STA 372-6 Project 5: Dynamic Programming Game

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1. **<u>States</u>:**
   This problem must have three states for the exact position in the game to be located. The three states are represented by:

   i: our accumulated score
   j: opponent's accumulated total
   k: current roll total

2. **<u>Actions</u>:**
   The actions are whether to roll or hold the dice; the computer will always choose the option that maximizes the probability of winning.

3. **<u>Objective:</u>**
   Maximize your (player *i* 's) probability of winning (reaching 100 first) based on your score, your opponent's score, and the current roll total.

4. **<u>Bellman Equation:</u>**

$$V(i,j,k) = \max\{\mathbf{roll}, \mathbf{hold}\}$$

$$\mathbf{roll} = p_1(1 - V(j, i+1, 0) + p_r\big(V(i,j,k+r)\big);\ \forall\, r\ \in \{2,3,4,5,6\} \text{ and } p_1 = p_r = \tfrac{1}{6}$$

$$\mathbf{hold} = \begin{cases} 1 - V(j, i+1, 0) & \text{if } k = 0 \\ 1 - V(j, i+k, 0) & \text{if } k > 0 \end{cases}$$

5. **<u>Intuition:</u>**

   a. **<u>Roll:</u>** The expected value of rolling with an unweighted die is 1/6 probability of rolling a 1 and 1/6 probability for each possible value of rolling *not* 1 (2, 3, 4, 5, 6). Rolling a 1 also ends your turn, which is shown in the matrix by switching from *i* to *j* and adding the 1 to your current score; that is the point of view switches between player and opponent. The value of *k* is set to zero as there is no total turn value at the beginning of a new turn.

   b. **<u>Hold:</u>** If *k* is zero then the player holds without rolling and adds 1 to their score before switching players. Otherwise, player *i* adds *k* to their score before ending their turn.

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c. **Terminal Conditions:**

  i. If i is 99 or greater then player i has won as both rolling and holding will cause them to win (100); thus the probability of winning is 1.
  ii. If j is 100 or greater then player j has won the previous turn and the probability of winning is 0.
  iii. If k is 100 or greater then player i will hold and win. The probability of winning is 1.
  iv. If k is at least as large as the difference between player i's current score and 100 then the player holds and wins (this supersedes terminal condition 3, but both are left for clarity) and the probability of winning is 1.

6. **Extra Credit:**

As the probability of rolling a 1 increases then the player will hold more often to reduce their chance of losing the accumulated score of that turn. Similarly if higher probabilities were given to higher roll values then the player would roll more and hold less. This could be simulated by making each p_i a random variable probability (that is each $p_i \geq 0$ and sum to 1) and recalculating the decisions matrices based on this new probability distribution.

A player could approximate the non-normal probability distribution by keeping track of all rolls and recalculating the distribution after each roll. This would then lead to a new decision matrix being recalculated after every roll to determine the most likely way to win according to the new estimated distribution.