Evan Johnston - eaj628
STA 372-6: Dr. Muthraman
Homework 3
30 September 2015

1) Max $[-x_1 + 4x_2]$ s.t. ① $-10x_1 + 20x_2 \leq 22$

② $5x_1 + 10x_2 \leq 49$

③ $x_1 \leq 5$

④ $x_i \in \mathbb{Z}$, $x_i \geq 0 \ \forall i = \{1, 2\}$

[LP_0: $x_1 = 3.8$, $x_2 = 3.0$, $z = 8.2$]

$x_1 \leq 3$ | $x_1 \geq 4$

[LP_1: $x_1 = 3.0$, $x_2 = 2.6$, $z = 7.4$]   [LP_2: $x_1 = 4.0$, $x_2 = 2.9$, $z = 7.6$]

$x_2 \leq 2$ | $x_2 \geq 3$ $x_2 \leq 2$ | $x_2 \geq 3$

[LP_3: $x_1 = 1.8$, $x_2 = 2.0$, $z = 6.2$]   [LP_4: $\emptyset$]   [LP_5: $x_1 = 4.0$, $x_2 = 2.0$, $z = 4.0$]   [LP_6: $\emptyset$]

lower bound

$x_1 \leq 1$ | $x_1 \geq 2$

[LP_7: $x_1 = 1$, $x_2 = 1.6$, $z = 5.4$]   [LP_8: $x_1 = 2$, $x_2 = 2$, $z = 6$]

lower bound, → optimal bundle with $x_1 = 2$, $x_2 = 2$, $z = 6$

	cost	profit
2.) fact A	6M	9M
fact D	3M	5M
Ware A	5M	6M
Ware D	2M	4M

Total available = 11M

logical constraints: ① only 1 factory or warehouse can be built in A or D

② only 1 warehouse in A or D

③ At least 1 factory should be built

let x_1 = fact A, x_2 = fact D, x_3 = ware A, x_4 = ware D

maximize $[3x_1 + 2x_2 + x_3 + 2x_4]$

subject to: $6x_1 + 3x_2 + 5x_3 + 2x_4 \leq 11$

$x_1 + x_3 \leq 1$, $x_2 + x_4 \leq 1$ — constraints on space (1)

$x_1 + x_2 \geq 1$ — 1 factory must exist (3)

$x_3, x_4 \leq 1$ — only 1 warehouse can exist in A or D (2)

$x_i \in \mathbb{Z}, x_i \geq 0 \ \forall i = \{1, \ldots, 4\}$

Solution: $[1, 0, 0, 1] \to 5$

$\Rightarrow$ 1 factory in Austin $\to -6 + 9 = 3$

1 warehouse in Dallas $\to -2 + 4 = 2$

$5 million

	Sn	M	Tu	W	Th	F	Sa
cost	90			60			90
3.) req.	5	13	12	10	14	8	6

workers per schedule $x_i \ \forall i \in \{1, \dots, 7\}$

number of workers per day $y_i \ \forall i$

① choose $x_i \ \forall i$

② to minimize:

$$\sum_{i=1}^{7} x_i \text{cost}_i$$

note: y_i exists for simplicity, it is not a decision variable

③ constrained to:

$y_i \geq \text{req}_i \ \forall i$

$x_i, y_i \in \mathbb{Z}$ and $x_i, y_i \geq 0$

total cost	day / var	Sn (90)	M (60)	Tu (60)	W (60)	Th (60)	F (60)	Sa (90)
330	x_1	0	0	1	1	1	1	1
360	x_2	1	0	0	1	1	1	1
	x_3	1	1	0	0	1	1	1
	x_4	1	1	1	0	0	1	1
	x_5	1	1	1	1	0	0	1
330	x_6	1	1	1	1	1	0	0
300	x_7	0	1	1	1	1	1	0
		y_1	y_2	y_3	y_4	y_5	y_6	y_7

let $y_1 = x_2 + x_3 + x_4 + x_5 + x_6$

$y_2 = x_3 + x_4 + x_5 + x_6 + x_7$

$\vdots$

$y_7 = x_1 + x_2 + x_3 + x_4 + x_5$

Solution: $x_1 = 2$

$x_2 = 0$

$x_3 = 3$

$x_4 = 0$

$x_5 = 1$

$x_6 = 1$

$x_7 = 8$ → most popular, also the cheapest schedule

$+$ ———

15

⇒ min. cost of \$4830 and 15 employees needed