M348-53610: Scientific Computing
Homework # 09
Handout: 03/29/2016, Tuesday
Due: 04/05/2016, Tuesday

Submission. Please make your homework neat and stapled. You have to submit your homework in ECJ 1.204 before **3:00 PM** on the due date. Note that *no late homework will be accepted without compelling reasons.*

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1 To be Graded

Problem 1. Evaluate the exact values of the following integrals and approximate them using Gaussian quadrature with (i) $n = 2$ and (ii) $n = 3$.

$$(a) \int_1^{1.5} x^2 \ln x dx, \qquad (b) \int_0^1 x^2 e^{-x} dx$$

Problem 2. Determine constants a, b, c, and d that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(1) + cf'(-1) + df'(1)$$

that has degree of precision 3.

Problem 3. Determine constants a, b, c, d and e that will produce a quadrature formula

$$\int_{-1}^{1} f(x)dx = af(-1) + bf(0) + cf(1) + df'(-1) + ef'(1)$$

that has degree of precision 4.

Problem 4. Use Euler's method to approximate the solutions for each of the following initial-value problems.

$$(a)\ y' = te^{3t} - 2y, \quad 0 \le t \le 1, \quad y(0) = 0, \quad \text{with } h = 0.5$$

$$(b)\ y' = 1 + (t - y)^2, \quad 2 \le t \le 3, \quad y(2) = 1, \quad \text{with } h = 0.5$$

Problem 5. Show that each of the following initial-value problems has a unique solution by showing that the $f(t, y)$ function in the problem is Lipschitz continuous.

$$(a)\ y' = y \cos t, \quad 0 \le t \le 1, \quad y(0) = 1.$$

$$(b)\ y' = \frac{2}{t} y + t^2 e^t, \quad 1 \le t \le 2, \quad y(1) = 0.$$

Problem 6 (Programming Assignment)**.** A full description of the Euler's method is provided in Algorithm 5.1 of Burden and Faires (also attached here in case you do not have the textbook).
(a) Implement the Euler's method in Matlab, C++ or any other program language that you prefer. Include the full program in your submission.
(b) Use the code developed in (a) to solve the initial value problem

$$y' = 1 + y/t, \quad 1 \leq t \leq 2, \quad y(1) = 2$$

with $h = 0.1$. In the $t - y$ coordinates, plot your numerical solution together with the true solution

$$y(t) = t \ln t + 2t.$$

You can use solid line to plot the true solution and dashed line to plot the approximation.
(c) Repeat (b) with $h = 0.001$.

2 Reading Assignments

- Review Sections 4.6, 5.1 and 5.2 of Burden & Faires or Sections 5.6, 6.1 and 6.2 of Epperson.

For this problem $f(t, y) = y - t^2 + 1$, so

$$\begin{aligned}
w_0 &= y(0) = 0.5;\\
w_1 &= w_0 + 0.5\left(w_0 - (0.0)^2 + 1\right) = 0.5 + 0.5(1.5) = 1.25;\\
w_2 &= w_1 + 0.5\left(w_1 - (0.5)^2 + 1\right) = 1.25 + 0.5(2.0) = 2.25;\\
w_3 &= w_2 + 0.5\left(w_2 - (1.0)^2 + 1\right) = 2.25 + 0.5(2.25) = 3.375;
\end{aligned}$$

and

$$y(2) \approx w_4 = w_3 + 0.5\left(w_3 - (1.5)^2 + 1\right) = 3.375 + 0.5(2.125) = 4.4375. \qquad \square$$

Equation (5.8) is called the **difference equation** associated with Euler's method. As we will see later in this chapter, the theory and solution of difference equations parallel, in many ways, the theory and solution of differential equations. Algorithm 5.1 implements Euler's method.

ALGORITHM
5.1

Euler's

To approximate the solution of the initial-value problem

$$y' = f(t, y), \quad a \le t \le b, \quad y(a) = \alpha,$$

at $(N + 1)$ equally spaced numbers in the interval $[a, b]$:

INPUT endpoints a, b; integer N; initial condition α.

OUTPUT approximation w to y at the $(N + 1)$ values of t.

Step 1 Set $h = (b - a)/N$;
$t = a$;
$w = \alpha$;
OUTPUT (t, w).

Step 2 For $i = 1, 2, \ldots, N$ do Steps 3, 4.

Step 3 Set $w = w + hf(t, w)$; (*Compute* w_i.)
$t = a + ih$. (*Compute* t_i.)

Step 4 OUTPUT (t, w).

Step 5 STOP. ■

To interpret Euler's method geometrically, note that when w_i is a close approximation to $y(t_i)$, the assumption that the problem is well-posed implies that

$$f(t_i, w_i) \approx y'(t_i) = f(t_i, y(t_i)).$$