Understanding the interplay between third-generation cryptocurrencies and stock portfolio returns using Monte Carlo simulations

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1 Introduction

Investment in stocks allows people to build their wealth. However, this type of investment does not come without risks. To mitigate these risks, it is common to invest in a diversified set of assets as a portfolio.

Prior studies on portfolio diversification have concluded that diversification depends on the type of asset[**1**]. Specifically, cryptocurrency have garnered interest when hedging portfolios. One study in 2019 looked into using cryptocurrency to diversify assets and increase portfolio returns. This paper reported that cryptocurrencies can increase returns and diversity for portfolios consisting of the top and bottom 10 stocks from varying S&P indices[**2**]. With the high volatility associated in cryptocurrency market (i.e. recent Black Swan effect) and the rise of third-generation cryptocurrencies, both vital components, yet missing from the paper's methodology and considerations, gives motivation to this case study to understand the hedging potential of third-generation cryptocurrencies.

In this paper we will be comparing the effects of adding third-generation cryptocurrencies and traditional assets (commodities) to a portfolio. For each set of Portfolios we first find the optimum weights of investments, then use a Monte Carlo to simulate 6 months of stock trading. We then judge these Portfolios on their return, risk and diversity, through the metrics of Return on Investment, Sharpe Ratio and Generalized Herfindahl-Hirschman Index. Using these metrics, we can conclude whether third-generation cryptocurrencies can be competitive hedging assets like traditional commodities.

2 Technical Approach and Methods

2.1 Portfolio

To understand this goal, ten of the top and bottom stocks are taken from the S&P500 index. The stocks are then hedged with either commonly traded third-generation cryptocurrencies and commodities. Stocks, cryptocurrencies and commodities of interest are as follows:

Top 10 Stocks	Bottom 10 Stocks	Cryptocurrencies	Commodities
Apple (AAPL)	Leggett and Platt (LEG)	Cardano (ADA)	Crude Oil (CL)
Microsoft (MSFT)	Ralph Lauren (RL)	Stellar Lumens (XLM)	Natural gas(NG)
Amazon (AMZN)	Hanesbrand (HBI)	Solana (SOL)	Gold (GC)
Tesla (TSLA)	IPG Photonics (IPGP)	Polkadot (DOT)	Silver (SI)
Google (GOOGL)	Fox Corporation (FOX)	Avalanche (AVAX)	Live cattle (LE)
Facebook (FB)	Gap (GAP)	Miota (IOTA)	Corn (ZC)
Nvidia (NVDA)	Under Armour A (UAA)	Cosmos (ATOM)	Coffee (KC)
Berkshire Hathaway (BRK)	Armour C (UA)	EOS (EOS)	Sugar (SB)
JP Morgan (JPM)	Discovery (DISCA)	Algorand (ALGO)	Copper (HG)
Johnson and Johnson (JNJ)	News Corporation (NWS)	Shiba Inu (SHIB)	Wheat (ZW)

The six portfolios will be composed of:

- Top 10 stocks
- Top 10 stocks with cryptocurrencies
- Top 10 stocks with commodities
- Bottom 10 stocks
- Bottom 10 stocks with cryptocurrencies
- Bottom 10 stocks with commodities

The historical data for each component is accessed with Pandas Yahoo Finance API[**3**] for the past year. Once retrieved, we calculate the mean returns, covariance matrix, and the correlation matrix to use in upcoming Monte Carlo simulations.

2.2 Metrics used to understand portfolio performance

To succinctly understand and compare portfolio performances, we employ three different metrics: Return on Investment (ROI), Sharpe ratio, and the Generalized Herfindahl-Hirschman Index (GHHI). These metrics analyze the returns, risk, and diversity of a portfolio, respectively.

2.2.1 Return on Investment (ROI)

Return on investment is a common and easily interpretive metric to evaluate and compare performance between different portfolios. This is described as the ratio between the net profit and the initial cost then commonly expressed as a percentage.

$$\text{ROI} = \frac{\text{Current value of investment - Initial cost}}{\text{Initial cost}} \times 100$$

2.2.2 Sharpe Ratio

The Sharpe Ratio measures the relationship between the net return and risk.

$$\text{Sharpe Ratio} = \frac{R_p - R_f}{\sigma_p}$$

where R_p is the portfolio returns, R_f is the risk-free rate, and σ_p is the standard deviation of the return on investment.

The risk free rate is the theoretical rate of return on an investment with zero risk. As there is always an inherent risk in any investment, this value must be approximated. A common proxy risk-free rate is the current three-month U.S. Treasury bill (T-bill) rate or government bonds. As of this study, the T-bill rate is 0.06%[**4**], very much close to zero, thus this number will be used when calculating the Sharpe ratio.

With the risk-free rate set to zero, the Sharpe ratio only considers the portfolio returns and the risk (in the form of the standard deviation). When interpreting this ratio, we can expect portfolios with a high Sharpe ratio to yield high returns at a minimal risk.

2.2.3 Generalized Herfindahl-Hirschman Index (GHHI)[**5**]

The GHHI is a measure of how diverse an investment portfolio is by summing the weights of each component in the portfolio.

$$\text{GHHI} = \sum_{i}^{n} c_i^2 + \sum_{i}^{n} \sum_{j \neq 1}^{n} 2 c_i c_j \rho_{ij}$$

where c_i is the corresponding proportions of investment in a stock and ρ is the correlation between assets.

This is an expansion on the Herfindahl-Hirschman Index (HHI). HHI is calculated as the sum of the squares of the proportions invested in each stock. It thereby measures the market concentration of an portfolio. The generalized form adds a term that includes the correlation between assets. This accounts for when assets are highly correlated and thus there is not as much diversity. (Consider that investing in two assets from the same company is not as diverse as two unrelated assets). Overall, a smaller GHHI indicates a higher level of diversification.

2.3 Optimal Weight Distribution

For each of the six portfolio's we first calculated their optimum weight distribution. We consider the optimum weights to be that which produce the Maximum Sharpe ratio. This method uses what is called the Efficient frontier[**6**]. That is, a line of portfolio's for which at the given returns, it has the minimum risk. From this line we then select the portfolio with the maximum Sharpe ratio.

We initially used a Monte Carlo simulation to sample different portfolio weights[**7**][**8**]. For this we randomly sampled the weight distribution. For each random weight we calculated the expected returns, standard deviation and Sharpe ratio. For each of the Portfolio's we ran 10000 iterations, then graphed the results. This method was relatively effective for the control groups that only contained 10 stocks. However, in the groups with 20 stocks the number of iterations was too small to effectively explore the full sample space.

For this reason we switched to using the python library PyPortfoliOpt[**9**] which solves this as a convex optimization problem. To do this we optimize a vector $\mathbf{w}$ as an array of weights corresponding to the portion invested in each stock. The constraints on this is that the weights must sum to 1 and each entry must be non-negative. PyPortfoliOpt then uses another python library CVXPY to solve this problem. From this method we were able to find the portfolio having a absolute maximum Sharpe ratio.

2.4 Monte Carlo Simulation Details

The repeated random sampling commonly attributed to Monte Carlo simulations is not sufficient when predicting financial markets since the correlations and trends between assets is lost. First, we assume that the market behaves in a multi-variate normal distribution. To introduce normally correlated random variables between each asset in the portfolio, the Cholesky decomposition method is implemented. We start with a correlation matrix of the portfolio returns. This correlation matrix Σ is decomposed with the Cholesky decomposition. By doing so, the resulting matrix R will satisfy $\Sigma = RR^*$ where R^* is the transpose matrix of R. Essentially, R^* is the upper triangular matrix and R is the lower triangular matrix. To uncover the correlations in the multi-asset portfolio, R is multiplied by a normal random distributed array[**10**].

The simulation starts by considering an initial investment of $10,000. This money is invested in the different assets according to the weights found as described in section 2.3. For every Monte Carlo run, a new normal random array is multiplied to the R matrix then used to calculate the correlated daily returns. This is repeated for a total of 4000 simulations. We found that 4000 simulations was a good convergence between computational cost and accuracy. From the simulations, we were able to calculate the metrics described in section 2.2.

```
initial_Portfolio = 10000
mc_sims = 4000
for m in range(0, mc_sims):
    Z = np.random.normal(size=(T, len(weights)))
    L = np.linalg.cholesky(covMatrix)
    dailyReturns = meanM + np.inner(L, Z)
    portfolio_sims[:,m] = np.cumprod(np.inner(weights,
        dailyReturns.T)+1)*initialPortfolio
```

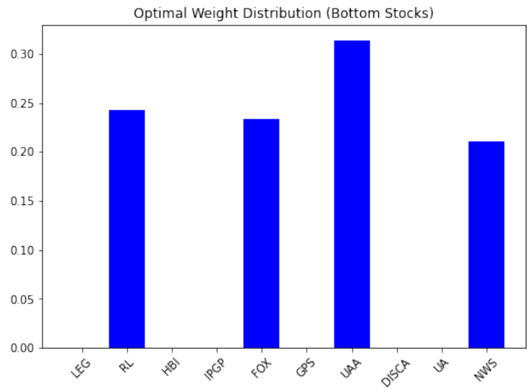
Example code of Monte Carlo simulation

3 Results and Discussion

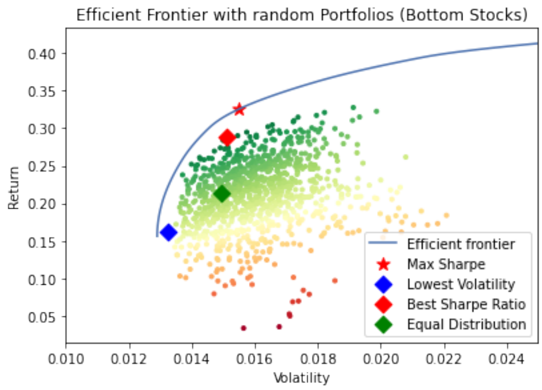
3.1 Portfolio Distribution

We initially used a Monte Carlo simulation to sample to Portfolio space and find the optimal weight distribution. This samples 1,000 portfolios, which are shown on a plot of reward (returns) vs. risk (volatility). The Sharpe ratio is then used as a color gradient. A portfolio with equal weights is marked in green, while the portfolio with the minimum risk is marked in blue and the portfolio with the maximum Sharpe ratio is marked in red.

While this method was reasonable effective, it was not able to find the absolute maximum Sharpe ratio. To find the absolute maximum we used the PyPortfolioOpt[**9**] to plot the Efficient frontier. This is represented by the blue line in Figures 1-6 (b) with the maximum Sharpe ratio marked with a red star. The distance from this optimal portfolio and the one found by the Monte Carlo simulation is show by the difference in position between the red diamond and the red star. For the control groups and the commodities these marks are close, but the optimal portfolio has a higher return. For the cryptocurrencies the convex optimization was able to a higher Sharpe ratio by significantly decreasing the volatility.

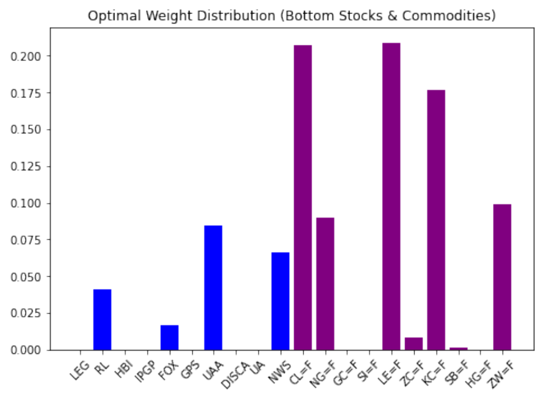


(a) Optimal weight distribution

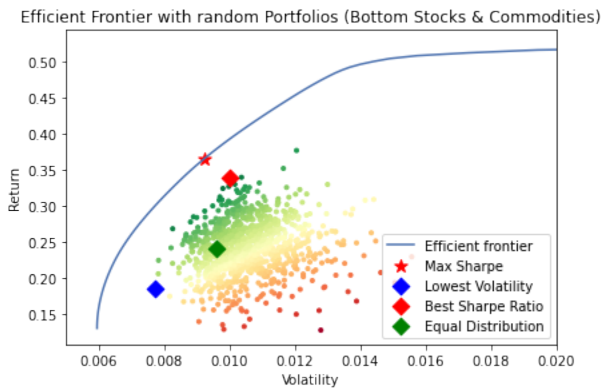


(b) Efficient frontier

Figure 1: Bottom stock portfolio



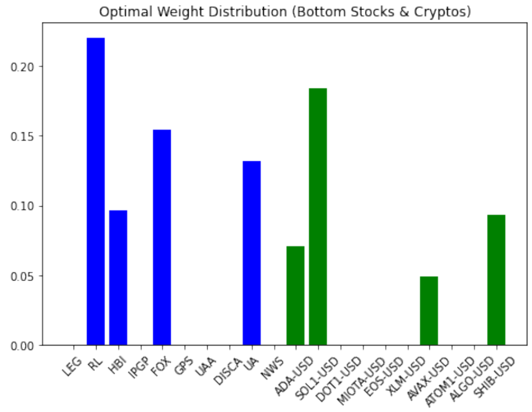
(a) Optimal weight distribution



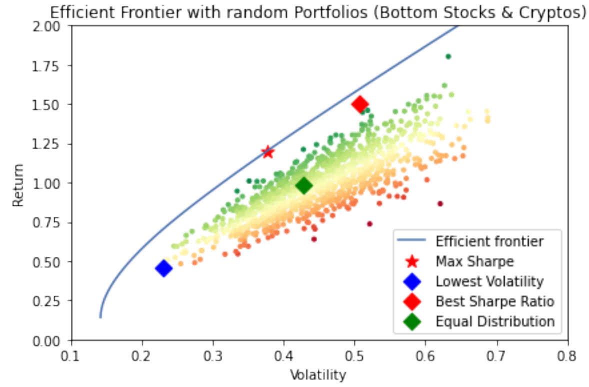
(b) Efficient frontier

Figure 2: Bottom stock portfolio with commodities

Using the optimal weight vector $\mathbf{w}$ found by the convex optimization we were able to calculate the GHHI using correlation matrix. The results for this as well as the projected Sharpe ratio from the Efficient frontier are shown below.

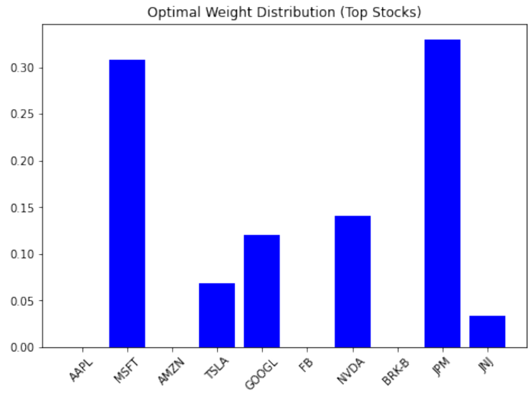


(a) Optimal weight distribution



(b) Efficient frontier

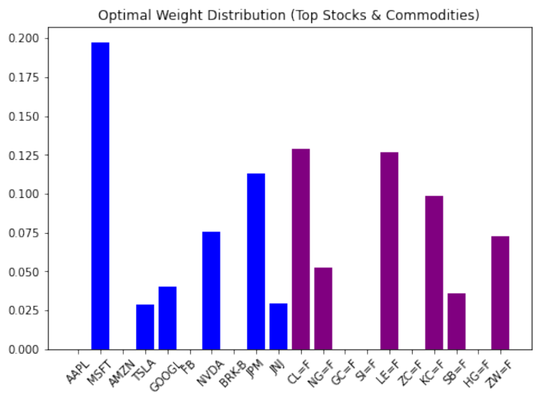
Figure 3: Bottom stock portfolio with cryptocurrencies



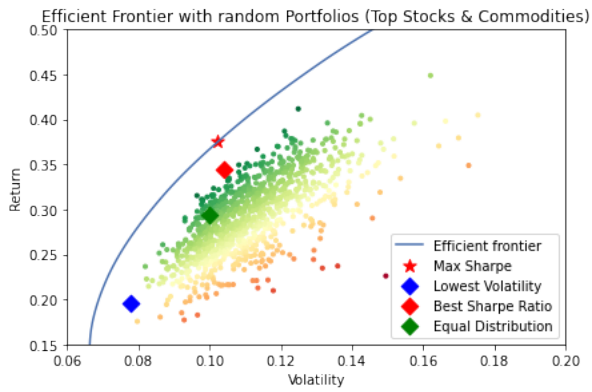
(a) Optimal weight distribution

(b) Efficient frontier

Figure 4: Top stock portfolio

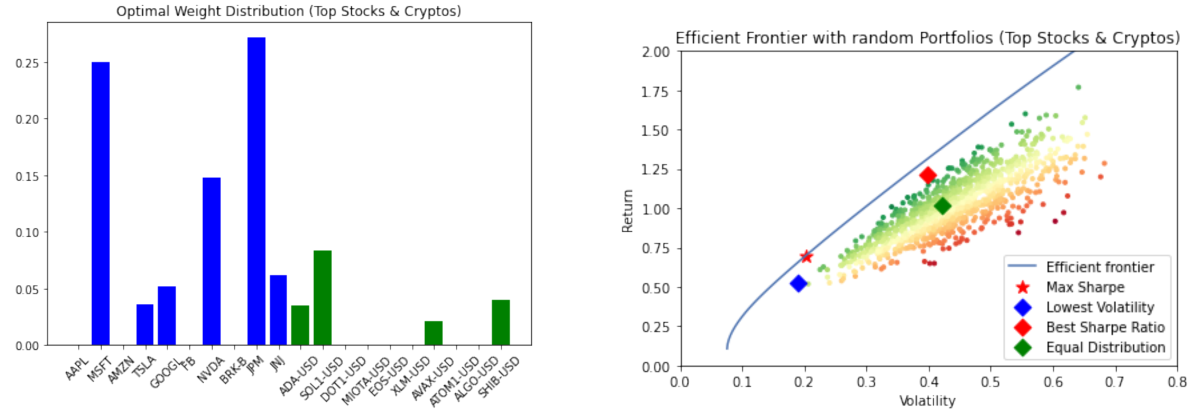


(a) Optimal weight distribution



(b) Efficient frontier

Figure 5: Top stock portfolio with commodities



(a) Optimal weight distribution

(b) Efficient frontier

Figure 6: Top stock portfolio with cryptocurrencies

	GHHI	Projected Sharpe Ratio
Bottom Stocks	1.7448	1.5656
Bottom Stocks w/ Commodities	1.8520	2.9569
Bottom Stocks w/ CryptoCurrencies	1.8508	3.1843
Top Stocks	1.7608	2.8075
Top Stocks w/ Commodities	1.88933	3.6828
Top Stocks w/ CryptoCurrencies	1.82533	3.4537

Table 2: GHHI and projected Sharpe Ratio of each investment portfolio.

Within the bottom stocks the control distribution included four stocks that were approximately equally weighted. Once commodities are added they contain have significantly more of the weights than the S&P stocks in the portfolio. This is in contrast to the cryptocurrencies which are more in balance with the stocks from the S&P. While we may prefer the more equal distribution shown by CryptoCurrencies the GHHI is essentially the same as for Commodities. This indicates that both distributions are avoiding correlations to the same extent.

Within the top stocks the control distribution includes six of the stocks. While this has a higher number of stocks than the Bottom S&P, they are not as evenly invested. This balance causes it's similar GHHI number to the Bottom control group. Once Commodities and cryptocurrenencies are added they make up less weight in the portfolio than they did when with the bottom stocks. While they still do improve the portfolio by increasing the Sharpe ratio it is less dramatically. This indicates that both commodities and cryptocurrencies do diversify a portfolio, but they add more to a portfolio when the stocks are not performing as well.

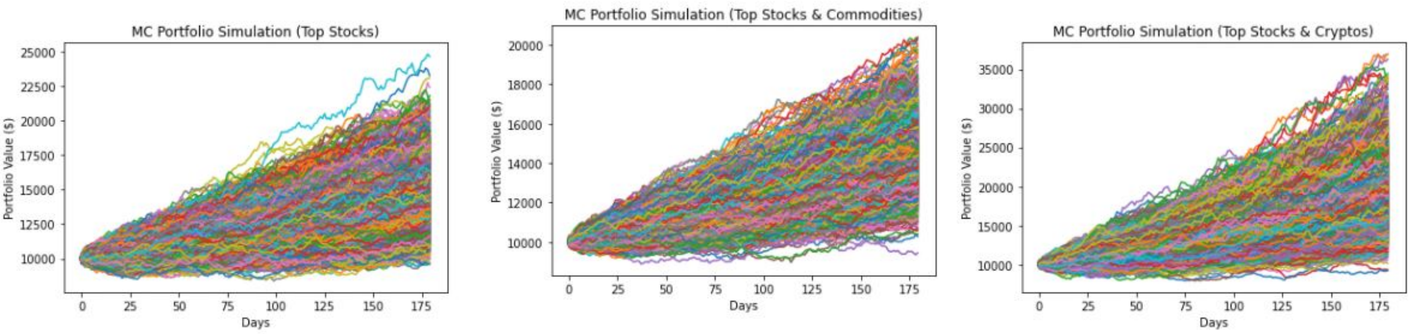
3.2 Monte Carlo Simulation Results

From 4000 Monte Carlo simulation runs over a period of six months, the portfolio consisting of top stocks and cryptocurrencies have the greatest ROI out of all the portfolios. Considering the contrast between portfolios with cryptocurrencies and commodities, it can be seen in Table 3 that cryptocurrencies hedge portfolios better than commodities when comparing the return on investment: 0.9617 and 0.4493, respectively. A general trend prevailing from our data shows that cryptocurrencies yield higher ROIs.

In contrast, the Sharpe Ratio between the two shows another element to investing to consider. The Sharpe Ratio of the portfolio with top stocks with commodities is greater than top stocks with cryptocurrency. There are varying angles to look at this result. First, investing in the cryptocurrency market, at least during the Black Swan effect (which was when this historical data range was taken from), carries great risk. Within the past year, the fervor around cryptocurrency has grown beyond the means of cryptography and permeating into government affairs, art (in the form of non-fungible tokens (NFTs)), and social media presences, affecting market prices and demand. This brings noticeable volatility into the market, especially since this asset is currently less orthodox than conventional investing mediums. Second, some of the commodities used such as gold, silver, copper, sugar, corn, and livestock serve as safe haven investments in times of market turbulence. Thus the higher Sharpe Ratio from the portfolio with commodities stem from their consistent, dependable nature compared to cryptocurrencies.

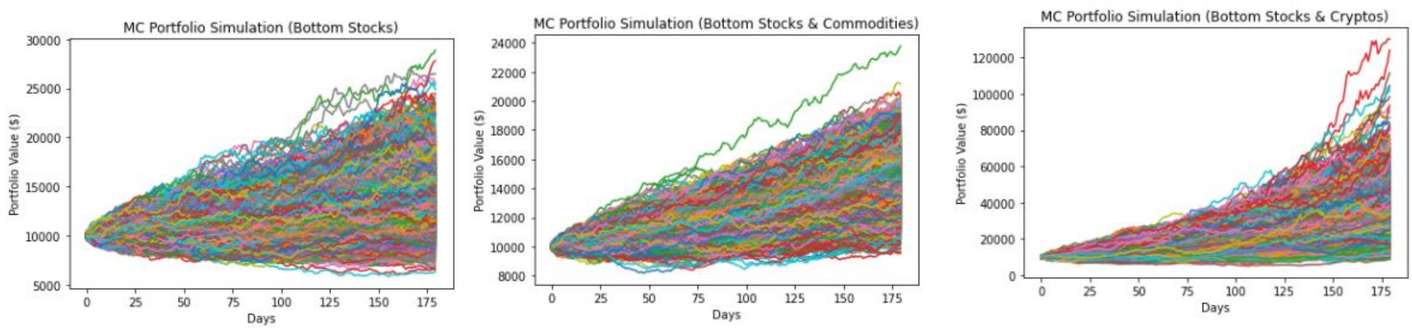
Previously mentioned in section 3.1, we observed top stock portfolios to primarily consist of top stocks while bottom stock portfolios primarily consist of hedging assets (if applicable) after calculating their optimal weights. Thus, we can expect bottom stock portfolios to perform similarly compared to top stock portfolios. Shown in Table 3, bottom and top stocks with commodities, the ROIs are almost equivalent. The Sharpe Ratio of the top stocks with commodities portfolio is greater than bottom stocks, understandably, due to top stocks yielding higher returns despite both hedged with commodities.

The bottom and top stocks with cryptocurrencies performed differently. When looking at the optimal weight distribution between the two portfolios, the bottom stock portfolio heavily relies on more cryptocurrency weights than the top stock portfolio. With more cryptocurrencies comes greater instability and potentially greater net returns. This is reflected in in practice. The bottom stocks with cryptocurrencies has a greater ROI and a lesser Sharpe Ratio than top stocks with cryptocurrencies.



(a) Stocks (b) Stocks with commodities (c) Stocks with cryptocurrencies

Figure 7: Monte Carlo results of top stock portfolios



(a) Stocks (b) Stocks with commodities (c) Stocks with cryptocurrencies

Figure 8: Monte Carlo results of bottom stock portfolios

	ROI (x100)	Standard deviation	Sharpe Ratio
Bottom Stocks	0.3777	0.2859	1.3211
Bottom Stocks w/ Commodities	0.4349	0.1741	2.483
Bottom Stocks w/ CryptoCurrencies	2.2518	1.2622	1.784
Top Stocks	0.4721	0.2002	3.3581
Top Stocks w/ Commodities	0.4493	0.1451	3.0945
Top Stocks w/ CryptoCurrencies	0.9617	0.3870	2.4849

Table 3: Returns, standard deviation, and Sharpe Ratio of each investment portfolio

4 Conclusions

In this study, we analyze the hedging effect of third-generation cryptocurrencies and traditional assets (commodities) to top and bottom stock portfolios, specifically when increasing asset diversity and portfolio returns while decreasing risk. This is motivated by a previous study where only the top cryptocurrencies were used to improve top and bottom stock portfolios and current understanding of potential promising third-generation cryptocurrencies as assets are limited.

When optimizing the individual weights in each portfolio, we noticed that top stock portfolios relied on more stocks than assets to maximize profit and reduce risk. Bottom stock portfolios mostly consisted of assets like commodities and cryptocurrency whenever possible and dependent on assets to be as competitive as top stock portfolios.

The results suggest that commodities and third-generation cryptocurrencies can increase diversification of a stock portfolio than a stock portfolio without any financial assets regardless if the stocks were taken from the top or bottom of S&P index. In terms of return on investment, one can expect greater returns when diversifying traditional stock portfolios with cryptocurrency than commodities. However, as expected, there is a greater risk when investing in cryptocurrency than commodities.

Following this case study, future work of interest may lie with exploring influence of political, cultural, and social phenomena on cryptocurrency market price. With the ever evolving landscape in financial markets, broader understanding of how external forces affect cryptocurrency can improve how we implement and control this potentially powerful hedging asset to be more advantageous and less risky.

5 Author Contributions

EA: Data retrieval, portfolio optimization, efficient frontier, simulations (code), metrics
CW: Project conception, literature review, status report slides, portfolios of interest, metrics, MC simulation results, conclusion

6 Acknowledgments

N/A

7 Code

A link to our repository: https://github.com/evana2/AtomicSimulationProject

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