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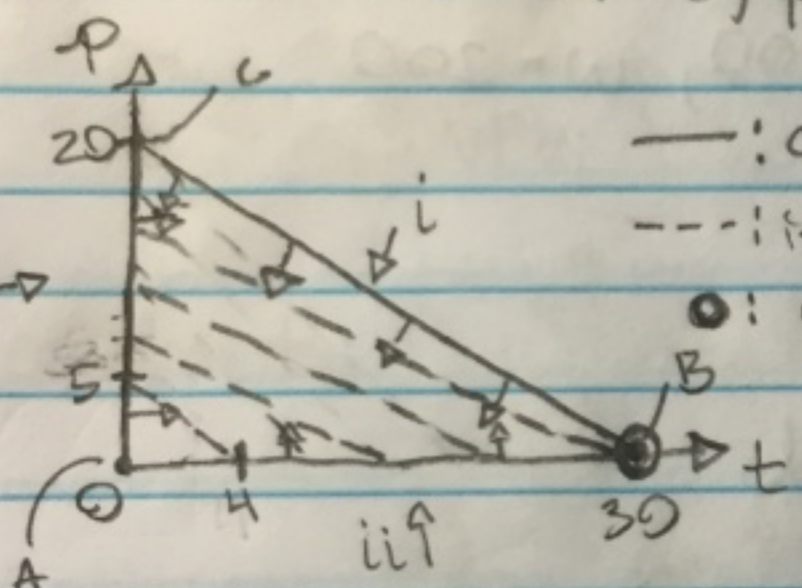
STA 372-6 - Dr. Muthraman
Homework 2
16 Sept. 2015

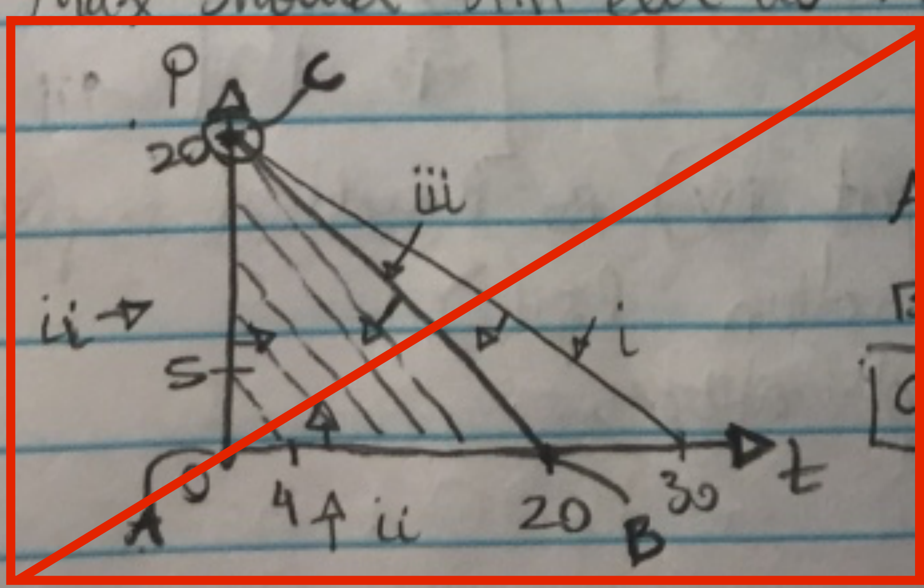
1) a.) let $T=60$ be the time limit in mins.
and t be the # of tarts and p be the # of pies
① decide t and p
② maximize points P such that $4t + 5p = P$
③ constraints: i. $2t + 3p \leq 60$
ii. $t, p \geq 0$



— : constraints
- - - : iso-profit lines
●: optimal point (30, 0) ≡ 30 tarts
0 pies

corner A: 0, 0 = 0
$\boxed{\text{corner B: 30, 0 = 120} \leftarrow \text{optimal}}$
corner C: 0, 20 = 100

b.) additional constraint iii. $p \geq t$
given the answer to a.) is to maximize t consumption,
Max should still eat as many t as possible while satisfying iii
which becomes zero



A: 0,0 → 0
B: 20,0 → 80
$\boxed{\text{C: 0, 20} \rightarrow 100 \leftarrow \text{optimal}}$
20 points less

2.) let w, c represent the number of acres of wheat and corn planted, respectively

a.) ① decide w and c

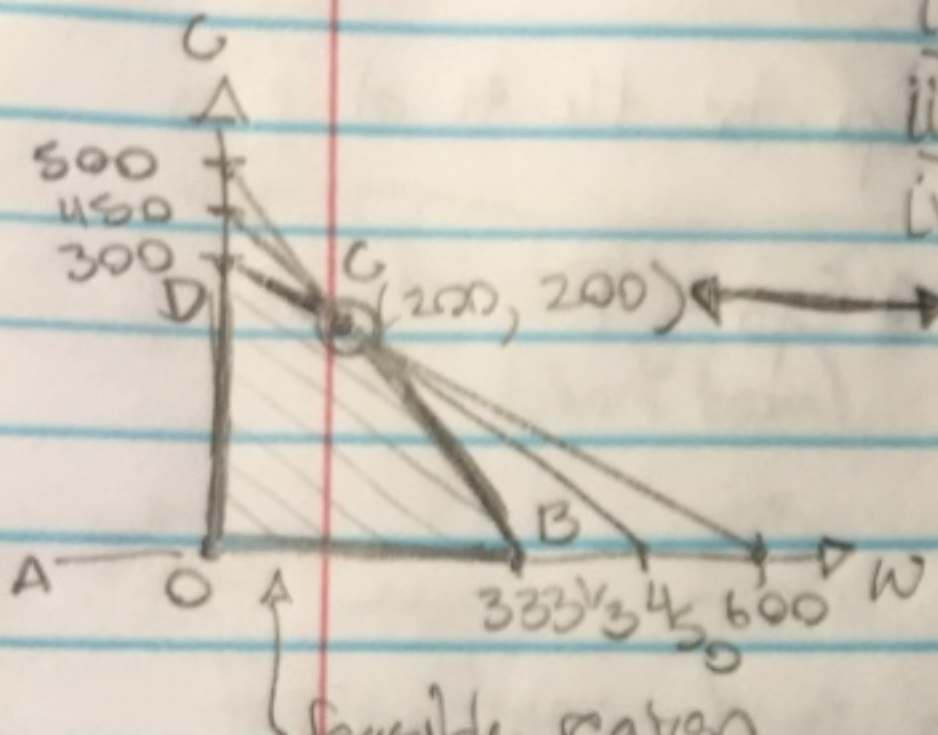
② maximize profit $P = 2000w + 3000c$

③ constrained to: i) $3w + 2c \leq 1000$ [workers]

ii) $2w + 4c \leq 1200$ [fertilizer]

iii) $w + c \leq 450$ [acres]

iv) $w, c \geq 0$



$(200, 200) \longleftrightarrow 3w + 2c = 2w + 4c - 200$

$w = 2c - 200$

$2(2c - 200) + 4c = 1200 = 8c = 1600$

$\Rightarrow c = 200, \; w = 200$

A: $0, 0 \to 0$

B: $(333\tfrac{1}{3}, 0) \to 666{,}666\tfrac{2}{3}$

$\boxed{\text{C: } (200, 200) \to 1{,}000{,}000}$ optimal

D: $(0, 300) \to 900{,}000$

b.) Matrix form of a): max $c^T x$ s.t. $Ax \leq b$

let $c^T = \begin{bmatrix} 2000 \\ 3000 \end{bmatrix}$ $A = \begin{bmatrix} 3 & 2 \\ 2 & 4 \\ 1 & 1 \end{bmatrix}$ $b = \begin{bmatrix} 1000 \\ 1200 \\ 450 \end{bmatrix}$

note that iv) is handled implicitly by the function lp()

c.) see code comments for explanations

project

3.)

	1	2	3	4	5
t_0	11	53	5	5	29
t_1	3	6	5	1	34
NPV	13	16	16	14	39

① decide how much to invest in each project at $t=0$ and $t=1$. let x_{ij} represent the amount invested where $i \in \{1, 2, 3, 4, 5\}$ and $j \in \{0, 1\}$ such that i is the project and j is the time
$\Rightarrow$ decision variables $(x_{10}, x_{11}, x_{20}, x_{21}, x_{30}, x_{31}, x_{40}, x_{41}, x_{50}, x_{51})$

② choose x_{ij} $\forall i, j$ to maximize total NPV, P

$$P = \sum_{i=1}^{5} x_{i0} NPV_i + x_{i1} NPV_i$$

where NPV_i is the NPV for each project

③ constraints: i) $\sum_{i=1}^{5} x_{i0} t_{i0} \leq 40$

t_{i0} is the outflow at $t=0$ $\forall i$

ii) $\sum_{i=1}^{5} x_{i1} t_{i1} \leq 20$

t_{i1} is the outflow at $t=1$ $\forall i$

iii) $x_{ij} \geq 0$ $\forall i, j$

Matrix form: max $c^T x$ s.t. $Ax \leq b$

note, P simplifies to:

$$P = \sum_{i=1}^{5} \left(NPV_i (x_{i0} + x_{i1}) \right)$$

$$A = \begin{bmatrix} 11 & 0 & 53 & 0 & 5 & 0 & 5 & 0 & 29 & 0 \\ 0 & 3 & 0 & 6 & 0 & 5 & 0 & 1 & 0 & 39 \end{bmatrix}_{2 \times 10}$$

$$b = \begin{bmatrix} 40 \\ 20 \end{bmatrix}_{2 \times 1}$$

$$x = \begin{bmatrix} x_{10} \\ x_{11} \\ x_{20} \\ x_{21} \\ x_{30} \\ x_{31} \\ x_{40} \\ x_{41} \\ x_{50} \\ x_{51} \end{bmatrix}_{10 \times 1}$$

let

$$C_T = \begin{bmatrix} 13 & 13 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 16 & 16 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 16 & 16 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 14 & 14 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 39 & 39 \end{bmatrix}$$