130.6	40.53	297.2	359	5.66	68.00	5.38
49	32.04	583.8	141.8	32.54	276.9	370	4.21	71.24	7.39
50	32.24	635.2	136.7	43.48	274.6	376	3.42	69.05	8.74
51	54.38	723.8	129.7	56.49	339.9	391	4.67	83.04	9.07
52	71.78	864.1	145.5	65.98	474.8	414	6.00	74.42	9.93
53	90.08	1193.5	174.8	66.11	496.0	443	6.53	63.51	11.68
54	68.60	1188.9	213.5	49.34	474.5	468	5.12	58.12	14.33

TABLE 11

COMPONENTS OF THE IMPLIED SAMPLING ERROR OF THE CONSTANT TERM BY FIRMS AND YEARS*

	G.M.	U.S. Steel	G. Elec.	Chrys.	Atl. Ref.	I.B.M.	U. Oil	West.	G. Year	D. Match	Total
1935	5471	−5	−158	586	−508	143	245	173	−47	5	5905
36	155	−315	66	−48	−1	−26	24	4	23	3	−113
37	6657	−242	245	351	−923	−139	−60	174	90	16	6169
38	−119	−3812	−846	265	162	159	91	−287	−225	−16	−4629
39	75	127	19	10	8	6	5	7	5	0	263
40	184	679	5	21	69	59	19	61	51	5	1154
41	−21	−3	−36	−1	−5	−9	−6	−15	−0	0	−97
42	3335	−27	546	−54	−513	299	−130	290	119	−9	3856
43	1082	−861	−379	−215	218	−149	33	−47	−85	−17	−419
44	1060	−938	−284	−82	−25	−106	271	−35	200	−8	52
45	−255	718	−13	−85	−30	20	30	56	−36	4	410
46	−1757	−952	−993	273	48	34	63	−98	−231	−6	−3620
47	480	1655	1388	−216	−217	−80	−116	474	401	52	3821
48	−1999	5389	1203	255	360	310	−193	113	−140	109	5408
49	−4262	961	−870	−273	156	26	−260	−424	−406	18	−5334
50	−1796	998	−1061	36	−226	−469	−522	−383	−10	−25	−3458
51	967	−841	40	−320	−19	1	18	−41	−53	−1	−248
52	989	−1830	−24	−35	−26	15	−48	−166	−50	−15	−1191
53	−2230	−4145	−229	280	−590	−623	−82	−508	−3	−44	−7679
54	−4931	4239	297	247	109	−74	−480	469	594	55	525
Total	3085	794	−1083	996	−1457	−602	−1100	−182	197	127	775

* All figures are to be divided by 100.

TABLE 12

COMPONENTS OF THE IMPLIED SAMPLING ERROR OF THE F_{-1}-COEFFICIENT BY FIRMS AND YEARS*

	G.M.	U.S. Steel	G. Elec.	Chrys.	Atl. Ref.	I.B.M.	U. Oil	West.	G. Year	D. Match	Total
1935	−4230	4	122	−453	393	−111	−190	−133	37	−4	−4565
36	−767	1553	−325	235	3	128	−119	−21	−115	−14	557
37	−9566	348	−352	−504	1326	200	86	−250	−130	−24	−8865
38	87	2775	616	−193	−118	−116	−66	209	164	11	3369
39	−1471	−2479	−377	−197	−163	−114	−96	−139	−100	−6	−5141
40	−452	−1666	−13	−53	−168	−146	−47	−150	−125	−13	−2832
41	299	48	505	8	64	129	88	214	5	−2	1358
42	−2718	22	−445	44	418	−244	106	−236	−97	7	−3143
43	−568	452	199	113	−115	78	−17	25	45	9	220
44	−237	210	63	18	6	24	−61	8	−45	2	−12
45	594	−1672	30	199	69	−47	−69	−131	83	−9	−955
46	2773	1502	1568	−431	−75	−54	−99	154	365	9	5713
47	−474	−1636	−1373	213	215	79	115	−469	−397	−51	−3778
48	2144	−5781	−1291	−273	−386	−333	207	−121	150	−117	−5801
49	4809	−1084	982	308	−176	−30	294	479	458	−20	6019
50	2116	−1176	1250	−42	267	553	615	451	12	29	4075
51	−657	571	−27	217	13	−1	−12	28	36	1	168
52	−550	1018	13	20	15	−8	27	93	28	8	663
53	1883	3501	194	−236	80	526	69	429	3	37	6485
54	3103	−2667	−187	−155	−69	46	302	−295	−373	−34	−330
Total	−3881	−6158	1152	−1164	1598	560	1134	142	2	−181	−6795

* All figures are to be divided by 10^6.

TABLE 13

COMPONENTS OF THE IMPLIED SAMPLING ERROR OF THE C_{-1}-COEFFICIENT BY FIRMS AND YEARS*

	G.M.	U.S. Steel	G. Elec.	Chrys.	Atl. Ref.	I.B.M.	U. Oil	West.	G. Year	D. Match	Total
1935	−1449	1	42	−155	135	−38	−65	−46	13	−1	−1565
36	1827	−3699	774	−560	−7	−305	284	51	275	34	−1326
37	10825	−394	399	570	−1501	−226	−97	283	147	27	10032
38	32	1037	230	−72	−44	−43	−25	78	61	4	1259
39	3509	5914	899	471	389	273	228	331	238	13	12265
40	742	2738	21	86	277	240	78	247	205	22	4656
41	−657	−106	−1110	−17	−140	−284	−194	−470	−10	5	−2949
42	345	−3	56	−6	−53	31	−13	30	12	−1	399
43	−464	369	163	92	−94	64	−11	20	37	7	180
44	−1100	973	294	86	26	110	−281	36	−207	8	−54
45	−767	2159	−39	−257	−89	61	89	170	−107	12	1233
46	−2620	−1419	−1481	407	71	51	94	−146	−345	−9	−5399
47	396	1365	1146	−178	−179	−66	−96	391	331	43	3153
48	−2101	5665	1265	268	378	326	−203	119	−147	115	5685
49	−5495	1239	−1122	−351	201	34	−336	−547	−523	23	−6878
50	−2937	1632	−1735	59	−370	−767	−854	−626	−17	−41	−5656
51	−3093	2688	−129	1023	60	−3	−56	130	168	4	792
52	−2836	5250	69	101	76	−43	138	478	143	42	3417
53	1595	2966	164	−200	68	446	59	364	2	31	5495
54	8283	−7121	−498	−415	−184	124	806	−789	−997	−92	−881
Total	4035	21255	−593	986	−982	−16	−459	104	−722	248	23856

* All figures are to be divided by 10^6.

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