SCS Senior Thesis Prospectus

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June 2, 2016

Abstract

We intend to study iterated transductions defined by a class of invertible transducers over the binary alphabet. Previous work has investigated orbit checking algorithms and the orbit rationality problem for a subclass of automata associated with Abelian free groups and monoids of finite rank.

We intend to answer these same questions for increasingly large subclasses of invertible transducers.

1 Problem Description / Significance

An invertible transducer is a Mealy automaton where all transitions are of the form $p^{a/\pi_p(a)} \to q$, where π_p is a permutation of the alphabet depending on the source state p. We only consider $\mathbf{2} = \{0, 1\}$ as input and output alphabet. Choosing an arbitrary start state p, we obtain a transduction from $\underline{2}^*$ to $\underline{2}^*$. These transductions form a semigroup $\mathcal{S}(A)$. Including inverses, we obtain groups $\mathcal{G}(A)$. These groups are called automata groups or self-similar groups, studied in great detail in group theory and symbolic dynamics. For instance, Grigorchuk's group of intermediate growth demonstrates the descriptive power of invertible transducers.

We're particularly interested in the connections between automata theory and group theory. Given a transductions $f \in \mathcal{S}(A)$, write $f^* \subseteq \mathbf{2}^* \times \mathbf{2}^*$ for the binary relation obtained by iterating f. While $\langle \mathbf{2}^*, f \rangle$ is clearly automatic and thus has a decidable first-order theory, it's difficult to determine when $\langle \mathbf{2}^*, f, f^* \rangle$ is automatic. It is this question that we seek to answer.

2 Proposed Research Plan

A good deal of background reading has already been completed. I've read 2 papers by Klaus, as well as Okano's thesis from last year. Further reading still needs to be done: in particular

I'll be working alongside Klaus to process original results. We expect to answer the orbit rationality question for subclasses of inverse transducers with associated Abelian semigroups and multiple toggle states; a class of automata not previously examined in this way.

We also hope to characterize the behaviour of several of the non-Abelian cases as well; though we expect these orbit relations will fail to be rational.

2.1 Timeline

- Prior to September: Characterize behavior of a particular automata. (A test case to determine how feasible the non-Abelian cases will be).

- September: Be able to reproduce all results from Okano's thesis from scratch.

3 Bibliography