FIN 372 / STA 372

Optimization Methods in Finance: Homework 4

This assignment is graded on Credit/No-Credit.

That is, if you complete the homework, and it is acceptable, you will get credit. If you do not submit or if the submitted work is not acceptable, you will not get credit. Getting a credit is required to obtain a grade for the group project that follows.

Please write a report that solves the following problems. Make sure that your report includes the mathematical formation of the optimization problem.

Problem 1:

It costs a company $12 to purchase an hour of labor and $15 to purchase an hour of capital. If L hours of labor and K units of capital are available, then $0.05L^{2/3}K^{1/3}$ machines can be produced. Suppose the company has $100,000 to purchase labor and capital.

What is the maximum number of machines it can produce?

Problem 2:

The file homework4stocks.csv contains historical monthly returns for 27 companies. The first row contains stock names, and the first column contains the dates. For each company, calculate the estimated mean return and the estimated variance of return. Then calculate the estimated correlations between the companies' returns.

Find a portfolio that achieves an expected monthly return of at least 1% and minimizes portfolio variance. What are the fractions invested in each stock? What are the portfolio's estimated mean, variance, and standard deviation?

Problem 3:

The file nflratings.csv contains the results of 256 regular-season NFL games from the 2009 season. The teams are indexed 1 to 32 as shown below:

Index	Team Name	Index	Team Name	Index	Team Name	Index	Team Name
1	Arizona Cardinals	9	Dallas Cowboys	17	Miami Dolphins	25	Pittsburgh Steelers
2	Atlanta Falcons	10	Denver Broncos	18	Minnesota Vikings	26	St. Louis Rams
3	Baltimore Ravens	11	Detroit Lions	19	New England Patriots	27	San Diego Chargers
4	Buffalo Bills	12	Green Bay Packers	20	New Orleans	28	San Francisco 49ers
5	Carolina Panthers	13	Houston Texans	21	New York Giants	29	Seattle Seahawks
6	Chicago Bears	14	Indianapolis Colts	22	New York Jets	30	Tampa Bay Buccaneers
7	Cincinnati Bengals	15	Jacksonville Jaguars	23	Oakland Raiders	31	Tennessee Titans
8	Cleveland Browns	16	Kansas City Chiefs	24	Philadelphia Eagles	32	Washington Redskins

The csv data file contains a matrix with the following columns:

- Week (1-17)

- Home Team Index (1-32 from the table above)
- Visiting Team Index (1-32 from the table above)
- Home Team Score
- Visiting Team Score

For example, the first game in the matrix is team 25 Pittsburgh versus team 31 Tennessee, played at Pittsburgh. Pittsburgh won the game by a score of 13 to 10, and the point spread (home team score minus visitor team score) is 3. A positive point spread means that the home team won; a negative point spread indicates that the visiting team won. The goal is to determine a set of ratings for the 32 NFL teams that most accurately predicts the actual outcomes of the games played. Use NLP to find the ratings that best predict the actual point spreads observed. The model will estimate the home team advantage and the ratings. The objective is to minimize the sum of squared prediction errors.

You will need to calculate the following:

- Actual Point Spread = Home Team Score – Visiting Team Score
- Predicted Spread = Home Team Rating – Visitor Team Rating + Home Team Advantage
- Prediction error = Actual Point Spread – Predicted Point Spread

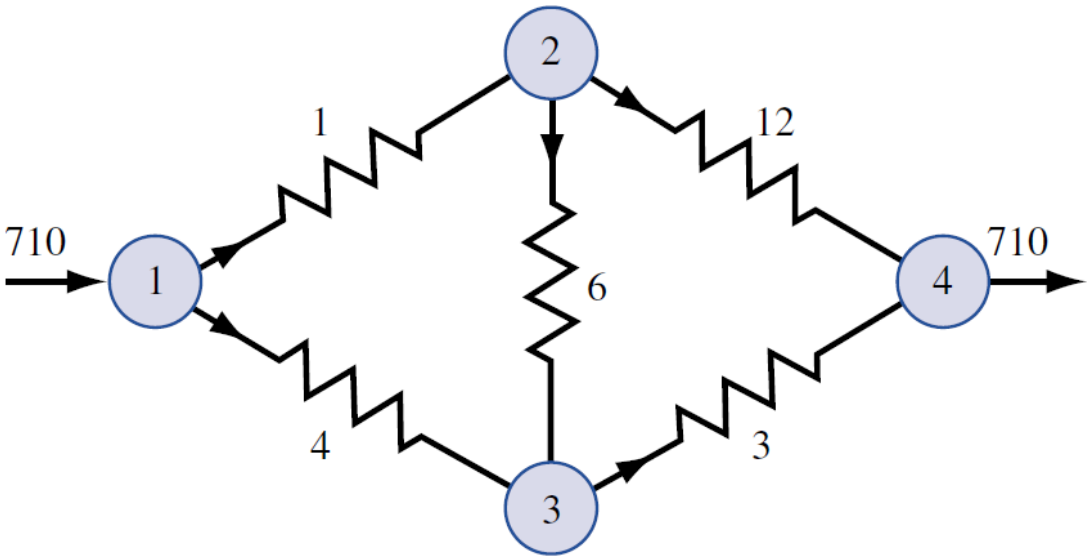
You will also need to normalize the ratings. To do this, you set the actual average of the ratings to be 85 (this is somewhat arbitrary but based on the well-known Sagarin rating system).

What do these ratings mean? If two teams had ratings of 82 and 91, then the second team would be predicted to win by 9 points if the game was played on a neutral field.

Problem 4 (Optional):

In an electrical network, the power loss incurred when a current of I amperes flows through a resistance of R ohms is I^2R watts. In the figure below, 710 amperes of current must be sent from node 1 to node 4. The current flowing through each node must satisfy conservation of flow. For example, for node 1, 710 = flow through 1-ohm resistor + flow through the 4-ohm resistor. Remarkably, nature determines the current flow through each resistor by minimizing the total power loss in the network.

1. Formulate a quadratic programming problem whose solution will yield the current flowing through each resistor.
2. Use R to determine the current flowing through each resistor.



Problem 5 (Optional):

The file 'variable_selection.csv' contains observations of variables y, x_1, x_2, and x_3. Here, y is the dependent variable. We want to choose a linear model that uses

at most two independent variables such that the sum of squared residuals is minimized. This can be formulated as a constrained quadratic programming problem.

$$\min \sum_{i=1}^{n} (y_i - x_1\beta_1 - x_2\beta_2 - x_3\beta_3)^2$$

$$Subject\ to: at\ most\ two\ of\ \beta_1, \beta_2, and\ \beta_3\ are\ nonzero$$

This is called best subset problem that is usually very hard to solve. We will solve this problem by enumeration. Run six OLS regressions (3 with one independent variable and three more with two variables each) and choose the regression that best fits the data. When we learn how to solve QPs with integer constraints you can revisit this problem and solve it as a QP.

Deliverables

You can either hand write or type your report, but make sure that you submit a PDF file. Please name your report as hw4_x.pdf(where x is your eid).

Please submit your code (.R) file online too.