Homework 9, due March 20th, 11:59pm

March 6, 2024

Again, you are required to attach to your report the code that you implemented for each problem, as text, not as picture. If you use some code from the web, also mention in your report where you obtained the code from.

1. Download the dataset `hmm_pb1.csv` from Canvas. It represents a sequence of dice rolls x from the Dishonest casino model discussed in class. The model parameters are exactly those presented in class. The states of Y are 1='Fair' and 2='Loaded'.

   a) Implement the Viterbi algorithm and find the most likely sequence y that generated the observed x. Use the log probabilities, as shown in the HMM slides from Canvas. Report the obtained sequence y of 1's and 2's for verification. (2 points)

   b) Implement the forward and backward algorithms and run them on the observed x. You should memorize a common factor u_t for the α_t^k to avoid floating point underflow, since α_t^k quickly become very small. The same holds for β_t^k. Report $\alpha_{138}^1/\alpha_{138}^2$ and $\beta_{138}^1/\beta_{138}^2$, where the counting starts from $t = 0$. (3 points)

2. Download the dataset `hmm_pb2.csv` from Canvas. It represents a sequence of 20000 dice rolls x from the Dishonest casino model but with other values for the a and b parameters than those from class. Having so many observations, you are going to learn the model parameters.

   Implement and run the Baum-Welch algorithm using the forward and backward algorithms that you already implemented for Pb 1. You can initialize the π, a, b with your guess, or with some random probabilities (make sure that π sums to 1 and that a_{ij}, b_k^i sum to 1 for each i). The algorithm converges quite slowly, so you might need to run it for 100 iterations or more for the parameters to converge.

   Report the values of π, a, b that you have obtained. (4 points)