Nside = 4: Covariance and likelihood.

Data

We use only CAMB generated scalar C_l values and CAMB generated maps. See Lewis, Challinor, Lasenby: astro-ph/9911177

Maps noiseless, seeded randomly, Gaussian.

If FITS files located in local "Downloads" directory after download/unzipped .tar file, everything works.

$\mathrm{N_{side}} = 4$ means $\mathrm{N_{pix}} = 12 \times \mathrm{side}^2 = 192$ total pixels.

C_l scalar values were set to $l_{\mathrm{max}} = 2\mathrm{N_{pix}}$.

Covariance results

The pixel-pixel temperature covariance matrix from CAMB maps is defined as

$$C_{ij} =< \Delta T_i \Delta T_j >= \frac{1}{N_{\mathrm{pix}}} \sum_{p=1}^{\mathrm{pix}} (T^i(p) - \bar{T}^i)(T^j(p) - \bar{T}^j)$$

The real-space (temperature space) covariance matrix is defined as

$$C_{ij} = \sum_{l=0}^{l} \frac{2l+1}{4\pi} C_l^{\mathrm{theor}} P_l(\cos\alpha_{ij})$$

where C_{ij} is the covariance between pixel i and pixel j, C_l^{theor} is the theoretical value of C_l, P_l are the Legendre polynomial, and α_{ij} is the angle between. The dot product is defined by unit vectors, $\cos\alpha_{ij} = \hat{n}_i \cdot \hat{n}_j$.

In spherical harmonic space, we have mean zero and covariance

$$\sum a_{lm} a^{\dagger}_{l'm'} = (2l+1) C_l \tag{1}$$

Likelihoods

Likelihood outputs will not be exact, because of some constants. They are approximate.

In spherical harmonic space, the expression is

$$-2\ln\mathcal{L} = \sum_l (2l+1)\Big[\ln\Big(\frac{C_l^{\mathrm{th}}}{\hat{C}_l}\Big) + \Big(\hat{C}_l / C_l^{\mathrm{th}}\Big) - 1\Big] \tag{2}$$

where

$$\hat{C}_l = \frac{1}{2l+1} \sum_m |\hat{a}_{lm}|^2 \tag{3}$$

from HEALPix anafast.

In temperature space (real space), the expression is

$$-2\ln\mathcal{L} \propto \mathbf{T}\mathbf{S}^{-1}\mathbf{T} + \ln\det\mathbf{S} + \mathrm{N}\ln 2\pi \tag{4}$$