SCS Senior Thesis Prospectus

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Abstract

We intend to study iterated transductions defined by a class of invertible transducers over the binary alphabet. Previous work has investigated orbit checking algorithms and the orbit rationality problem for a subclass of automata associated with Abelian free groups and monoids of finite rank.

We intend to answer these same questions for increasingly large subclasses of invertible transducers.

1 Problem Description / Significance

An invertible transducer is a Mealy automaton where all transitions are of the form $p^{a/\pi_p(a)} \to q$, where π_p is a permutation of the alphabet depending on the source state p. We only consider $\mathbf{2} = \{0, 1\}$ as input and output alphabet. Choosing an arbitrary start state p, we obtain a transduction from $\mathbf{2}^*$ to $\mathbf{2}^*$. These transductions form a semigroup $\mathcal{S}(A)$. Including inverses, we obtain groups $\mathcal{G}(A)$. These groups are called automata groups or self-similar groups, studied in great detail in group theory and symbolic dynamics. For instance, Grigorchuk's group of intermediate growth demonstrates the descriptive power of invertible transducers.

2 Proposed Research Plan

A good deal of background reading has already been completed. I've read 2 papers by Klaus, as well as Okano's thesis from last year. Further reader still needs to be done, in particular [1].

I'll be working alongside Klaus to produce original results. We expect to answer the orbit rationality question for subclasses of inverse transducers with associated Abelian semigroups and multiple toggle states; a class of automata not previously examined in this way.

We also hope to characterize the behaviour of several of the non-Abelian cases as well; though we expect these orbit relations will fail to be rational.

2.1 Timeline

- September: reading, absorbing known techniques.
- October: study special cases with a view towards handling simple non-commutative automata.
- November: extend orbit characterization in commutative cases.
- December-January: develop and implement decision algorithm for orbit rationality for as large a class of automata as feasible.
- February - March: further develop connections between group theoretic and model theoretic properties of invertible automata.
- April: Writeup.

References

[1] V. Nekrashevych and S. Sidki. *Automorphisms of the binary tree: state-closed subgroups and dynamics of 1/2-endomorphisms*. Cambridge University Press, 2004.

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[3] Y. Muntyan. *Automata Groups*. PhD thesis, Texas A&M University, May 2009.

[4] A. J. Cain. Automaton semigroups. *TCS*, 410(47–49):5022–5038, 2009.

[5] O. Kharlampovich, B. Khoussainov, and A. Miasnikov. From automatic structures to automatic groups. *ArXiv e-prints*, July 2011.

[6] L. Bartholdi and P. V. Silva. Groups defined by automata. *CoRR*, abs/1012.1531, 2010.