Problem Set 8

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Problem 1

We model the probability that a person will recognize the phrase "Be Kind, Rewind" as a function of age with a logistic function:

$$p(x) = \frac{1}{1 + e^{-(\beta_0 + \beta_1 x)}} \tag{1}$$

where β_0 and β_1 are fit parameters. We the values of β_0 and β_1 by minimizing the negative log of the likelihood, which we will call $\mathcal{L}$:

$$\mathcal{L} = -\sum_{i=0}^{N} y_i \times \log(p(x_i)) + (1 - y_i) \times \log(1 - p(x_i)) \tag{2}$$

where y_i is the survey result of a person with age x_i. Minimizing $\mathcal{L}$ using scipy.optimize.minimize, starting with $\beta_0 = -50$ and $\beta_1 = 1$, gives $\beta_0 = -5.62023141$ and $\beta_1 = 0.10956337$. It also gives the inverse hessian matrix, an approximation of the covariance matrix:

$$\begin{bmatrix} 1.10569735 & -2.08727178 \times 10^{-02} \\ -2.08727178 \times 10^{-02} & 4.29988075 \times 10^{-04} \end{bmatrix} \tag{3}$$

where the diagonal values are the squares of the errors. Thus, the error in β_0 is 1.0515214459960391 and the error in β_1 is 0.02073615381192488. The minimum value of the log likelihood is 34.72562248675183. A plot of the model along with the original data is shown in figure 1.

Problem 2

The waveforms of a piano and trumpet playing a single note are shown in figures **??** and **??** respectively. We can determine which note each instrument is playing by studying the Fourier spectra of their waveforms, which are shown in figures 4 and 5. The dominant frequency in each spectra is the note that the instrument is playing. For the piano, the frequency with the highest amplitude is 525.231Hz, which is about a C5. For the trumpet, the dominant frequency is 1043.847Hz with is about a C6.

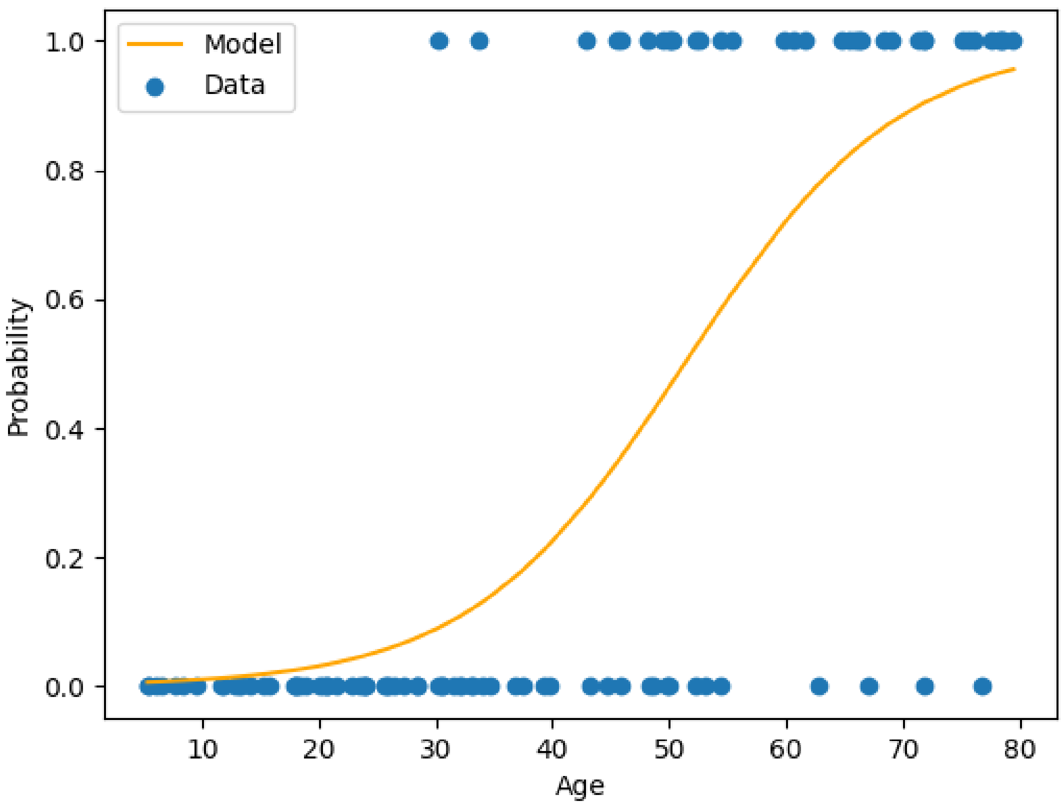

Figure 1: Model and survey data of the probability that a person will recognize the phrase ”Be Kind, Rewind” as a function of age.

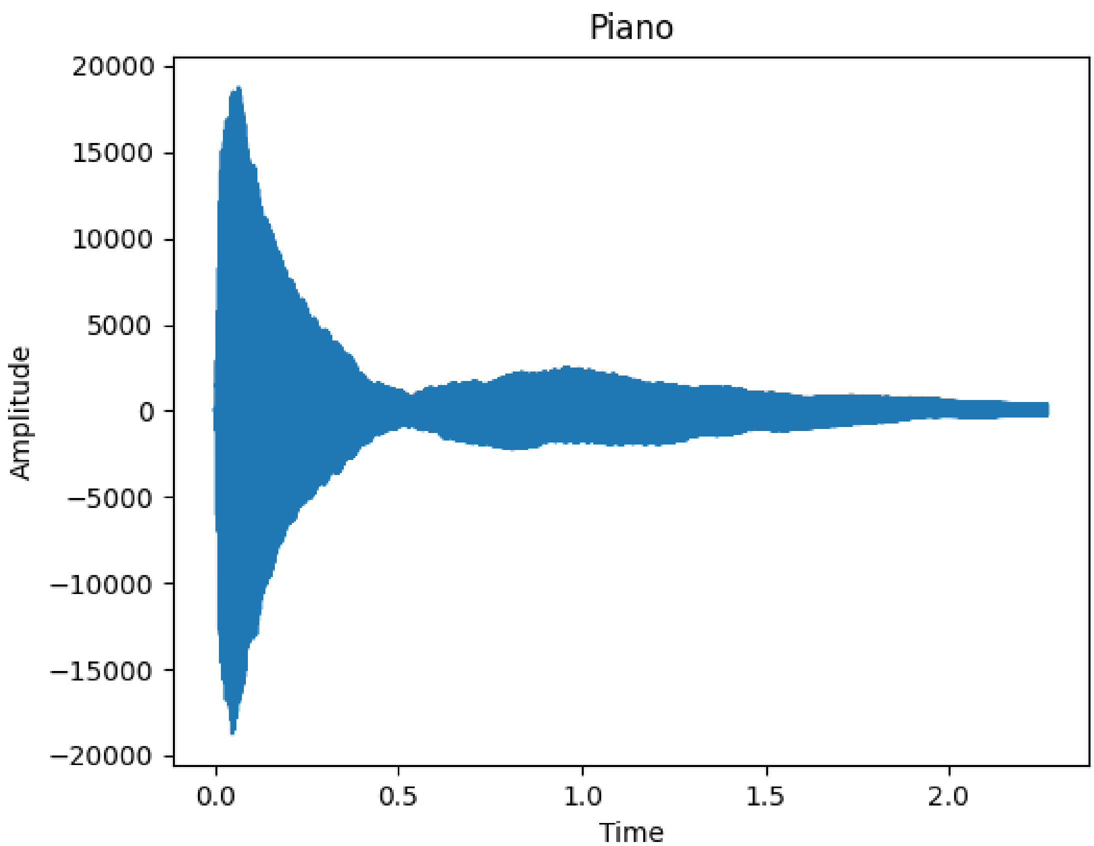

Figure 2: Waveform of a piano note

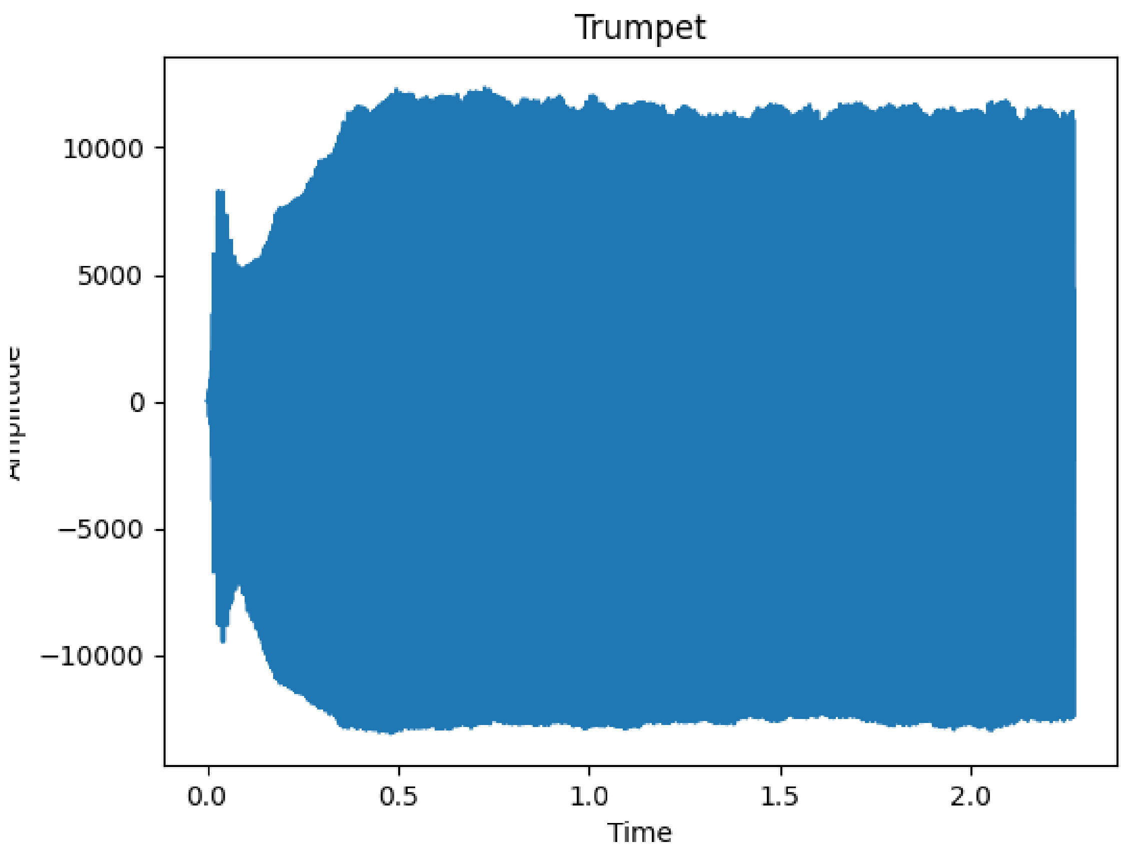

Figure 3: Waveform of a trumpet note

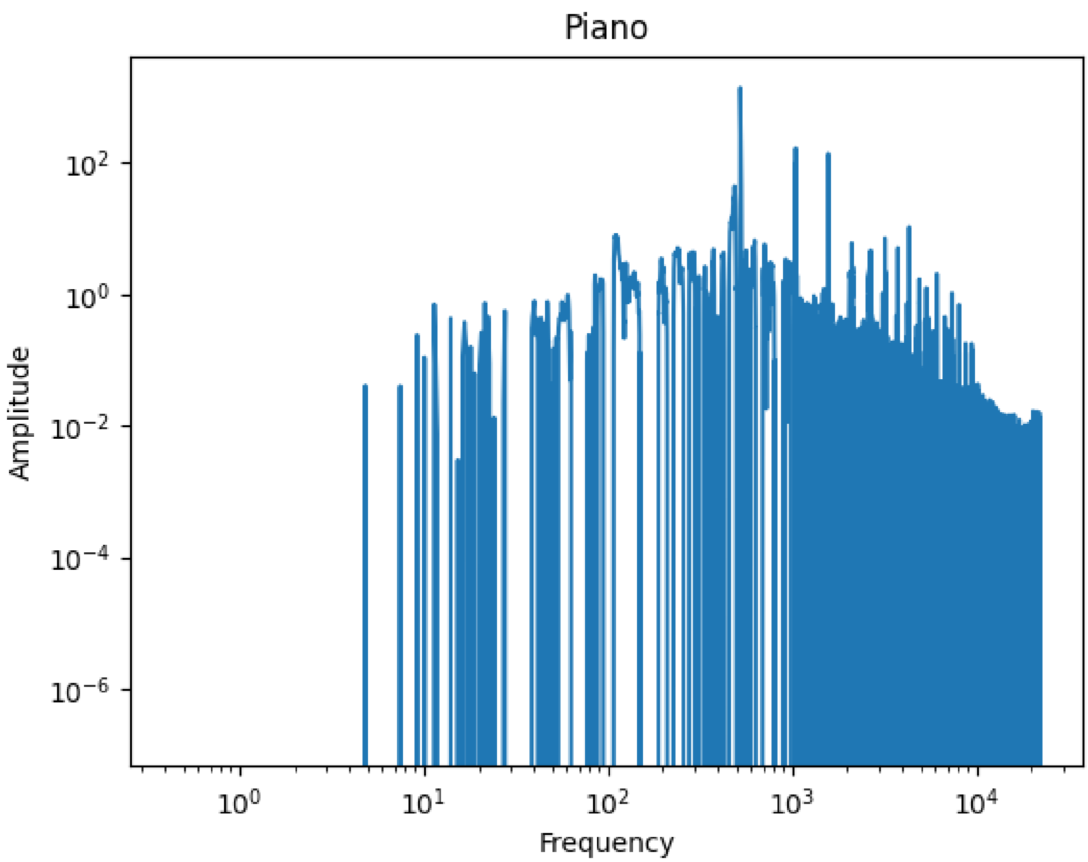

Figure 4: Fourier spectra of the piano note waveform shown in figure 2.

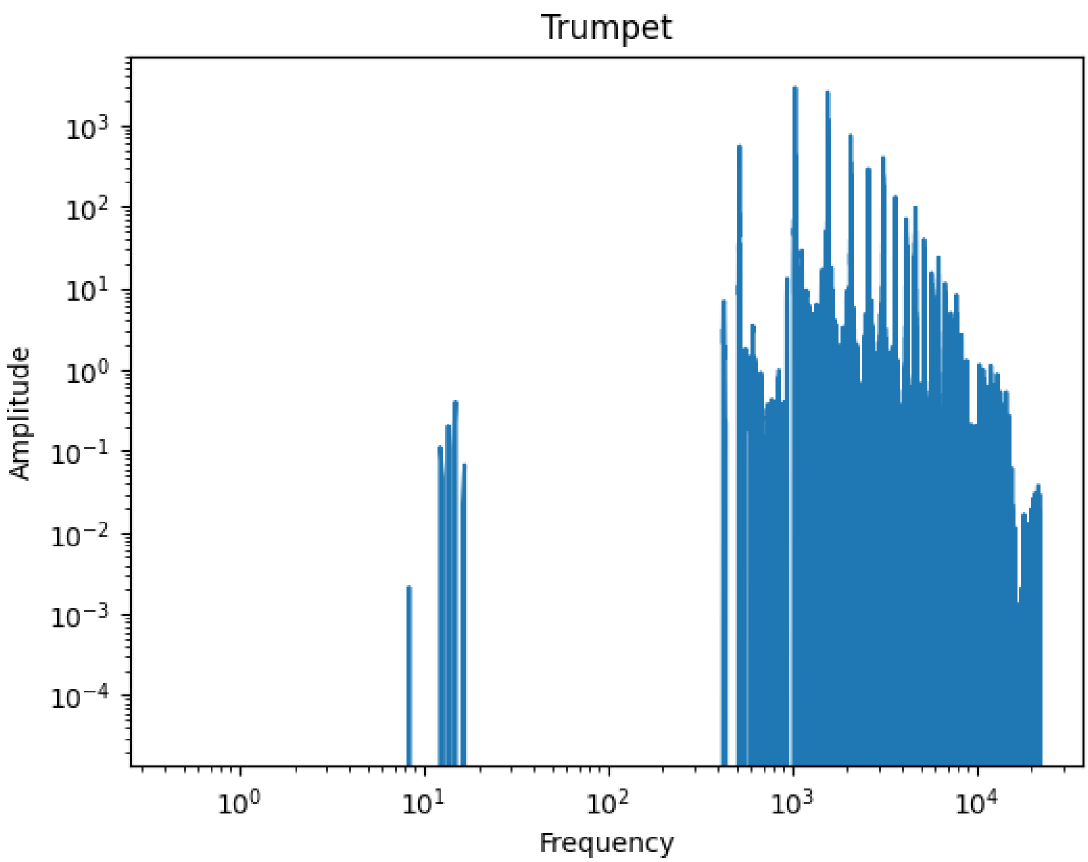

Figure 5: Fourier spectra of the piano note waveform shown in figure 3.

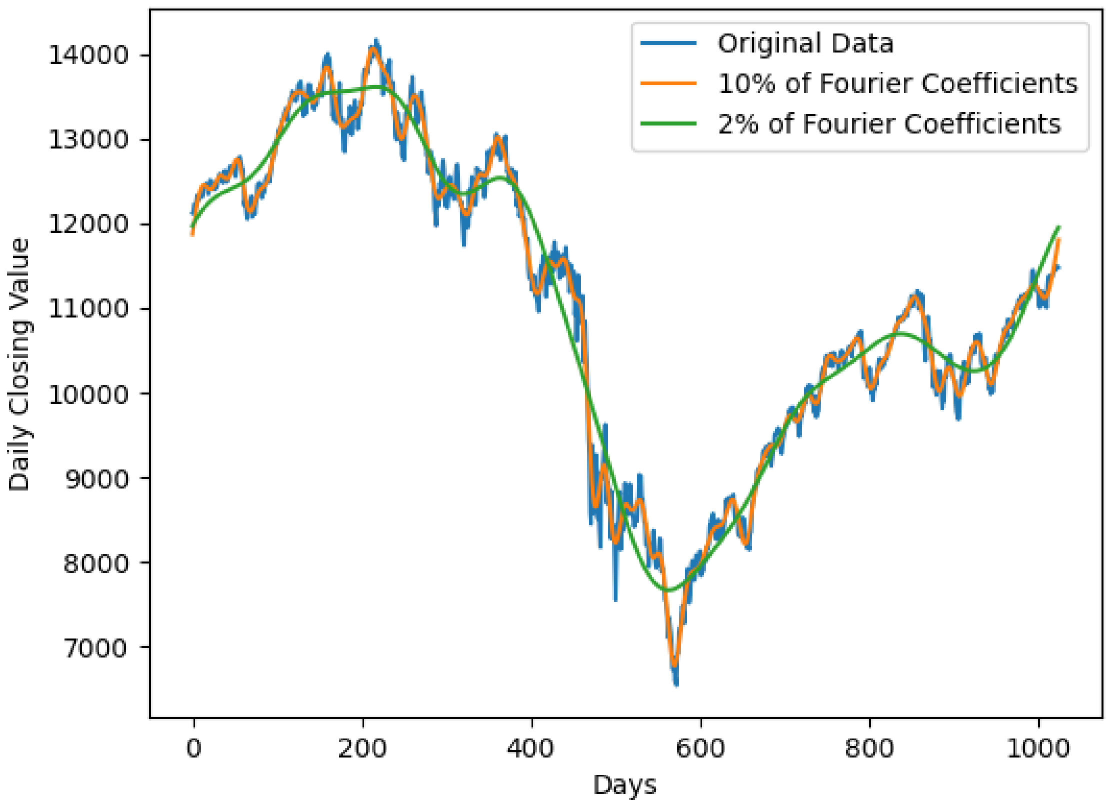

Figure 6: Daily closing value of the Dow Jones Industrial Average and two approximations of it using 10% and 2% of the Fourier amplitudes.

Problem 3

We can generate approximations of time series data by taking the Fourier spectrum, and setting the higher frequency Fourier amplitudes to 0. Figure 6 shows the daily closing value of the Dow Jones Industrial Average and approximations of it using the first 10% and the first 2% of the Fourier amplitudes. We notice that we loose larger and larger fluctuations as we set lower frequency amplitudes to zero.