Exercise 1.

finding F...

$$x_k = f(x_{k-1}, u_k) + w_k \; ; \; x_t = \left[X_t \; Y_t \; \Psi_t \; m_x^1 \; m_y^1 \; \dots \; m_x^h \; m_y^h \right]^T$$

f_1 : $x_{t+1} = x_t + \delta t \left(\dot{x}_t \cos(\Psi_t) - \dot{y}_t \sin(\Psi_t) \right)$

f_2 : $Y_{t+1} = Y_t + \delta t \left(\dot{x}_t \sin(\Psi_t) + \dot{y}_t \cos(\Psi_t) \right)$

f_3 : $\Psi_{t+1} = \Psi_t + \delta t \, \dot{\psi}_t$

$f_{(2+2j)}$: $m^j_{x,t+1} = m^j_{x,t}$ ← (m is global position of static landmark)

$f_{(3+2j)}$: $m^j_{y,t+1} = m^j_{y,t}$

Taking Partial Derivatives ...

$$\frac{df_1}{dx} = 1 \; ; \; \frac{df_1}{dY} = 0 \; ; \; \frac{df_1}{d\Psi} = \delta t(-\dot{x}\sin\Psi - \dot{y}\cos\Psi) \; ; \; \frac{df_1}{dm_x^j} = 0 \; ; \; \frac{df_1}{dm_y^j} = 0$$

$$\frac{df_2}{dx} = 0 \; ; \; \frac{df_2}{dY} = 1 \; ; \; \frac{df_2}{d\Psi} = \delta t(\dot{x}\cos\Psi - \dot{y}\sin\Psi) \; ; \; \frac{df_2}{dm_x^j} = 0 \; ; \; \frac{df_2}{dm_y^j} = 0$$

$$\frac{df_3}{dx} = 0 \; ; \; \frac{df_3}{dY} = 0 \; ; \; \frac{df_3}{d\Psi} = 1 \; ; \; \frac{df_3}{dm_x^j} = 0 \; ; \; \frac{df_3}{dm_y^j} = 0$$

$$\frac{df_{(2+2j)}}{dx} = 0 \; ; \; \frac{df_{(2+2j)}}{dY} = 0 \; ; \; \frac{df_{(2+2j)}}{dt} = 0 \; ; \; \frac{df_{(2+2j)}}{dm_x^j} = 1 \; ; \; \frac{df_{(2+2j)}}{dm_y^j} = 0$$

$$\frac{df_{(3+2j)}}{dx} = 0 \; ; \; \frac{df_{(3+2j)}}{dY} = 0 \; ; \; \frac{df_{(3+2j)}}{d\Psi} = 0 \; ; \; \frac{df_{(3+2j)}}{dm_x^j} = 0 \; ; \; \frac{df_{(3+2j)}}{dm_y^j} = 1$$

Also: $\frac{df_{(2+2j)}}{dm_x^i} = 0 \; ; \; \frac{df_{(3+2j)}}{dm_y^i} = 0$ where $i \neq j$

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Exercise 1 (continued)

Assembling PD's in F matrix and evaluating at $\hat{x}_{k-1|k-1}$ ...

$$F = \begin{bmatrix} 1 & 0 & \delta t(-\dot{x}\sin\psi - \dot{y}\cos\psi) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \delta t(\dot{x}\cos\psi - \dot{y}\sin\psi) & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 1 \end{bmatrix}_{\hat{x}_{k-1|k-1},\, u_k}$$

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Exercise 1 (continued)

Finding H...

$$Y_k = h(x_k) + v_k$$

distance: $h_d = \|m^j - p_t\| = \sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}$

bearing: $h_b = \text{atan2}(m_y^j - Y, m_x^j - X) - \Psi$

Taking Partial Derivatives of distance measurement system...

$$\frac{\partial h_d}{\partial X} = \frac{X - m_x^j}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}} ; \frac{\partial h_d}{\partial Y} = \frac{Y - m_y^j}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}}$$

$$\frac{\partial h_d}{\partial \Psi} = 0 ; \frac{\partial h_d}{\partial m_x^j} = \frac{m_x^j - X}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}} ; \frac{\partial h_d}{\partial m_y^j} = \frac{m_y^j - Y}{\sqrt{(m_x^j - X)^2 + (m_y^j - Y)^2}}$$

$$\frac{\partial h_d}{\partial m_x^{i \neq j}} = 0 ; \frac{\partial h_d}{\partial m_y^{i \neq j}} = 0$$

Taking Partial derivatives of bearing measurement system...

$$\frac{\partial h_b}{\partial X} = \frac{m_y^j - Y}{(m_x^j - X)^2 + (m_y^j - Y)^2} ; \frac{\partial h_b}{\partial Y} = \frac{X - m_x^j}{(m_x^j - X)^2 + (m_y^j - Y)^2}$$

$$\frac{\partial h_b}{\partial \Psi} = -1 ; \frac{\partial h_b}{\partial m_x^j} = \frac{Y - m_y^j}{(m_x^j - X)^2 + (m_y^j - Y)^2} ; \frac{\partial h_b}{\partial m_y^j} = \frac{m_x^j - X}{(m_x^j - X)^2 + (m_y^j - Y)^2}$$

$$\frac{\partial h_b}{\partial m_x^{i \neq j}} = 0 ; \frac{\partial h_b}{\partial m_y^{i \neq j}} = 0$$

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Exercise 1 (continued)

Assembling Partial Derivatives into H matrix...

$$H = \begin{bmatrix} \frac{x-m_x^1}{\sqrt{(m_x^1-x)^2+(m_y^1-y)^2}} & \frac{y-m_y^1}{\sqrt{(m_x^1-x)^2+(m_y^1-y)^2}} & 0 & \frac{m_x^1-x}{(m_x^1-x)^2+(m_y^1-y)^2} & \frac{m_y^1-y}{\sqrt{(m_x^1-x)^2+(m_y^1-y)^2}} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & & \\ \frac{x-m_x^n}{\sqrt{(m_x^n-x)^2+(m_y^n-y)^2}} & \frac{y-m_y^n}{\sqrt{(m_x^n-x)^2+(m_y^n-y)^2}} & 0 & 0 & \cdots & 0 & \frac{m_x^n-x}{\sqrt{(m_x^n-x)^2+(m_y^n-y)^2}} & \frac{m_y^n-y}{\sqrt{(m_x^n-x)^2+(m_y^n-y)^2}} \\ \frac{m_y^1-y}{(m_x^1-x)^2+(m_y^1-y)^2} & \frac{x-m_x^1}{(m_x^1-x)^2+(m_y^1-y)^2} & -1 & \frac{y-m_y^1}{(m_x^1-x)^2+(m_y^1-y)^2} & \frac{m_x^1-x}{(m_x^1-x)^2+(m_y^1-y)^2} & 0 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & & & \\ \frac{m_y^n-y}{(m_x^n-x)^2+(m_y^n-y)^2} & \frac{x-m_x^n}{(m_x^n-x)^2+(m_y^n-y)^2} & -1 & 0 & \cdots & 0 & \frac{y-m_y^n}{(m_x^n-x)^2+(m_y^n-y)^2} & \frac{m_x^n-x}{(m_x^n-x)^2+(m_y^n-y)^2} \end{bmatrix}$$

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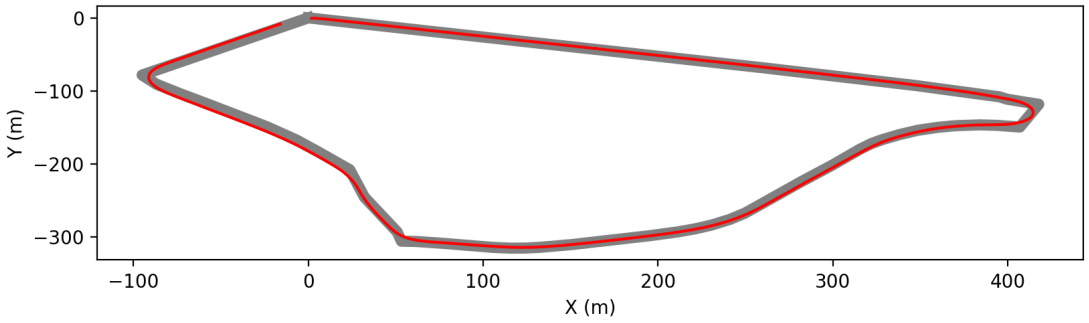
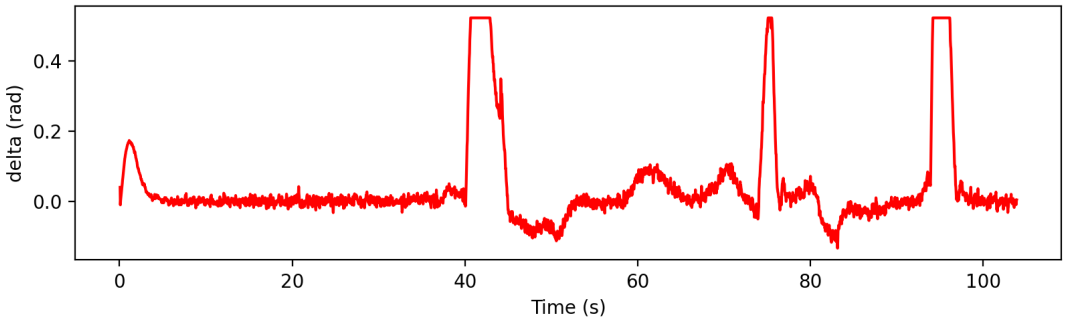
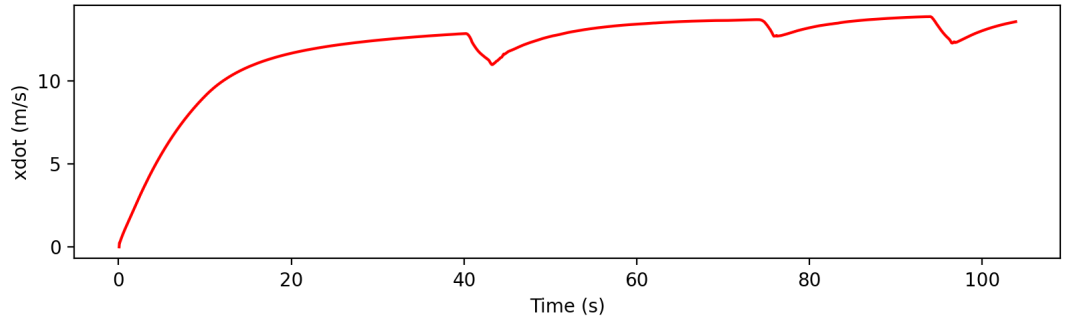
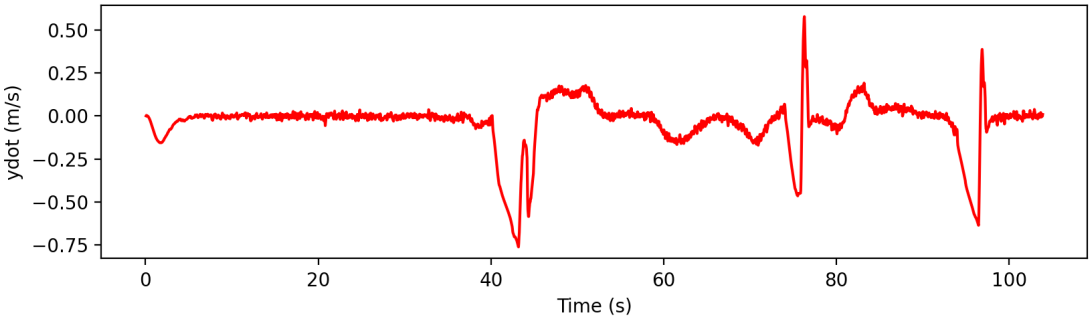
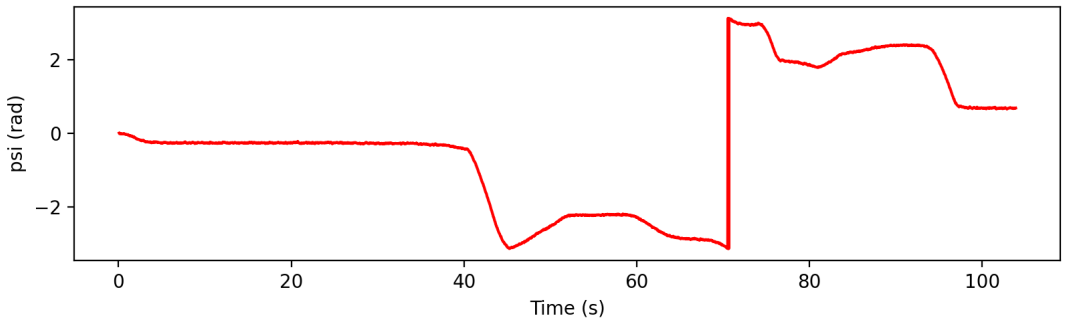
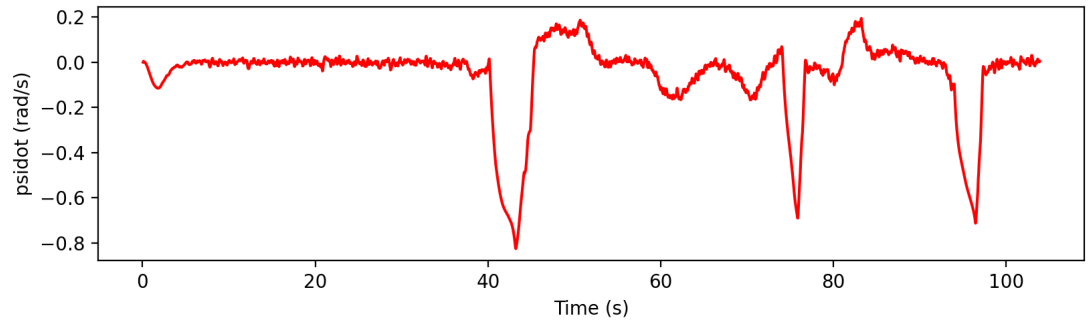

Y (m)
X (m)

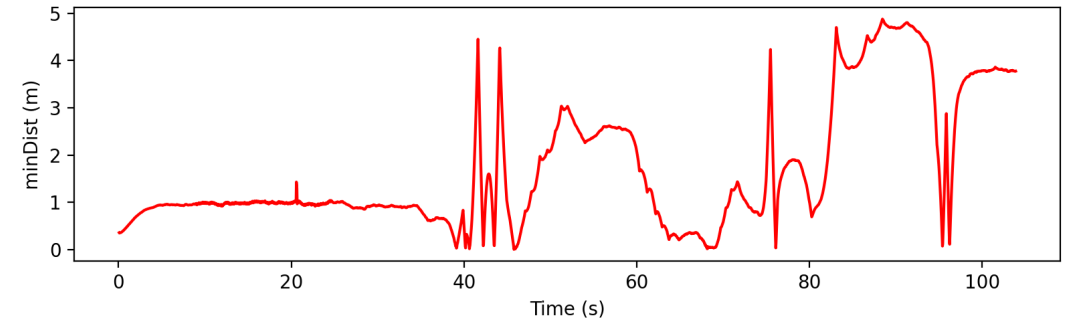

delta (rad)
Time (s)

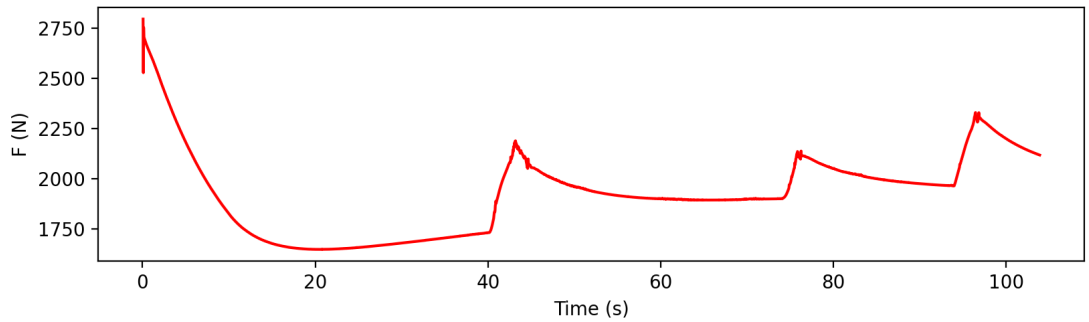

xdot (m/s)
Time (s)


ydot (m/s)
Time (s)


psi (rad)
Time (s)


psidot (rad/s)
Time (s)


minDist (m)
Time (s)


F (N)
Time (s)