FIN 372 / STA 372

Optimization Method in Finance: Project 3

The Portfolio Optimization Problem

An efficient portfolio can be constructed by minimizing the portfolio's variance (a measure of risk) for a given expected return (a measure of reward). The following formulates this:

$$\begin{aligned} \min \quad & x'Cx \\ \text{s.t.} \quad & m'x = R \\ & \sum_{i}^{n} x_i = 1 \\ & x_i \geq 0 \ \forall \ i \end{aligned} \tag{1}$$

In the above formulation, C is the covariance matrix, x is the proportion of funds invested in each of the stocks, R is some level of required expected return, and m is a vector of returns. Finally, we also have the non-negativity constraints that prohibit shorting of any stock. Plotting the standard deviation (square root of the objective above) against various values of R is termed the efficient frontier. See the class notes for more information.

Several issues arise from this formulation, however. One of the primary disadvantages is that we cannot factor in beliefs that we hold for the future into our formulation.

Black-Litterman Model

The Black-Litterman (BL) model aims to combine the investor's views and beliefs about the returns into the problem. As in the example discussed in class, to incorporate beliefs (or views) into the model, we adjust our expectation of next year's returns using the following equation

$$\widehat{m} = [(\tau C)^{-1} + P^T \Omega^{-1} P]^{-1} [(\tau C)^{-1} m + P^T \Omega^{-1} q] \tag{2}$$

where,

τ	Scalar representing a small constant
C	Covariance matrix of excess returns
P, q	Matrix P and vector q summarize your beliefs on the expected return as Pm=q
Ω	A diagonal matrix with the diagonal values standing to represent confidence levels. Smaller the value, stronger your view.
m	Historical average returns

The Assignment

We will be constructing a portfolio containing eight asset classes – essentially trying to decide what fraction of our money should go into each asset class. We will plot the efficient frontiers for both the simple mean-variance portfolio optimization problem and the BL model. We consider the following asset classes and historical returns:

Asset Class	**m**
US Bonds	0.08%
Int'l Bonds	0.67%
US Large Growth	6.41%
US Large Value	4.08%
US Small Growth	7.43%
US Small Value	3.70%
Int'l Dev. Equity	4.80%
Int'l Emerg. Equity	6.60%

Table 5 Covariance Matrix of Excess Returns (Σ)

Asset Class	US Bonds	Int'l Bonds	US Large Growth	US Large Value	US Small Growth	US Small Value	Int'l Dev. Equity	Int'l. Emerg. Equity
US Bonds	0.001005	0.001328	-0.000579	-0.000675	0.000121	0.000128	-0.000445	-0.000437
Int'l Bonds	0.001328	0.007277	-0.001307	-0.000610	-0.002237	-0.000989	0.001442	-0.001535
US Large Growth	-0.000579	-0.001307	0.059852	0.027588	0.063497	0.023036	0.032967	0.048039
US Large Value	-0.000675	-0.000610	0.027588	0.029609	0.026572	0.021465	0.020697	0.029854
US Small Growth	0.000121	-0.002237	0.063497	0.026572	0.102488	0.042744	0.039943	0.065994
US Small Value	0.000128	-0.000989	0.023036	0.021465	0.042744	0.032056	0.019881	0.032235
Int'l Dev. Equity	-0.000445	0.001442	0.032967	0.020697	0.039943	0.019881	0.028355	0.035064
Int'l Emerg. Equity	-0.000437	-0.001535	0.048039	0.029854	0.065994	0.032235	0.035064	0.079958

First, use the above data to formulate and solve (1) repeatedly for various values of R in the appropriate range (20 different values in the range will suffice). Using the solutions, plot the efficient frontier and the fraction of wealth in each asset for various values of R. The covariance matrix C is provided in Table 5 and in a csv file called C_Mat.csv on Canvas.

For the BL model, investors are not required to specify views on all assets. In the case of 8 assets, you could have just 3 views in which case the P matrix would be 3 x 8 and the q vector would be 3 x 1.In other words, for each view you will have a 1 x n row vector in P (where n is the number of assets). Views can be absolute or relative. An example of an absolute view is one in which the investor believes the return of an asset class will be > 5%. A relative view on the other might compare two asset classes (e.g. class 1 will outperform class 2 by 7%). See the in class examples.

For this assignment, consider the following views:

1. You have the view that US Small Cap Value will earn a premium of 6.10%
2. You have the view that International Emerging Markets will outperform International Developed Equity by 3.00%
3. You have the view that US Small Cap Growth will outperform US Large Cap Growth by 0.20%.

We also use the following assumptions for the Black-Litterman model:

- Set $\tau = 1$
- Your confidence levels for the three views are summarized using the diagonal elements of matrix Ω. Use .000801, .009546, and .000884 as the corresponding diagonal values for views 1, 2 and 3 above. The rest of the elements of matrix Ω are zeros.

Also, note that we will assume there is no shorting allowed.

We will plot the efficient frontier and the fractions in each asset for the BL model, too.

Specifics

1. Solve formulation (1) in R (This is the software, not the return ☺.) and report the amount invested in each class using the data given in the project. Repeatedly solve for various values of R (return) in the appropriate range (20 different values in the range will suffice). Plot the mean-variance frontier. Also, plot one visualization of how the fraction in each asset changes with respect to R.

2. Solve the formulation again using the Black-Litterman model (2). The following steps should serve as a guideline to solving step 2.
    a. Use the views to define matrix **P** and vector **q**.
    b. Use (2) to calculate $\widehat{m}$. Note that to invert a matrix you can use solve(matrix). For example, use solve(Ω) to get the inverse of the matrix $\boldsymbol{\Omega}$.
    c. Solve for BL portfolio optimization problem with the new expected returns $\widehat{m}$ replacing the m in (1).

3. In Black-Litterman model, set $\tau = .025$ and use procedure 2 to calculate the efficient frontier again.
4. Compares the three portfolios/frontiers you developed. Plot the three efficient frontiers on the same graph and interpret the results.

Deliverables

Your report which includes the following.

a. The plot of the efficient frontier for all models.
b. One visualization of how the fraction in each asset changes with respect to R.
c. Interpretations (half a page to one page is sufficient)

Name your report **project3_gZ.pdf** (where Z is your group number). Note you're your project report will be the main document used for grading. You are also encouraged to submit any R code that you might have used (this will be helpful for partial credits and additional insights for us if something is missing on your PDF). Compress the files in a zip file called **project3_gZ.zip**. Submit the zip file to Canvas.