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STA 372-6 - Dr. Muthraman
Homework 4
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1.) $p_L = 12/hr$ $p_K = 15/hr$ Prod. $P = 0.05 L^{2/3} K^{1/3}$

$100{,}000 \leq 12L + 15K$

calculus method

① choose L, K

② maximize $P = 0.05 L^{2/3} K^{1/3}$

③ s.t. $100{,}000 \leq 12L + 15K$

$\Rightarrow 15K = 100{,}000 - 12L$

$K = \frac{100{,}000}{15} - \frac{4L}{5}$

$$\Rightarrow P = 0.05 L^{2/3} \left[\frac{100{,}000}{15} - \frac{4L}{5}\right]^{1/3}$$

$$\frac{dP}{dL} = \frac{536.383 - 0.0965489L}{[25000 - 3L]^{2/3} L^{1/3}}, \quad \frac{dP}{dL} = 0 \Rightarrow L = 5555.56$$

$\Rightarrow K = 2222.21867$

$\Rightarrow \boxed{P_{max} = 204.668}$

2.) Choose $x_i \; \forall i \in \{1, \dots, 27\}$ representing the share of wealth in each stock x_i

Obj. min[variance], return ≥ 0.01 per month

① $= \min[\sigma_i^2] \; \forall i$

② $= \sum_{i=1}^{27} \mu_i x_i \geq 0.01$

$$= \min\left[\sum_{i=1}^{27} \sum_{j=1}^{27} x_i x_j \sigma_{ij}\right]$$

Constraints ① $\sum_{i=1}^{27} x_i = 1$, sum of shares is 1

② $x_i \geq 0 \; \forall i$, no shorting formulation

solveQP: $-d^T x + \frac{1}{2} x^T D x$ s.t. $Ax \geq b$

let, $x = [x_1, \ldots, x_{27}]$, $\underset{27\times1}{d} = \vec{0}$, $\underset{27\times27}{D} = [\sigma_{ij}] \cdot 2$ obj ①

$$A = \begin{bmatrix} 1 & -1 & \mu_1 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & -1 & \mu_{27} & 0 & \cdots & 1 \end{bmatrix} \Big\} 27 \qquad b = \begin{bmatrix} 1 \\ -1 \\ 0.01 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \Big\} 27$$

$\underbrace{\qquad}_{27}$

3.) Choose $x_1, \ldots, x_{32}$, htadv

where x_i is the rating of a team i

and htadv is the home team advantage

Objective, $\min \left[\sum_i^n (\text{actual spread}_i - \text{predicted spread}_i)^2 \right]$

where i is a game $i = \{1, \ldots, n\}$