M348-53610: Scientific Computing
Homework # 07
Handout: 03/08/2016, Tuesday
Due: 03/22/2016, Tuesday

Submission. Please make your homework neat and stapled. You have to submit your homework in ECJ 1.204 before **3:00 PM** on the due date. Note that *no late homework will be accepted without compelling reasons.*

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1 To be Graded

Problem 1. Derive a method for approximating $f'''(x)$ whose error term is of order h^2 by expanding the function f in a fourth Taylor polynomial about x and evaluating at $x = \pm h$ and $x = \pm 2h$.

Problem 2. Derive an $O(h^4)$ five-point formula to approximate $f'(x)$ that uses $f(x-h)$, $f(x)$, $f(x+h)$, $f(x+2h)$, and $f(x+3h)$. [*Hint*: Consider the expression $Af(x-h)+Bf(x+h)+Cf(x+2h)+Df(x+3h)$. Expand in fourth Taylor polynomials, and choose A, B, C, and D appropriately.]

Problem 3. The values for $f(x) = \tan(x)$ are given below for 6 different values of x. Use all the applicable formulas, including at least (a) Forward-Difference, (b) Backward-Difference, (c) two Three-Point formulas and (d) a Five-Point Formula, to approximate $f'(2.4)$.

2.1	2.2	2.3	2.4	2.5	2.6
-1.70985	-1.37382	-1.11921	-0.91601	-0.74702	-0.60160

Problem 4. The forward-difference formula can be expressed as

$$f'(x) = \frac{1}{h}[f(x+h) - f(x)] - \frac{h}{2}f''(x) - \frac{h^2}{6}f'''(x) + O(h^3).$$

Use extrapolation to derive an $O(h^3)$ formula for $f'(x)$.

Problem 5. Suppose that $N(h)$ is an approximation to M for every $h > 0$ and that

$$M = N(h) + K_1 h + K_2 h^2 + K_3 h^3 + \cdots,$$

for some constants K_1, K_2, K_3, $\cdots$. Use the values $N(h)$, $N(\frac{h}{3})$, and $N(\frac{h}{9})$ to produce an $O(h^3)$ approximation to M.

Problem 6. Let $f(x) = e^x$, $x_0 = 1.0$, and $h = 0.1$.
(a) Use the forward-difference scheme and the Richardson's extrapolation method to get a second order approximation to $f'(1)$.
(b) Use the backward-difference scheme and the Richardson's extrapolation method to get a second order approximation to $f'(1)$.

2 Reading Assignments

- Review Sections 4.1, 4.2 and 4.3 of Burden & Faires or Sections 2.2, 5.7 and 5.2 of Epperson.