ESE 351 Spring 2023 Case Study #2 PAM communication

Assigned Tuesday 4/11/23, Due Tuesday 4/25/23

You may work in groups of up to 3 students.

In this case study, you will extend your binary pulse-amplitude modulation (PAM) simulation from Homework 6 and explore other types of pulse shapes to see how their properties relate to the performance and bandwidth of a digital communication channel. The main tasks in this case study are to

- Up-convert PAM signals with a carrier frequency on transmit and down-convert on receive to send independent signals simultaneously over adjacent frequency bands,
- Design additional pulse shapes to minimize required spectral bandwidth while also minimizing time-domain duration and inter-symbol interference.
- Evaluate the communication performance (error rate) of each pulse shape due to interference from adjacent spectral bands and due to additive noise,
- Demonstrate your findings for transmission of a custom message, e.g., text or other data, converted to binary for transmission and back to its original form after reception.

Background

In Homework 6, you explored the use of a triangular pulse shape in a binary Pulse Amplitude Modulation (PAM) scheme. As we have discussed, a time-limited function requires infinite bandwidth, and a band-limited function requires infinite time. To encode messages with a pulse, we would like a signal that has a narrow frequency bandwidth to minimize spectral needs but is also time-limited to minimize time required to send.

In Homework 6, you should have found that the triangular pulse shape was successful for communication as long as the symbol rate was slow enough that adjacent pulses didn't interfere, but the triangular pulse requires a wide bandwidth in the frequency domain. A more relaxed condition to reduce inter-symbol interference (ISI) is that the pulse shape, $p(t)$, is zero-valued at sample times for all other symbols, i.e., for symbol period $T_s, p(nT_s) = \begin{cases} 1, & n = 0 \\ 0, & else \end{cases}$. This condition implies no constraint on the pulse shape duration and, in a system with no noise, adjacent pulses do not interfere and symbols can be retrieved without error. In the frequency domain, this criterion relaxes the spectral constraint. The condition can be satisfied by any frequency-domain function, $P(j\omega)$, that can be expressed as

$$P(j\omega) = \begin{cases} 1 + P_1(j\omega), & |\omega| \le \dfrac{2\pi}{T_s} \\ 0, & else \end{cases}$$

$$P_1\left(-j\omega + \frac{j\pi}{T_s}\right) = -P_1\left(j\omega + \frac{j\pi}{T_s}\right) \quad 0 \le \omega \le \frac{\pi}{T_s}$$

where $P_1(j\omega)$ is a function with odd symmetry around a point equal to half the symbol frequency $2\pi/T_s$. (For reference, see section 8.6 in Oppenheim and Wilsky, equations 8.28 and 8.29, and Wikipedia entries for Nyquist ISI criterion and raised-cosine filters. Note that equation 8.28 has been changed above to match the relation in Figure 8.31, which satisfies the Nyquist filtering criterion below). More general versions of this constraint are also possible but beyond the scope of this case study.

Note that pulse shapes satisfying the above criteria can be placed at adjacent bands in the frequency domain and summed to produce a constant amplitude, also known as the Nyquist filtering criterion, i.e.,

$$\sum_k P\left(\omega - \frac{k2\pi}{T_s}\right) = const.$$

In a typical communication system, the signal is up-converted, e.g., by multiplication with a sinusoid at much higher frequency than the bandwidth of the signal. The up-converted signal is transmitted across a channel (e.g., over-the-air broadcast), then received and down-converted to retrieve the original signal. A simple scheme for up-conversion and down-conversion is to multiply the signal by $\cos \omega_c t$, a cosine at the carrier frequency ω_c, to up-convert, then down-convert by multiplying again by $\cos \omega_c t$ and filtering with a lowpass filter having bandwidth appropriate to the original signal (as discussed in class, see also Sections 8.1-8.3 in the text covering synchronous modulation and demodulation, and frequency multiplexing).

Tasks

Build on your simulation tool from Homework 6. In this case study, you will do the following:

- add up-conversion and down-conversion of signals using a carrier frequency (see demo code),
- add frequency multiplexing to send multiple (three) messages simultaneously over adjacent frequency bands,
- add the ability to use pulse shapes longer than the symbol period, using the above criteria,
- develop a demonstration of your simulation by communicating a custom message and illustrating performance with varying additive noise levels,
- evaluate performance (error rate) for simultaneous communication while minimizing the required bandwidth and pulse duration.

Details:

1. Note that the bit rate is fixed in this model with symbol period T_s, so results will not depend on bit rate. For simplicity, use $T_s = 0.1$ seconds as in Homework 6.
2. Pulse shapes - Modify (or develop) your previous binary-PAM simulation to allow arbitrary pulse shapes with variable length. Implement at least two pulse shapes:
    a. The most common extended pulse shape with regular zero crossings is the sinc() function. Implement a finite-length (truncated in time, with variable length) sinc pulse.
    b. Implement at least one additional pulse shape that meets the Nyquist filtering criteria (see references above for examples). These shapes are typically designed in the frequency domain (meeting the frequency-domain criteria above) then transformed to the time domain. Note: the triangular pulse (as in Homework 6) meets the time-domain criteria. If you use it, you should pay particular attention to demonstrating that it meets the frequency-domain criteria.
    c. For each pulse shape you use, characterize in both the time domain and the frequency domain, and include plots that demonstrate whether the shape satisfies the Nyquist filtering criterion.
    d. Note that although PAM communication is implemented as a continuous-time system, you are effectively simulating as a discrete-time system by sampling the continuous-time signal so be aware of these implications for your frequency-domain assessments.

3. Add up-conversion and down-conversion of your signal to your simulation. Verify operation in the time- and frequency-domains for sending three signals over adjacent bands. For symbol period $T_s = 0.1$ seconds, the Nyquist filtering criteria requires bandwidth of $\frac{1}{T_s} = 10Hz$, thus three signals could be communicated at frequencies of 20, 30 and 40 Hz.
   a. Confirm that the sampling rate you use (sampling period dt from Homework 6) allows up-conversion and down-conversion without issues.
   b. For recovering individual signals sent simultaneously over adjacent bands, down-conversion is accomplished by multiplication with a synchronized sinusoid at the respective frequency, then processing with a lowpass filter. Note that the matched filter may fill this role, depending on the pulse shape you use.
4. PAM simulation – use the same structure as in Homework 6 to evaluate the communication performance with different pulse shapes. Your simulation code should be able to plot or report the following to illustrate (note: use only the matched-filter receiver in this case study):
   a. Pulse shape, $p(t)$
   b. Noise-free PAM signal, $y(t)$
   c. Up-converted signal, $y(t)\cos(\omega_c t)$
   d. Noisy received signal, $r(t) = y(t)\cos(\omega_c t) + n(t)$ (noise model from Homework 6)
   e. Down-converted signal $y_{rec}(t) = [r(t)\cos(\omega_c t)] * h_{LPF}(t)$
   f. Sent message, x_n, and decoded message, $\hat{x}_n$ for matched filter receiver
   g. Bandwidth used per channel/message (decoding should work well with 10Hz per channel, but how does performance vary if less bandwidth is used)
   h. Report noise levels, signal-to-noise ratio (SNR), and error rate for each simulation
5. Performance analysis - use your simulation to evaluate the overall performance using each pulse shape. Again, the objective is to minimize error rate, bandwidth, and temporal pulse duration. Include plots in your report that illustrate that performance.
6. Demonstrate with a text message or other data that can be interpreted in terms of error rate. The code snippet below converts ASCII text to a vector of binary data, then converts the binary data back to ASCII text:

```
message = 'I love ESE 351! :)';
binary = str2num(reshape(dec2bin(message)',1,[])');
messageOut = char(bin2dec(num2str(reshape(binary,7,[])')))';
```

Document your design in a 4-page report using the IEEE journal template (here for Microsoft Word and here for LaTeX, a typesetting system that is particularly useful for academic publications). Use of LaTeX is not required but you're welcome to use it. See an introduction to LaTeX here.

Your final case study submission will include:

- A writeup in IEEE style which includes all of the following sections, as well as any other sections you decide to include. (Each section can be as long or as brief as it needs to be)
  - An Abstract describing your findings in brief
  - A Background section describing the context of your work
  - A Methods section describing your filter design and implementation
  - A Results section including figures that illustrate how well you accomplished the above objectives
  - A Conclusion section summarizing what you learned
- Your MATLAB code and any dependent functions
- A published pdf of your MATLAB output

Note: please ensure that your demo code can be run by the grader (include all needed files) and that your code generates most (at least the main) figures in the report. If possible, submit to canvas as a .zip file so filenames don't get changed by canvas.

Projects will be graded based on the following items

- Study design - 25%
  - Strategy and rationale for system design
- MATLAB implementation - 25%
  - Development of PAM simulation with up- and down-conversion and for at least two pulse shapes
  - Demonstration of PAM communication with a custom message
- Results – 25%
  - Demonstration of time- and frequency-domain properties of pulse shapes
  - Evaluation of performance with varying pulse shape and SNR
  - Evaluation of performance with varying spectral bandwidth per channel
- Report - 25%
  - Report is well-organized, concise, and clearly written.
  - Plots are easy to read and interpret, with appropriate font sizes, line widths, labels, etc.
  - Results are interpreted relative to the design objectives.
- Video (bonus 5%, strongly encouraged)
  - Create a 3-4 minute video demonstrating your results. Note: demo of Matlab code is fine during the video, but slides are preferred for the bulk of the presentation.

Demo code: fft of sinc pulse with modulation/up-conversion and demodulation/down-conversion at a carrier frequency

```
%% demo - Sinc pulse shape

Ts = .1; % symbol period (rate 1/Ts)
dt = .01; % sample period
t = -5*Ts:dt:5*Ts; % time vector
x = sinc(t/Ts); % define sinc, note Matlab convention sinc(x) =
sin(pi*x)/(pi*x)

figure
subplot(2,1,1), plot(t,x)
xlabel('time (s)'), ylabel('x(t)'), title('Truncated sinc')

fs = 1/dt; % sample frequency
Nfft = 1024; % length of fft
f = [0:fs/Nfft:fs-fs/Nfft];
subplot(2,1,2), plot(f,abs(fft(x,Nfft)))
xlabel('frequency (Hz)'), ylabel('|X(j\omega)|')

%% modulated sinc
wc = 2*pi*20; % 20Hz modulation
y = x.*cos(wc*t);

figure
subplot(2,1,1), plot(t,y)
xlabel('time (s)'), ylabel('y(t)'), title('Modulated sinc')

fs = 1/dt; % sample frequency
Nfft = 1024; % length of fft
f = [0:fs/Nfft:fs-fs/Nfft];
subplot(2,1,2), plot(f,abs(fft(y,Nfft)))
xlabel('frequency (Hz)'), ylabel('|Y(j\omega)|')

%% demodulated
xr = y.*cos(wc*t);

figure
subplot(2,1,1), plot(t,xr)
xlabel('time (s)'), ylabel('x_r(t)'), title('Demod (without LPF)')

fs = 1/dt; % sample frequency
Nfft = 1024; % length of fft
f = [0:fs/Nfft:fs-fs/Nfft];
subplot(2,1,2), plot(f,abs(fft(xr,Nfft)))
xlabel('frequency (Hz)'), ylabel('|X_r(j\omega)|')

% then LPF, e.g., lowpass()
```