Wine Fermentation Tank Capacity

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Wine is an important drink for adults. Wine is in everyday human lives, not only for their pleasures but can also serve as a health benefit (not excessive).The process of producing wine can be tough and extraneous. An important step in the production of wine, is going through a scientific process known as Fermentation. Fermantation can be defined as "the process of fermentation involved in the making of beer, wine, and liquor, in which sugars are converted to ethyl alcohol"(Google Dictionary 2018). This happens inside of Fermantation tanks, which is what I am about to talk about. I am talking specifically with the tanks made by Sonoma Stone in mind.

These tanks have complicated engineering put into them. The interior of the tank is interior of the surface described by rotating that curve about the y-axis. This paper will talk about the volume of these tanks, using integration to find the volume of the interior of a surface of rotation. The boundary of the central cross section of the interior of the tank is

$$0.0002017404542x^2 + \frac{0.0001303290910y^2}{20.9520444748 + \alpha y} = 1.$$

All of these distances are measured in centimeters. Recall that the volume of a tank is in cm^3 units which are equivalent to mL in a 1:1 ratio. Most tanks made by Sonoma Stone have $\alpha \approx 0.005$, which will be used in examples later.

Volume of the Tank

The volume of these tanks cannot be found by the typical cylinder volume formula:

$$V = \pi r^2 h$$

where r is the radius and h is the height of the cylinder.

We know that we have a solid figure that is rotated about the y-axis. There are 2 methods for finding the volume of a solid of revolution: Disc integration and Shell integration(cylinder method). For the Disk method, it can be visualized as a ring/disc that is revolving about the y-axis. For the cylinder method, it can be visualized as cylindrical shell revolving about the y-axis The easiest way to decide which method will work the best, is to draw the graph of the equation that is being questioned. See Figure:1 for the Tanks central cross section when

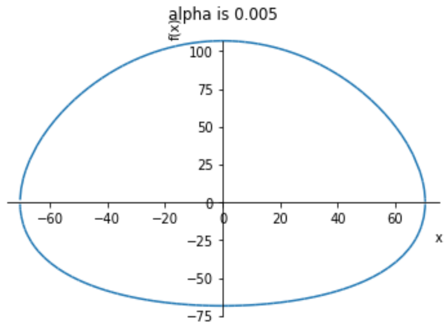

Figure 1: Cross section when α is 0.005

α is 0.005. After visualizing a disc revolving about the y-axis the method of integration that seems the most appropriate is the Disc method.

The disc method of integration for finding volume is defined to be:

$$V = \pi \int_a^b f(x)^2 dx.$$

The problem we face is that our equation is not in a form that is easily just plug and evaluate. Recall our equation given:

$$0.0002017404542x^2 + \frac{0.0001303290910y^2}{20.9520444748 + \alpha y} = 1.$$

Firstly, We need a value for α in order to do anything. We then solved for the equation for the variable y and setting the equation $= 0$. We then must find the endpoints for our definite integral, which this was found by solving the new equation for x. The minimum and maximum (a and b, respectively) were found to be $-70.4050000066484, 70.4050000066483$. Now we know our a and b for our integration. We only need to square our new-equation (solved for y) and we can plug it into our integration method and evaluate.

Significance of α

The overall volume of the tank is changed significantly due to the role that α plays in the cross section formula. After running trials with α ranging from 0.0to0.02by steps of 0.001 we can see that the volume increases as α rises. See Figure: 2 for a plot of the α values with respect to volume. You can see that the volume almost appears to grow exponentially as α increases. Notice that with our highest value α at 0.02 the volume was 9346230.49815142cm^3, while at the lowest α (0.0) the volume was 2154308.93603807cm^3.

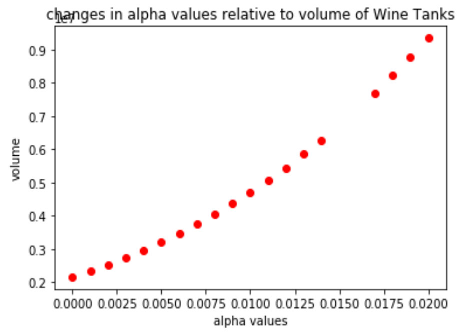

Figure 2: α with respect to Volume

Conclusion

The volume of these fermentation tanks is heavily dependent upon the α value as represented earlier. These approximations were done through the use of the sympy package through Python. The conclusions were drawn from these approximations, since I do not have the mathematical skill, nor patience to be able to perform these by hand. Given all these prior calculations and evaluations, we can determine how many bottles of wine that are expected to be produced from a tank that has $\alpha = 0.005$. The calculation is:

$$\begin{aligned} Total_{bottles} &= \frac{Total_{volume}}{750\text{mL}} \\ &= \frac{3200091.44151188}{750\text{mL}} \\ &= 4266.78858868250 \text{ bottles} \\ &\approx 4266 \text{ bottles} \end{aligned}$$

The tanks can produce 4266 bottles of wine given that the bottles hold 750mL of wine. We cannot round up because we would only fill a little over $\frac{3}{4}$ of another whole bottle, which cannot be distributed to buyers.

There are other methods to finding the volume that could of been used for this. This was the easiest method for me to visualize, therefore I used it.

Lastly, on Figure: 2 notice there are two data points missing. These data points were for when $\alpha = 0.015, 0.016$. They are missing because the were complex numbers, which python would not allow me to plot complex number. However, if we kept the real number portion of the volumes they would follow the trend of the previous volumes prior to 0.015.