M348-53610: Scientific Computing
Homework # 08
Handout: 03/22/2016, Tuesday
Due: 03/29/2016, Tuesday

Submission. Please make your homework neat and stapled. You have to submit your homework in ECJ 1.204 before **3:00 PM** on the due date. Note that *no late homework will be accepted without compelling reasons.*

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1 To be Graded

Problem 1. Approximate the following integrals using (i) the Trapezoidal rule and (ii) Simpson's rule.

$$(a) \int_{0.5}^{1} x^4 dx, \qquad (b) \int_{1}^{1.6} \frac{2x}{x^2-4} dx$$

Problem 2. The *degree of accuracy*, or *precision*, of a quadrature formula is the largest positive integer n such that the formula is exact for x^k, for each $k = 0, 1, \cdots, n$. For instance, we showed in class that the Trapezoidal rule has degree of precision one. Prove that Simpson's rule has precision of degree three (which is actually greater than two!).

Problem 3. The quadrature formula $\int_{-1}^{1} f(x)dx = c_0 f(-1) + c_1 f(0) + c_2 f(1)$ is exact for all polynomials of degree less than or equal to 2. Determine c_0, c_1 and c_2.

Problem 4 (Programming Assignment)**.** A full description of the composite Simpson's rule is provided in Algorithm 4.1 of Burden and Faires (also attached here in case you do not have the textbook).
(a) Implement the composite Simpson's rule in Matlab, C++ or any other program language that you prefer. Include the full program in your submission.
(b) Implement the composite Trapezoidal rule in Matlab, C++ or any other program language that you prefer. Include the full program in your submission.
(c) Use the (i) the composite Simpson's rule and (ii) the composite Trapezoidal rule codes to evaluate the integral $\displaystyle\int_0^2 e^{2x}\sin(3x)dx$ with $n = 10$, $n = 100$ and $n = 1000$ intervals for $[0, 2]$.

Problem 5. Use Romberg integration to compute $R_{3,3}$ for the following integrals.

$$(a) \int_{1}^{1.5} x^2 \ln x dx, \qquad (b) \int_{0}^{1} x^2 e^{-x} dx$$

Problem 6 (Programming Assignment)**.** A full description of the Romberg integration method is provided in Algorithm 4.2 of Burden and Faires (also attached here in case you do not have the textbook).

(a) Implement the Romberg integration method in Matlab, C++ or any other program language that you prefer. Include the full program in your submission.

(b) Use the code developed in (a) to approximate $\int_{-1}^{1} (\cos x)^2 dx$ to within 10^{-6} by computing the Romberg table until $|R_{n-1,n-1} - R_{n,n}| < 10^{-6}$.

2 Reading Assignments

- Review Sections 4.3, 4.4 and 4.5 of Burden & Faires or Sections 5.2, 5.3 and 5.8 of Epperson.