Exercise 1.

finding F...

$$x_k = f(x_{k-1}, u_k) + w_k \; ; \; x_t = [X_t \; Y_t \; \psi_t \; m_x^1 \; m_y^1 \; \dots \; m_x^n \; m_y^n]^T$$

f_1: $X_{t+1} = X_t + \delta t(\dot{x}_t \cos(\psi_t) - \dot{y}_t \sin(\psi_t))$

f_2: $Y_{t+1} = Y_t + \delta t(\dot{x}_t \sin(\psi_t) + \dot{y}_t \cos(\psi_t))$

f_3: $\psi_{t+1} = \psi_t + \delta t \, \dot{\psi}_t$

$f_{(2+2j)}$: $m^j_{x,t+1} = m^j_{x,t}$ (m is global position of static landmark)

$f_{(3+2j)}$: $m^j_{y,t+1} = m^j_{y,t}$

Taking Partial Derivatives ...

$$\frac{\partial f_1}{\partial X} = 1 \; ; \; \frac{\partial f_1}{\partial Y} = 0 \; ; \; \frac{\partial f_1}{\partial \psi} = \delta t(-\dot{x}\sin\psi - \dot{y}\cos\psi) \; ; \; \frac{\partial f_1}{\partial m_x^j} = 0 \; ; \; \frac{\partial f_1}{\partial m_y^j} = 0$$

$$\frac{\partial f_2}{\partial X} = 0 \; ; \; \frac{\partial f_2}{\partial Y} = 1 \; ; \; \frac{\partial f_2}{\partial \psi} = \delta t(\dot{x}\cos\psi - \dot{y}\sin\psi) \; ; \; \frac{\partial f_2}{\partial m_x^j} = 0 \; ; \; \frac{\partial f_2}{\partial m_y^j} = 0$$

$$\frac{\partial f_3}{\partial X} = 0 \; ; \; \frac{\partial f_3}{\partial Y} = 0 \; ; \; \frac{\partial f_3}{\partial \psi} = 1 \; ; \; \frac{\partial f_3}{\partial m_x^j} = 0 \; ; \; \frac{\partial f_3}{\partial_y^j} = 0$$

$$\frac{\partial f_{(2+2j)}}{\partial X} = 0 \; ; \; \frac{\partial f_{(2+2j)}}{\partial Y} = 0 \; ; \; \frac{\partial f_{(2+2j)}}{\partial \psi} = 0 \; ; \; \frac{\partial f_{(2+2j)}}{\partial m_x^j} = 1 \; ; \; \frac{\partial f_{(2+2j)}}{\partial m_y^j} = 0$$

$$\frac{\partial f_{(3+2j)}}{\partial X} = 0 \; ; \; \frac{\partial f_{(3+2j)}}{\partial Y} = 0 \; ; \; \frac{\partial f_{(3+2j)}}{\partial \psi} = 0 \; ; \; \frac{\partial f_{(3+2j)}}{\partial m_x^j} = 0 \; ; \; \frac{\partial f_{(3+2j)}}{\partial m_y^j} = 1$$

Also: $\dfrac{\partial f_{(2+2j)}}{\partial m_x^{\,i}} = 0 \; ; \; \dfrac{\partial (f_{3+2j})}{\partial m_y^{\,i}} = 0$ where $i \neq j$

Contined on next page...

Exercise 1 (continued)

Assembling PD's in F matrix and evaluating at $\hat{x}_{k-1|k-1}$ ...

$$F = \left.\begin{bmatrix} 1 & 0 & \delta t(-\dot{x}\sin\psi - \dot{y}\cos\psi) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \delta t(\dot{x}\cos\psi - \dot{y}\sin\psi) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}\right|_{\hat{x}_{k-1|k-1},\, u_k}$$

Exercise 1 (continued)

Finding H...

$$y_k = h(x_k) + v_k$$

distance: $h_d = \| m^j - p_t \| = \sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}$

bearing: $h_b = \operatorname{atan2}(m_y^j - Y, m_x^j - X) - \psi$

Taking partial derivatives of distance measurement system...

$$\frac{\partial h_d}{\partial X} = \frac{X - m_x^j}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}} \; ; \; \frac{\partial h_d}{\partial Y} = \frac{Y - m_y^j}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}}$$

$$\frac{\partial h_d}{\partial \psi} = 0 \; ; \; \frac{\partial h_d}{\partial m_x^j} = \frac{m_x^j - X}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}} \; ; \; \frac{\partial h_d}{\partial m_y^j} = \frac{m_y^j - Y}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}}$$

$$\frac{\partial h_d}{\partial m_x^{i \neq j}} = 0 \; ; \; \frac{\partial h_d}{\partial m_y^{i \neq j}} = 0$$

Taking partial derivatives of bearing measurement system...

$$\frac{\partial h_b}{\partial X} = \frac{m_y^j - Y}{(m_x^j - X)^2 + (m_y^j - Y)^2} \; ; \; \frac{\partial h_b}{\partial Y} = \frac{X - m_x^j}{(m_x^j - X)^2 + (m_y^j - Y)^2}$$

$$\frac{\partial h_b}{\partial \psi} = -1 \; ; \; \frac{\partial h_b}{\partial m_x^j} = \frac{Y - m_y^j}{(m_x^j - X)^2 + (m_y^j - Y)^2} \; ; \; \frac{\partial h_b}{\partial m_y^j} = \frac{m_x^j - X}{(m_x^j - X)^2 + (m_y^j - Y)^2}$$

$$\frac{\partial h_b}{\partial m_x^{i \neq j}} = 0 \; ; \; \frac{\partial h_b}{\partial m_y^{i \neq j}} = 0$$

Exercise 1 (Continued)

Assembling Partial Derivatives into H matrix...

$$
H = \begin{bmatrix}
\frac{X-m_x^1}{\sqrt{(m_x^1-X)^2+(m_y^1-Y)^2}} & \frac{Y-m_y^1}{\sqrt{(m_x^1-X)^2+(m_y^1-Y)^2}} & 0 & \frac{m_x^1-X}{\sqrt{(m_x^1-X)^2+(m_y^1-Y)^2}} & \frac{m_y^1-Y}{\sqrt{(m_x^1-X)^2+(m_y^1-Y)^2}} & 0 & 0 & \cdots & 0 \\
\vdots & \vdots & \vdots & 0 & 0 & \ddots & & & \\
\frac{X-m_x^n}{\sqrt{(m_x^n-X)^2+(m_y^n-Y)^2}} & \frac{Y-m_y^n}{\sqrt{(m_x^n-X)^2+(m_y^n-Y)^2}} & 0 & 0 & \cdots & 0 & 0 & \frac{m_x^n-X}{\sqrt{(m_x^n-X)^2+(m_y^n-Y)^2}} & \frac{m_y^n-Y}{\sqrt{(m_x^n-X)^2+(m_y^n-Y)^2}} \\
\frac{m_y^1-Y}{(m_x^1-X)^2+(m_y^1-Y)^2} & \frac{X-m_x^1}{(m_x^1-X)^2+(m_y^1-Y)^2} & -1 & \frac{Y-m_y^1}{(m_x^1-X)^2+(m_y^1-Y)^2} & \frac{m_x^1-X}{(m_x^1-X)^2+(m_y^1-Y)^2} & 0 & \cdots & & 0 \\
\vdots & \vdots & \vdots & 0 & 0 & \ddots & & & \\
\frac{m_y^n-Y}{(m_x^n-X)^2+(m_y^n-Y)^2} & \frac{X-m_x^n}{(m_x^n-X)^2+(m_y^n-Y)^2} & -1 & 0 & \cdots & 0 & 0 & \frac{Y-m_y^n}{(m_x^n-X)^2+(m_y^n-Y)^2} & \frac{m_x^n-X}{(m_x^n-X)^2+(m_y^n-Y)^2}
\end{bmatrix}
$$