CO 370: Deterministic Operations Research Models, Winter 2022

Practice Test #1

There are four problems, each worth 25 marks.

Name ________________________________

ID # ________________________________

1. (25 Marks). An ice cream firm is under contract to deliver d_i tons of their product at the end of each month i over the next year, that is, $i = 1, 2, \ldots, 12$. Ice cream produced and sold in the same month does not incur storage cost, but ice cream can also be stored in inventory for use in future months at the cost of \$20 per month for each ton. The firm can backlog orders at a cost of \$25 per month for each ton. If the firm produces x_i units in month i and x_{i-1} units in month $i-1$, it incurs a cost of $\$50|x_i - x_{i-1}|$, reflecting the cost of switching to a new production level. (There are no other production costs in the model.) The firm begins the year with production at 120 tons, that is, $x_0 = 120$. The firm has no inventory at the start of the year and it is required that no inventory remain at the end of year. Formulate an LP model to minimize the total cost for the firm to meet their contracted deliveries over the year.

2. (25 marks). For a positive integer n, let a_j, b_j, and d_j be rational numbers for $j = 1, \ldots, n$ and let f be a rational number. Show how to convert the follow optimization problem into an LP model.

$$\begin{gathered}
\text{maximize} \;\; (\textstyle\sum_{j=1}^{n} a_j x_j)/(\sum_{j=1}^{n} b_j x_j) \\
\text{subject to} \\
\textstyle\sum_{j=1}^{n} b_j x_j \geq 1 \\
\textstyle\sum_{j=1}^{n} d_j x_j \geq f \\
x_j \geq 0, \text{ for } j = 1, \ldots, n.
\end{gathered}$$

3. (25 Marks) The following table lists the age, obesity, and systolic blood pressure for three patients.

Patient	Age (Years)	Obesity (BMI)	Systolic Blood Pressure (mmHg)
1	73	29	131
2	44	38	125
3	62	22	120

Build an LP model, implementing L_1-regression to attempt to explain the observed blood pressure values (the *response* values) by the recorded age and obesity values (the *control* values).

4. (25 Marks). Consider the following LP problem written in standard form

$$
\begin{gathered}
\text{minimize} \quad z = -4x_1 - 4x_2 - 6x_3 \\
\text{subject to} \\
2x_1 + 2x_2 + 3x_3 + 1x_4 + 0x_5 = 6 \\
-2x_1 + 2x_2 - 3x_3 + 0x_4 + 1x_5 = 2 \\
x_1 \geq 0, x_2 \geq 0, x_3 \geq 0, x_4 \geq 0, x_5 \geq 0.
\end{gathered}
$$

and the basis header $B = [1, 2]$ and non-basic indices $N = [3, 4, 5]$. We have

$$A_B \equiv \mathbb{B} = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} \text{ and } \mathbb{B}^{-1} = \begin{bmatrix} 1/4 & -1/4 \\ 1/4 & 1/4 \end{bmatrix}.$$

The general formula for a simplex tableau is

$$
\begin{aligned}
x_B + \mathbb{B}^{-1} A_N x_N \quad & = \mathbb{B}^{-1} b \\
0x_B + (c_N^T - c_B^T \mathbb{B}^{-1} A_N) x_N \quad & = z - c_B^T \mathbb{B}^{-1} b
\end{aligned}
$$

Part A. (5 Marks) Verify that the indicated B yields an optimal basic solution.

Part B. (10 Marks) Find the range of values Δ such the objective value for variable x_1 can be changed to $-4 + \Delta$ and the simplex tableau continues to be optimal.

Part C. (10 Marks) Find the range of values Δ such that the right-hand-side value for the first constraint can be changed to $6 + \Delta$ and the simplex tableau continues to be optimal.