FIN372 / STA 372

Optimization Methods in Finance: Homework 3

b

This assignment is graded on Credit/No-Credit.

That is, if you complete the homework and it is acceptable, you will get credit. If you don't submit or if the submitted work is not acceptable, you will not get credit. Getting a credit is required to obtain a grade for the group project that follows.

Please write a report that solves the following problems. Make sure that your report includes the mathematical formations and the solutions of these optimization problems.

Problem 1:

Use the branch and bound method to manually solve the following question.

$$Max\ Z = -x_1 + 4x_2$$

Subject to

$$\begin{aligned} -10x_1 + 20\,x_2 &\le 22 \\ 5x_1 + 10\,x_2 &\le 49 \\ x_1 &\le 5 \\ x_i \ge 0, x_i\ &are\ integers \end{aligned}$$

1) Like what Kumar did in class, your result should be a tree.
2) Check your result using R. Please also count how many feasible solutions are there.
3) What's the difference between the number of branches and the number of feasible solutions?

Problem 2:

A company is thinking about building new facilities in Austin and Dallas. Here is the relevant data.

	Capital Needed	Expected Profit
Factory in Austin	$6 M	$9 M
Factory in Dallas	$3 M	$5 M
Warehouse in Austin	$5 M	$6 M
Warehouse in Dallas	$2 M	$4 M

Total capital available for investment is $11M. At most one factory (warehouse) can

be built in one place. At most one of the warehouse in Austin and the warehouse in Dallas can be built. On the other hand, at least one of the factory in Austin and the factory in Dallas should be built.

What is the optimal investment strategy?

Problem 3:

The days-off scheduling problem must be solved routinely by businesses that operate 6 or 7 days a week. Examples include hospitals, airlines, municipal transportation companies, and the postal service. The most common example is the (5,7)-cyclic staffing problem. The objective of it is to minimize the cost of assigning workers to a 7-day cyclic schedule so that

(1) Sufficient workers are available every day.
(2) Each person works 5 consecutive days and is idle for the remaining 2 days.

Here is the table showing the cost of having an employee for each day and the number of employees required for each day.

	Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
Cost	90	60	60	60	60	60	90
#Required	5	13	12	10	14	8	6

For example, the pattern that one works from Sunday to Thursday costs 90 + 60*4 = 330.

1) Formulate an integer programming problem to represent this problem.

2) Solve the problem in R.

3) Which working pattern is the most popular?

Problem 4 (optional):

A paper mill cuts the rolls of paper into different widths to satisfy customers' demand. In this problem, assume the original rolls of paper are 120 inches wide. The table below shows the orders received by the paper mill.

Width	Quantity
25	233
37	148
54	106

A 120 inch roll can be cut in a lot of ways. For example, we can cut four 25-inch rolls while wasting the remaining 20 inches; we can also cut one 25-inch, one 37-inch, and one 54-inch. In the second case, only 4 inches is wasted.

1) Develop an integer programming model to minimize the waste. Write out the formulation.

2) Formulate and solve the problem in R.

For more information, please check the cutting stock problem.

Problem 5 (optional):

Western Airlines wants to design a hub system in the United States. Each hub is used for connecting flights to and from cities within 1000 miles of the hub. Western runs flights among the following cities: Atlanta (ATL), Boston (BOS), Chicago (CHI), Denver (DEN), Houston (HOU), Los Angeles (LAX), New Orleans (NO), New York (NY), Pittsburgh (PIT), Salt Lake City (SLC), San Francisco (SF), and Seattle (SEA). The company wants to determine the smallest number of hubs it needs to cover all these cities, where a city is covered if it is within 1000 miles of at least one hub. Table 1.1 lists which cities are within 1000 miles of other cities. For example, if a hub was placed at Boston (BOS), it could cover the cities of Boston, New York, and Pittsburgh.

Table 1.1: Western Airlines

	ATL	BOS	CHI	DEN	HOU	LAX	NO	NY	PIT	SLC	SF	SEA
ATL	X		X		X		X	X	X			
BOS		X						X	X			
CHI	X		X				X	X	X			
DEN				X						X		
HOU	X				X		X					
LAX						X				X	X	
NO	X		X		X		X					
NY	X	X	X					X	X			
PIT	X	X	X					X	X			
SLC				X		X				X	X	X
SF						X				X	X	X
SEA										X	X	X

1) Develop a binary integer programming model to find the minimum number of hub locations to cover all cities. Write out the formulation.

2) Solve the problem in R.

Deliverables

You can either hand write or type your report, but make sure that you submit a PDF file. Please name your report as **hw3_x.pdf**(where x is your eid).

Please also submit your R code. You may not get full credit without it.