Problem Set 6

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Problem 1

For an object orbiting in the Lagrange point $L1$ between the Earth and the Moon, the following must be true

$$F_{\text{Earth}} + F_{\text{Moon}} = F_C, \tag{1}$$

where F_{Earth} is the gravitational force on the object due to the Earth, F_{Moon} is the gravitational force on the object due to the Moon, and F_C is the centripetal force on the object. F_{Earth} and F_{Moon} are given by Newton's gravitational law and $F_C = \omega^2 r$. Equation 1 thus becomes

$$\frac{GM}{r^2} - \frac{Gm}{(R-r)^2} = \omega^2 r, \tag{2}$$

where M is the mass of the Earth, m is the mass of the Moon, r and ω are the radius and angular frequency of the orbit at $L1$, R is the radius of the Moon's orbit, and G is the gravitational constant. The angular frequency ω can be expressed in terms of G, R, and M,

$$\omega^2 = \frac{GM}{R^3}. \tag{3}$$

We can replace our variables with the dimensionless quantities $r' = \frac{r}{R}$ and $m' = \frac{m}{M}$. This gives

$$\frac{m'}{(1-r')^2} + r' - \frac{1}{r'^2} = 0. \tag{4}$$

Using Newton's method and jax for the analytic derivative, we can compute the distance to $L1$, r, for two objects of any masses m, M, orbiting at a distance R by finding the roots of equation 4. From a plot of equation 4, (fig 1) we can see that the root is somewhere near the middle of the interval (0, 1). Thus, we will choose a starting point for Newton's method near 0.

Using following constants

Earth Mass	5.974×10^{24}kg
Moon Mass	7.384×10^{22}kg
Solar Mass	1.988×10^{30}kg
Jupiter Mass	1.898×10^{27}kg
Lunar Orbit Radius	3.844×10^{8}m
Earth Orbit Radius	149.60×10^{9}m

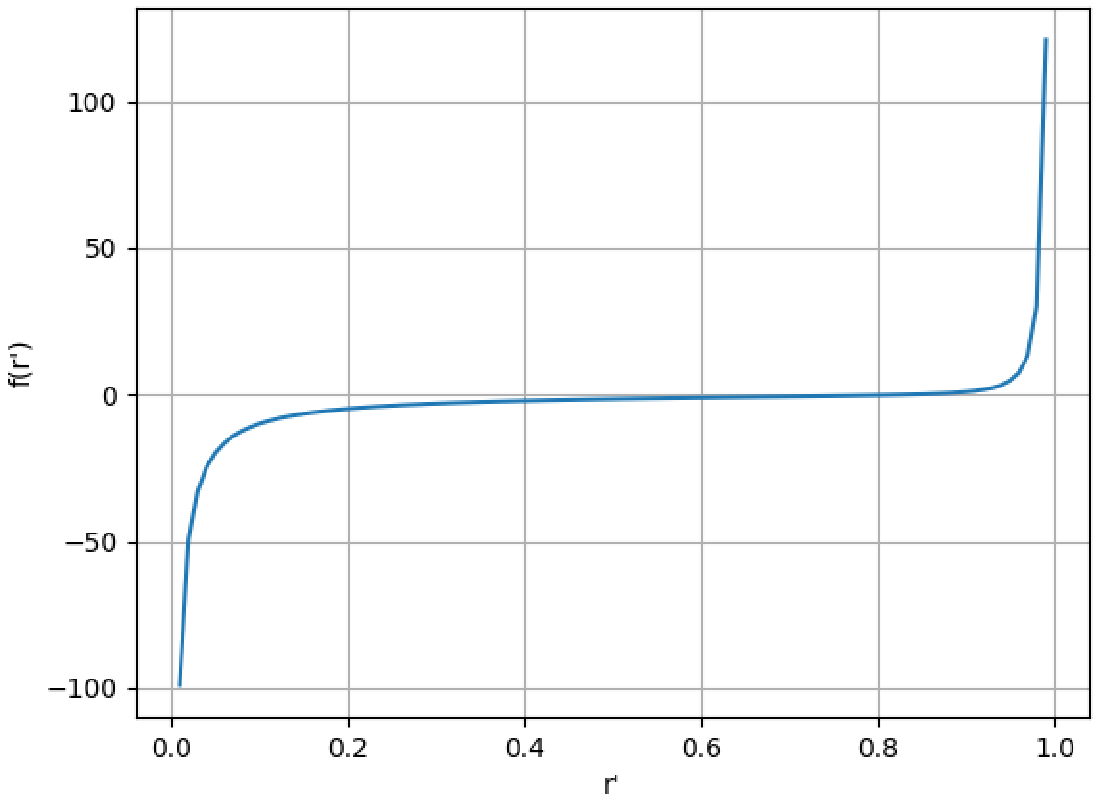

Figure 1: Plot of equation 4 as a function of r' with $m' = \frac{m}{M}$

we calculate $L1$ for the Earth-Moon system to be at a distance of 3.262×10^8 meters. For the Earth-Sun system, $L1$ is at 1.481×10^{11}. For a Jupiter sized planted orbiting the sun in place of Earth, $L1$ is at a distance of 1.396×10^{11}.

Problem 2

We want to minimize

$$f(x) = (x - 0.3)^2 e^x \tag{5}$$

using Brent's method. A plot of equation 5 is shown in figure 2. Brent's method uses a combination of parabolic minimization and golden section minimization. My implementation of Brent's method in python gives a minimum of 0.3000259015536602 with a tolerance of 0.0001. Scipy's built in Brent's minimization gives a minimum of 0.2999996417851 with a tolerance of 0.0001. The difference between these two methods is $2.6259768560188412 \times 10^5$.

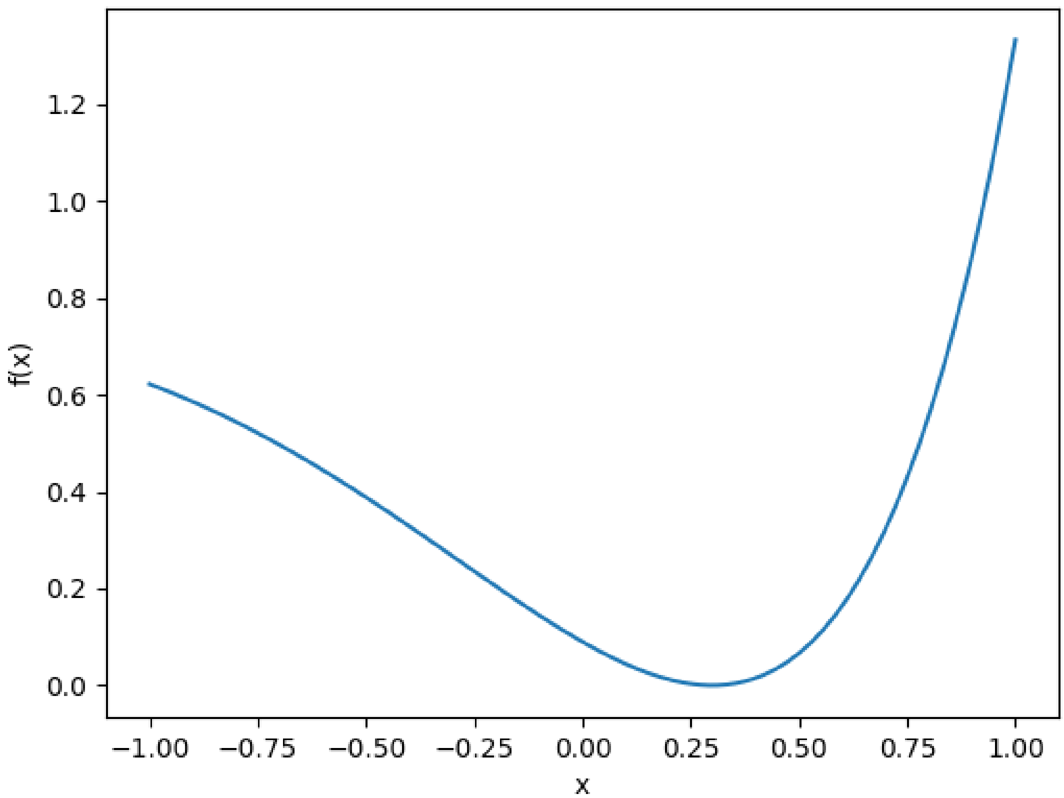

Figure 2: Plot of equation 5 over the integral -1, 1.