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—Evan Chow

DYNAMIC SPILLOVERS IN PRICE VOLATILITY ACROSS BITCOIN EXCHANGES

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Abstract. Quantitative work on Bitcoin's price volatility has analyzed returns aggregated daily across exchanges, yet the specific price dynamics between these institutions have gone unstudied. This research examines the existence and behavior of spillover effects in price volatility across Bitcoin exchanges. Consistent with the financial literature, we use GARCH (Generalized Autoregressive Conditional Heteroskedasticity) variants to capture spillover. We find that spillovers propagate efficiently between large exchanges, but generally weaken when traveling to small ones. We also show that correlations in volatility evolve over time.

Keywords: Bitcoin, MGARCH, volatility spillovers

1 Introduction

Bitcoin is a peer-to-peer virtual currency introduced by the pseudonymous Satoshi Nakamoto (2009) that uses cryptography to verify payments, record transactions, and store monetary value. The perceived benefits of Bitcoin are privacy, convenient foreign currency exchange, and political freedom from centralized governance. Although the currency has not yet become mainstream in the US, Bitcoin nevertheless wields a market capitalization of several billion USD, with 40 million to 100 million USD in transaction volume per day (Blockchain.info). In recent years, the cryptocurrency gained some traction: the massive payments platform PayPal introduced support for Bitcoin (Cutler 2014), and Bitcoin ATMs appeared in the US (Jervis 2014).

Over the last five years, however, skyrocketing price volatility has curbed Bitcoin's growing popularity. For instance, in 2014, the USD value of Bitcoin

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plummeted from \$1000 to \$300, with some months experiencing drops of almost \$300 (Blockchain.info). Some parties, including some economists (Yermack 2013) and the press (Tucker 2013), argue that high volatility will ultimately invalidate the currency as a unit of account. Others (Lee 2014) believe Bitcoin will eventually stabilize as mainstream adoption increases and the novelty of cryptocurrencies wears off.

Currently, around 80 Bitcoin exchanges facilitate conversion between Bitcoin and fiat currencies (Planet Bitcoin). The largest are Bitfinex, Bitstamp, and BTC-e, which each transfer between \$100,000 and \$1,000,000 in Bitcoin daily (Rivera 2014). Although relatively smooth over long time periods, short-run price movements and volatilities can differ dramatically across exchanges. This exchange-specific BTC/USD data is vast, structured, and updated daily (Coindesk.com). However, to our knowledge, nothing in the current literature has thoroughly studied these large datasets. Past work on Bitcoin volatility has historically aggregated the price series across exchanges, thereby losing microeconomic information from the cross-exchange dynamics. Moreover, the scant existing work on Bitcoin exchanges generally does not venture beyond interpreting summary statistics (Petrov & Schuffla 2013). This paper aims to introduce familiar volatility analysis into the understudied field of Bitcoin exchanges and thereby lay a foundation for future quantitative study.

2 Literature Review

Cryptocurrencies are very much a new area of research, particularly in economics. Many papers (Meiklejohn et al. 2013; Reid & Harrigan 2013; Christin 2013) are concerned with transaction anonymity, while others focus on Bitcoin's unique computational characteristics, such as the blockchain[1] (Barber et al. 2012). Felten et al. (2013), however, examine from a game theoretic perspective how miner collusion might threaten the integrity of the Bitcoin protocol.

Although Bitcoin's infamous price volatility is frequently mentioned in cryptocurrency literature, little quantitative analysis exists. Some Reed College students (Buchholz et al. 2012) used GARCH variants to test whether volatility affected the price mean. They found statistically significant effects for certain historical bubble periods, concluding that positive shocks generally produced more volatility than negative ones. Yermack (2013) finds little correlation between daily movements in Bitcoin and those in European fiat currencies, while another paper (Sapuric & Kokkinaki 2014) adjusts the effect of price volatility for daily transaction volume and consequently finds that the price movements smooth. Nevertheless, past volatility work has neither analyzed intraday or high-frequency data nor tested for cross-exchange effects.

Even more meager research exists on specific exchanges. Moore and Christin (2013) studied daily data from 40 of these in order to determine why some close down. With a proportional hazards model, they find that large transaction

[1] The blockchain is a distributed public ledger of Bitcoin's entire transaction history.

volume is the best predictor of long lifetimes for exchanges. Petrov and Schuffla (2013) discuss potential arbitrage opportunities for 2-3 exchanges, but do not investigate beyond summary statistics.

Despite this dearth of Bitcoin-specific analysis, one can find much discussion about cross-market volatility effects in the general financial literature. In a highly-cited paper, Hamao et al. (1990) use a modified GARCH (Generalized Autoregressive Conditional Heteroskedasticity) to examine "spillover effects" for prices in three US, UK, and Japanese stock exchanges. They find that volatility in the larger New York exchange significantly impacts volatility in the smaller Tokyo one[2]. Since then, many multivariate GARCH (MGARCH) models have been developed to capture more complex effects, such as asymmetric movements and time-varying correlations, and have been applied to markets for stocks, bonds, and commodities (Ledoit et al. 2003; Xiao & Dhesi 2010).

In this paper, we test for spillover effects in price volatility across eight Bitcoin exchanges of various sizes. In particular, we draw upon the work of Chevallier (2012), who studies time-varying correlations in oil, gas, and carbon dioxide prices through the BEKK, CCC, and DCC variants of MGARCH. In addition to searching for volatility spillover, we show that leakages across the larger exchanges demonstrate high market efficiency, and that these effects usually weaken when propagating to smaller institutions. We show that correlations in volatility change over time. Finally, we interpret the spillover coefficients to determine which exchanges are the largest volatility transmitters, and in general, understand where volatility travels within the Bitcoin ecosystem.

3 Data

Most Bitcoin papers focus on daily data aggregated across exchanges, which is easier to find, usually through popular cryptocurrency data providers such as blockchain.info. However, we examine higher-frequency data for 7 major USD/Bitcoin exchanges as well as a smaller Chinese one: Bitfinex, ANXBTC, Bitstamp, LakeBTC.com, HitBTC, itBit, Kraken, and 1coin.com. We rank them by transaction volume in USD in the last 30 days, and for computational ease, omit OKCoin, BTC-e, BTCChina, Huobi, and the defunct Mt. Gox. Table 1 below describes the total market cap for each of our exchanges (Bitcoinity.org).

[2] Hamao et. al, p. 2. The definition of spillover effects: "... whether changes in price volatility in one market are positively related to changes in price volatility observed in the next market to trade."

Table 1: Bitcoin exchanges by 30-day transaction volume (4/15/15).

Exchange	Location	BTC Volume	Market Cap
Bitfinex	British Virgin Islands	908,382	6.77%
LakeBTC	Shanghai	350,590	2.61%
Bitstamp	Slovenia	251,934	1.88%
ANXBTC	Hong Kong	236,798	1.76%
Kraken	San Francisco	76,421	0.57%
itBit	Singapore	61,024	0.45%
HitBTC	United Kingdom	7,558	0.06%
1coin	China	unlisted	unlisted

In each exchange, the full history of the BTC/USD exchange rate is recorded as an irregular time series. This data is updated daily at bitcoincharts.com, and can be accessed in a backend file archive for full bulk download (Bitcoincharts.com). The name of each exchange's BTC/USD dataset contains the exchange name followed by the currency: for example, Bitfinex's file is `.bitfinexUSD.csv.gz` indicating a compressed .gz form. A quick cross-check of the recent BTC/USD prices for Bitstamp on both bitcoincharts.com and Bitstamp's official Twitter feed ensures the reliability of the data (`http://www.twitter.com/bitstampusd`). Market caps can also be viewed in a graphical pie chart online (`http://www.bitcoincharts.com/charts/volumepie`).

Intervals in these irregular time series range from every few seconds to several hours or more. As of March 2015, the datasets ranged from approximately 40,000 to 6.5 million observations. Table 2 below shows summary statistics for each series up through March 2015.

Table 2: Summary statistics for the exchanges.

Name	Start Date	End Date	Min.	1st Q.	Median	Mean	3rd Q.	Max.
1coin	3/9/14	3/21/15	0.20	315.5	378.7	407.9	501.5	679.0
ANXBTC	8/20/13	3/20/15	98.00	238.5	276.7	316.5	327.7	4980.0
Bitfinex	3/31/13	3/20/15	0.01	250.9	363.4	405.7	521.4	1175.0
Bitstamp	9/13/11	3/20/15	2.20	233.1	445.1	442.1	619.2	1163.0
HitBTC	12/27/13	3/20/15	179.40	449.1	573.9	547.4	624.4	1014.0
itbit	8/24/13	3/21/15	100.00	275.0	369.3	394.3	480.0	1200.0
kraken	1/7/14	3/20/15	175.00	385.0	559.7	533.4	648.1	918.1
lake	3/1/14	3/20/15	162.70q	294.2	372.0	382.5	472.4	1000.0

Although all series follow the same general moving price average, exchange-specific variance explains why summary statistics sometimes differ dramatically. Figure 2 (Appendix) shows the Bitstamp and Kraken series during the same time period (11/15/14–11/18/14). As expected, both exhibit similar trend but dissimilar volatility, and the time between transactions is much longer in the smaller Kraken exchange.

Many time series methods require perfectly aligned intervals across multiple series. We use intraday averages (every 3 hours) to align the Bitcoin datasets. We study the recent period 4/1/14–3/20/15 when all exchanges were active, which produces eight series of approximately 3,000 observations each. Nevertheless, simple intraday means may produce complex biases, and so future work should investigate more sophisticated approaches for handling this systematic difficulty (Eckner 2014).

4 Methodology

Recall that the univariate GARCH(p, q) model expresses the conditional variance at time t as a linear function of past mean-corrected returns (the residual returns) ϵ_t and conditional variances h_t:

$$r_t = \mu_t + \epsilon_t \tag{1}$$

$$\epsilon_t = h_t^{\frac{1}{2}} z_t \tag{2}$$

$$h_t = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j h_{t-j} \tag{3}$$

The model assumes $z_t \sim N(0, 1)$ is a white noise process (Bollerslev 1986; Engle 2001). Thus the GARCH$(1, 1)$ model is given by:

$$h_t = \alpha_0 + \alpha_1 \epsilon_{t-1}^2 + \beta_1 h_{t-1} \tag{4}$$

In the above equation, the variable a_0 measures the average long-term (unconditional) volatility, while β_1 describes how each volatility shock feeds into the next one. The quantity $\alpha_1 + \beta 1$ is called the *persistence*, which measures how quickly volatility shocks decay (Campbell 1997). When the persistence is equal to 1, volatility does not fade over time (Beg & Anwar 2014).

The multivariate case models volatility relationships across multiple series [3].

$$\boldsymbol{r_t} = \boldsymbol{\mu_t} + \boldsymbol{\epsilon_t} \tag{5}$$

$$\boldsymbol{\epsilon_t} = \boldsymbol{H_t^{\frac{1}{2}}} \boldsymbol{z_t} \tag{6}$$

Above, $\boldsymbol{r_t}$ is a $(m, 1)$ vector of returns for m series, and $\boldsymbol{\mu_t}, \boldsymbol{\epsilon_t}$ are $(m, 1)$ vectors of the means and residuals. The $(m, 1)$ i.i.d. vector z_t is subject to $E[z_t] = 0, Var[z_t] = I_m$. As dimensionality increases, it becomes computationally infeasible to estimate the number of parameters and verify the involved covariance matrices are positive-definite. Consequently, a variety of methods for estimating the conditional covariance matrix H_t (of r_t) have appeared in the MGARCH literature. Some (VEC, BEKK) attempt to directly model H_t, while others (factor models) an save computation by assuming the returns are

[3] Bold text indicates vectors or matrices.

generated by implicit conditionally-heteroskedastic, GARCH-like processes (Silvennoinen & Teräsvirta 2009).

Correlation models instead decompose the conditional covariance matrix H_t into conditional standard deviations and correlations. Conditional correlation is defined as follows (Engle 2002):

$$\rho_{12,t} = \frac{E_{t-1}[r_{1,t}, r_{2,t}]}{\sqrt{E_{t-1}[r_{1,t}^2]E_{t-1}[r_{2,t}^2]}} \tag{7}$$

The Constant Conditional Correlation (CCC-) GARCH model, introduced by Bollerslev (1990), assumes the conditional correlations to be time-invariant. This allows the conditional covariance matrix to be partitioned into two (m, m) matrices $\boldsymbol{D_t}$ and $\boldsymbol{P}$:

$$\boldsymbol{H_t} = \boldsymbol{D_t P D_t} \tag{8}$$

The matrix $\boldsymbol{D_t} = diag(h_{1t}^{\frac{1}{2}}, ..., h_{mt}^{\frac{1}{2}})$ contains the conditional standard deviations on the diagonals. The conditional correlation matrix $\boldsymbol{P}$ is a time-invariant, positive-definite matrix with every diagonal element equal to 1. Then each non-diagonal element of the conditional covariance matrix can be expressed as:

$$[H_t]_{ij} = h_{it}^{\frac{1}{2}} h_{jt}^{\frac{1}{2}} \rho_{ij} \mid i \neq j \tag{9}$$

To model the conditional variances $\boldsymbol{h_t}$, the individual returns r_{it} can be expressed as in the univariate GARCH(p,q) case. Then as in the univariate case, one can write for the vector of conditional variances [4]:

$$\boldsymbol{h_t} = \boldsymbol{\omega} + \sum_{i=1}^{p} \boldsymbol{A_i}[\boldsymbol{r_{t-i}} \circ \boldsymbol{r_{t-i}}] + \sum_{j=1}^{q} \boldsymbol{B_j h_{t-j}} \tag{10}$$

Finally, $\boldsymbol{H_t}$ satisfies the positive-definite requirement if $\boldsymbol{P}$ is positive-definite, the diagonal elements of the diagonal matrices $\boldsymbol{A_j}, \boldsymbol{B_j}$ are positive, and the elements of $\boldsymbol{\omega}$ are also positive (Silvennoinen & Teräsvirta 2009). Although CCC-GARCH measures constant correlations and conditional variances, the non-diagonal elements of the ARCH and GARCH parameter matrices $\boldsymbol{A_j}, \boldsymbol{B_j}$ are 0 by construction. This means that the vanilla CCC-GARCH model cannot directly estimate the one-way spillover effects of foreign innovations and volatilities. Some econometricians have used the computationally expensive Baba-Engle-Kraft-Kroner (BEKK) model to estimate these non-diagonal parameter terms (Engle & Kroner 1995; Xiao and Dhesi 2010; Teplova & Asturov n.d.).

For many assets, the assumption of a time-invariant conditional correlation matrix $\boldsymbol{P}$ may not be flexible enough. In Engle's (2002) widely-used Dynamic Conditional Correlation (DCC-) GARCH model, $\boldsymbol{P_t}$ (now time-variant) is specified as follows:

[4] The symbol $\circ$ denotes the Hadamard product, or the element-wise product of two matrices.

$$\boldsymbol{P_t} = (\boldsymbol{I} \circ \boldsymbol{Q_t})^{-}\frac{1}{2}\boldsymbol{Q_t}(\boldsymbol{I} \circ \boldsymbol{Q_t})^{-}\frac{1}{2} \tag{11}$$

$$\boldsymbol{Q_t} = (1 - \lambda_1 - \lambda_2)\boldsymbol{S} + \lambda_1 \boldsymbol{\epsilon_{t-1}}\boldsymbol{\epsilon'_{t-1}} + \lambda_2 \boldsymbol{Q_{t-1}} \tag{12}$$

This includes the *adjustment parameters* $\lambda_1, \lambda_2 : \lambda_1 + \lambda_2 < 1$, and $\boldsymbol{S}$ which is the unconditional correlation matrix of the standardized errors $\boldsymbol{\epsilon_t}$ (Silvennoinen & Teräsvirta 2009). The adjustment parameters determine how the conditional quasicorrelations (elements of $\boldsymbol{Q_t}$) change. These quasicorrelations can be regarded as the non-standardized correlations, and like the correlations, are symmetric between two variables: $q_t(A, B) = q_t(B, A)$. The presence of dynamic correlation can be tested with the null hypothesis $H_0 : \lambda_1 = \lambda_2 = 0$, since if the null hypothesis is true, $\boldsymbol{Q_t}$ is equal to the time-invariant $\boldsymbol{S}$ matrix and the $\boldsymbol{P_t}$ equation simply reduces to that of Constant Conditional Correlation (StataCorp 2013). With this method for calculating the conditional covariance matrix $\boldsymbol{H_t}$, the DCC-GARCH model can be run in three steps (Engle 2002). First, fit univariate GARCH models to the individual time series. Next, calculate the parameters with maximum likelihood estimation. Finally, estimate the coefficients for the conditional correlations. Subsequently, the conditional correlations between series may be graphed over time for visual analysis.

With additional computation, extended forms for CCC-GARCH and DCC-GARCH can be substituted for the BEKK model to directly estimate the nonzero elements of the ARCH and GARCH parameter matrices – precisely the spillover effects of interest. The diagonal elements of $\boldsymbol{D_t}$, the conditional variances $h_{ii,t}$ can be computed with a vectorized GARCH equation that allows for cross-exchange effects. For example, with two exchanges we can write the following (He & Teräsvirta 2004; Teplova & Asaturov n.d.):

$$\begin{pmatrix} h_{11,t} \\ h_{22,t} \end{pmatrix} = \begin{pmatrix} c_1 \\ c_2 \end{pmatrix} + \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} * \begin{pmatrix} \epsilon^2_{1,t-1} \\ \epsilon^2_{2,t-1} \end{pmatrix} + \begin{pmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{pmatrix} * \begin{pmatrix} h_{11,t-1} \\ h_{22,t-1} \end{pmatrix} \tag{13}$$

Before the ARCH/GARCH analysis, we show with the Augmented Dickey-Fuller test for unit roots that a first difference renders the Bitcoin series stationary at the 5% significance level. We use Engle's (1984) standard Lagrange Multiplier Test to test for ARCH effects, i.e. volatility clustering. The test statistic for the method is of the form TR^2, where the first quantity T is the sample size and R is the sample multiple correlation coefficient from a regression of the residuals: for q lags, it has a $\chi^2(q)$ asymptotic distribution. To confirm the result, we also calculate the Ljung-Box Q-Statistic for the McLeod-Li test, which is given by (McLeod & Li 1983; Wang et al. 2005):

$$Q = N(N + 2) \sum_{k=1}^{L} \frac{\hat{r_k}(\epsilon^2)}{N - k} \tag{14}$$

This statistic has a $\chi^2(L)$ asymptotic distribution and can be used to test the null hypothesis of no ARCH effect. Therefore, both of these statistical tests can reveal whether the Bitcoin data exhibits the conditional heteroskedasticity necessary for ARCH/GARCH models.

Subsequently, to follow Chevallier (2012) we fit a VAR(1) model to the set of Bitcoin series in order to remove linear serial dependence and trend. The general equation for a VAR(p) model is given by the following equation:

$$\boldsymbol{y_t} = \boldsymbol{\alpha_0} + \sum_{i=1}^{p} \boldsymbol{\alpha_i y_{t-i}} + \boldsymbol{\epsilon_t} \tag{15}$$

The quantity y_t is a vector containing the observations for every time series at time t. we run CCC-GARCH on the residuals to examine correlations in volatility. We run a Wald test on the adjustment parameters λ_1 and λ_2 to test whether these correlations change over time (dynamic correlations). We run DCC-GARCH to obtain the quasicorrelations and p-values to study the strength of time-varying spillovers. We plot the dynamic conditional correlations to understand the hierarchy of volatility correlations. Finally, with the extended DCC-GARCH model, we estimate the specific coefficients for ARCH/GARCH spillovers onto volatility.

5 Results

Because all series cluster around the same trend, we start by applying a simple first difference to the eight Bitcoin series to ensure stationarity (Wang et al. 2005). To test this, we run the Augmented Dickey-Fuller test, which checks for unit roots in a time series. If the null hypothesis of existing unit roots is rejected, then the time series may be regarded as stationary at some significance level (Said & Dickey 1984). For each of the series, the ADF test yields a p-value of approximately ≤ 0.01, a strong sign of stationarity. The ACF plots of the differenced series, shown in Figure 3 (Appendix), also confirm this through clear exponential decay in the autocorrelations. The small exception is the Kraken exchange, which has a few insignificant longer lags. A seasonal-trend decomposition by LOESS (Figure 4, Appendix), with the original frequency of eight intraday units, reveals a volatile but seemingly unmoving trend for the Kraken exchange. Nevertheless, because the ADF test still confirmed stationarity, we use the first-differenced Kraken data without modification. Each time series is approximately integrated of order 1, fulfilling the requirements for a VAR(p) model.

Before testing the differenced data for ARCH effects, it is important to remove linear serial dependence, usually with an AR-based model (Chevallier 2012). We use a VAR(1) model, which also gives some insight into the mean-corrected returns. Tables 5 and 6 in the Appendix show the coefficients and the p-values for each of the regressions. For example, the row for 1coin in Table 3 represents below regression for our $N = 8$ total exchanges:

$$y_{1coin,t} = \alpha_{0,1coin} + \sum_{i=1}^{N} \alpha_{i,1coin} y_{exchange_i,t-1} \tag{16}$$

We generally see significant and negative coefficients on the main diagonal, indicating that an exchange's return at time t will move opposite to the return at time $t-1$. The autoregressive coefficients for itBit and ANXBTC are particularly high in absolute value, and their p-values are extremely low (on the orders of 10^{-78} and 10^{-115}). The returns for ANXBTC are significantly affected by only itself, Bitfinex, and the growing Kraken exchange. This scarcity of significant effects may be attributed to the omitted effects of other large Chinese exchanges such as BTCChina and Huobi. Although Bitstamp was previously the largest exchange, after a catastrophic breach in January 2015, it quickly yielded to the growing Bitfinex exchange, which in the above table wields significant influence over all other exchanges (Bitcoinity.org). This suggests that Bitcoin investors, following the accident, moved their assets away from Bitstamp into more secure institutions. As a result, while Bitstamp, Bitfinex, and LakeBTC all influence each other, Bitstamp's effects are notably weaker. Finally, it is evident larger exchanges have a generally stronger impact on smaller ones, rather than the other way around.

Figure 5 (Appendix) shows the residual series for each individual regression estimated by VAR(1). The Bitcoin data exhibits clear volatility clustering: alternating periods of turmoil and calm. To detect ARCH effects, one can use Engle's (1982) well-known Lagrange Multiplier test, which yields a χ^2 statistic. Running the test on the various lags for the eight Bitcoin exchanges yields high χ^2 values:

Table 3: χ^2 statistics & p-values for ARCH effects (Engle 1982).

Term	$t-1$	$t-2$	$t-3$	$t-4$	$t-5$
1coin, χ^2	9.19	9.20	9.25	84.18	188.21
Bitfinex, χ^2	73.93	97.21	154.08	184.47	200.65
LakeBTC, χ^2	79.54	100.56	183.19	220.41	234.90
ANXBTC, χ^2	351.18	351.08	357.90	358.78	358.68
Bitstamp, χ^2	75.90	103.10	183.66	208.17	220.86
HitBTC, χ^2	67.26	86.05	123.97	148.79	155.20
itBit, χ^2	457.15	457.18	474.41	476.61	476.48
Kraken, χ^2	292.64	293.68	295.41	297.01	299.47
1coin, p_{χ^2}	0.00	0.01	0.03	0.00	0.00
Bitfinex, p_{χ^2}	0.00	0.00	0.00	0.00	0.00
LakeBTC, p_{χ^2}	0.00	0.00	0.00	0.00	0.00
ANXBTC, p_{χ^2}	0.00	0.00	0.00	0.00	0.00
Bitstamp, p_{χ^2}	0.00	0.00	0.00	0.00	0.00
HitBTC, p_{χ^2}	0.00	0.00	0.00	0.00	0.00
itBit, p_{χ^2}	0.00	0.00	0.00	0.00	0.00
Kraken, p_{χ^2}	0.00	0.00	0.00	0.00	0.00

High test statistics and low corresponding p-values suggests extremely significant ARCH effects, with 1coin's third lag being the least significant (but still well under the 5% level). To confirm this, we also run McLeod & Li's (1983) test for conditional heteroskedasticity. Plots of the p-values are in Figure 6 (Appendix), which similarly confirm the existence of volatility clustering. Although all lags are significant, many studies use only GARCH(1,1) for the univariate GARCH models involved in CCC- and DCC-GARCH, including Engle (2002) in his original work. Therefore, we follow the literature with a parsimonious GARCH(1,1).

The regular CCC-GARCH and DCC-GARCH models do not directly estimate one-way spillover coefficients, but do measure symmetric correlations in volatility. Therefore, one can find which exchanges are the focal points of volatility spillovers simply through the number and magnitude of their correlation coefficients. For example, if Bitfinex demonstrates comparatively high correlations in volatility with every other exchange, we may deduce that Bitfinex is a central transmitter or receiver of volatility. Table 7 (Appendix) shows the conditional correlations between the series, estimated with CCC-GARCH under the assumption of constant conditional correlations. All correlations are positive and extremely significant. We would expect this under CCC's constant correlation assumption, since historically, major shocks have generally been followed by high volatility simultaneously across exchanges. The largest correlation coefficients, with the exception of HitBTC, belong to the Bitfinex/Bitstamp/LakeBTC trading pairs. However, pairs involving smaller exchanges generally have much lower correlations. This suggests that price information updates near-instantaneously across the highly efficient large exchanges, while propagating more slowly to update inefficient smaller ones.

The individual univariate GARCH(1,1) models, fitted for CCC-GARCH, show generally high persistence ($\alpha + \beta$), as seen below in Table 4.

Table 4: α,β terms estimated by CCC-GARCH.

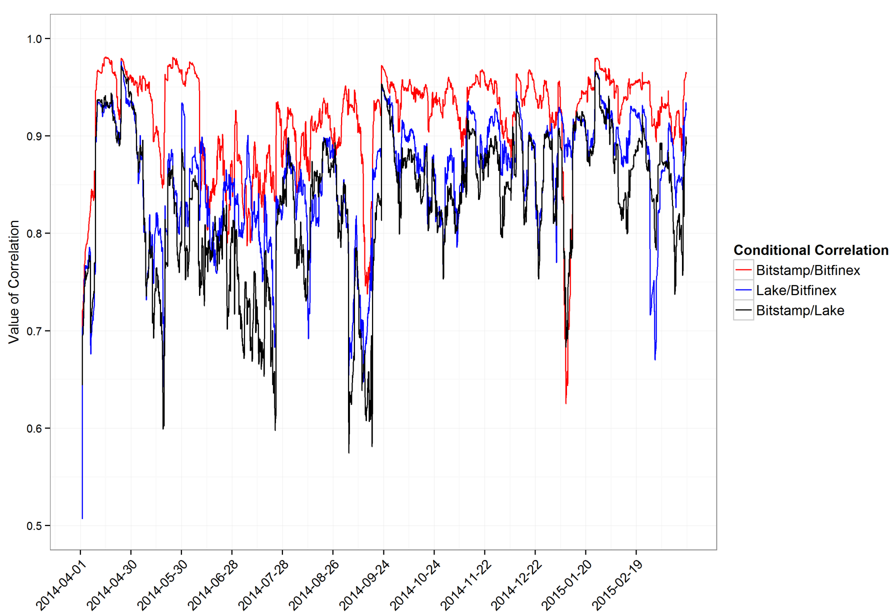
Term	StdErr.	p	z
α_{1coin}	0.040	0.009	4.865
β_{1coin}	0.707	0.042	16.643
$\alpha_{Bitfinex}$	0.050	0.003	14.509
$\beta_{Bitfinex}$	0.919	0.005	181.301
$\alpha_{LakeBTC}$	0.046	0.004	12.519
$\beta_{LakeBTC}$	0.917	0.006	154.610
α_{ANXBTC}	0.100	0.008	12.661
β_{ANXBTC}	0.914	0.005	175.572
$\alpha_{Bitstamp}$	0.060	0.004	14.491
$\beta_{Bitstamp}$	0.903	0.006	149.896
α_{HitBTC}	0.057	0.005	12.556
β_{HitBTC}	0.903	0.006	131.660
α_{itBit}	0.246	0.012	19.800
β_{itBit}	0.871	0.005	189.499
α_{Kraken}	0.277	0.025	10.914
β_{Kraken}	0.686	0.025	27.766

The persistence of larger exchanges in particular is generally close to 1. This is intuitive, since volatility shocks in these high-volume institutions may take a very long time to disappear. Smaller exchanges also display medium-high persistence, which may be attributed to spillovers from their larger neighbors.

The results of Dynamic Conditional Correlation GARCH are shown in Table 8 (Appendix).The coefficients in the table are the conditional quasicorrelations in the time-variant DCC-GARCH model. While almost all the coefficients are statistically significant at the 5% level, they are also roughly increasing in the quasicorrelation coefficients. Therefore, larger differences in quasicorrelation magnitude allows us to infer stronger causality. The largest, most significant quasicorrelations are between Bitfinex/Bitstamp/LakeBTC, perhaps due to the market efficiency in big exchanges discussed earlier. We also observe fairly high coefficients in larger/smaller pairs, while the smaller/smaller pairs tend to have the lowest coefficients. These results strongly suggest that volatility from a large exchange indeed leaks into its neighbors, although surprisingly, larger neighbors are impacted far more than smaller ones. One can therefore infer the existence of a positive feedback loop that amplifies volatility as it circulates across the large exchanges, allowing shocks to persist for long periods of time.

A simple Wald test with the hypothesis $H_0 : \lambda_1 = \lambda_2 = 0$ gives a very large $\chi^2(2) = 1 + E06$ statistic and a very small corresponding p value. We can therefore reject the CCC-GARCH assumption of constant conditional correlations in volatility across these Bitcoin series, which can be confirmed with visual analysis. In Figure 1 below, we see the conditional correlations plotted over time for the top 3 exchanges: Bitstamp, Bitfinex, and LakeBTC.

Fig. 1: Dynamic correlations between Bitstamp/Bitfinex/LakeBTC.



The conditional correlations are all very high, and move similarly: Bitstamp and Bitfinex have a mean conditional correlation of approximately 0.916. Such similar co-movements in the conditional correlations lend support to the market efficiency proposed earlier. Evidence for this appears in the plots for the smaller exchanges: the conditional correlations between smaller exchanges and Bitstamp/Bitfinex are noticeably lower, and likely reflect the delay in price information from those large exchanges. Figure 7 (Appendix) confirms that correlations change drastically over time, since CCC-GARCH would model the correlations as a flat horizontal line. Note that the conditional correlations with Bitstamp/Bitfinex vary dramatically based on the exchange: for example, right after the 2015 Bitfinex breach the correlations of itBit remain stable around 0.7, while those of HitBTC plummet. It seems, then, that these small exchanges react very differently to movements in large exchanges. For instance, one possible reason for itBit's resilience to movements in the large exchanges is that itBit has offered incentives to lure traders away from other platforms, perhaps creating an insulated community (Cawrey 2014).

The Extended DCC-GARCH model explicitly estimates the spillover effects of foreign returns and volatilities in the non-diagonal elements of its ARCH and GARCH matrices. Table 9 (Appendix) shows the ARCH parameter matrix calculated by EDCC-GARCH. We see that, in general, the lagged returns have little if any effect on their corresponding conditional variances $h_{ii,t}$. However, returns for the large Bitfinex and Bitstamp exchanges do have a weak impact on the volatility of the other exchanges. The lagged returns for Bitfinex have a relatively powerful effect on the volatility of itBit, whose lagged returns impact the other exchanges: perhaps Bitfinex affects these primarily through itBit. In general, domestic volatilities are impacted more by foreign spillovers in volatility than spillovers in returns. The GARCH parameter matrix in Table 10 (Appendix), which estimates the spillover effects $h_(ij,t) : i \neq j$ of foreign volatilities on domestic ones, has generally higher values than the previous ARCH matrix. The smallest exchange 1coin neither impacts nor is impacted by the other exchanges (except for itBit). Volatility in the large exchanges Bitfinex, LakeBTC, and ANXHK spills over strongly into other exchanges, as evidenced by the high coefficients for their lagged conditional volatilities. BitStamp and the smaller HitBTC demonstrate weaker spillovers, while itBit and Kraken exert almost unnoticeable effects. Surprisingly, ANXBTC generally demonstrates stronger spillover effects than any of its larger peers. In particular, large GARCH coefficients appear in both directions between BitFinex and ANXBTC, suggesting a positive feedback loop in volatility between those two exchanges. Finally, we see generally large coefficients on the main diagonal for the larger exchanges, affirming that volatilities in Bitcoin exchanges tend to be highly persistent.

Through CCC-GARCH, DCC-GARCH, and EDCC-GARCH analysis, we may thus conclude that volatility usually updates near-instantaneously across large exchanges (efficient markets hypothesis), and that in these large exchanges, volatility lasts for long periods of time (GARCH persistence) due to positive feedback loops in volatility spillovers. While the magnitude of volatility spillovers is not especially large, the large exchanges clearly leak volatility into one another and, to a lesser extent, into smaller exchanges. Small exchanges do not exhibit such positive amplification loops, as suggested by the CCC-GARCH and EDCC-GARCH results. A Wald Test on the DCC-GARCH adjustment parameters gives highly significant evidence for time-varying correlations in volatility, which appears in the visual results of DCC-GARCH. Correlations in volatility may also be impacted by shocks, but the results may vary across smaller institutions. Finally, our preliminary analysis also shows that mean-corrected returns for the larger exchanges significantly impact those of smaller ones.

6 Concluding Remarks

The decentralization and convenience of Bitcoin has allowed it to grow rapidly among black market merchants, political dissidents, and the tech elite. Yet despite fast growth in recent years, the virtual currency still suffers from high price volatility, and must arguably stabilize further before serving as a reliable store of value. To our current knowledge, this paper is the first to study price volatility dynamics across Bitcoin exchanges.

In this paper, we have statistically demonstrated the existence of price volatility spillovers in the Bitcoin ecosystem. While these leakages might promise lucrative arbitrage opportunities, more work is required for reliable volatility forecasting. Future work may wish to explore more advanced methods for aligning irregular time series, as well as other MGARCH variants that can employ exogenous Bitcoin-specific variables such as mining activity. Decentralization, blockchain authentication, and other quirks of the currency present theoretical and computational challenges for econometricians, and may thus require novel methods of inference.

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Fig. 2: Comparison of two exchanges of different sizes over 3 days.

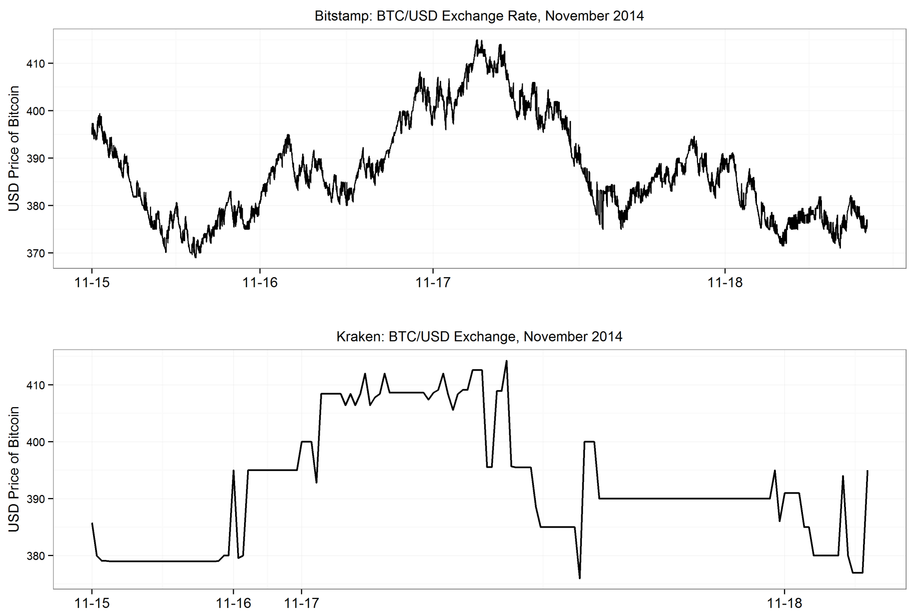

Fig. 3: ACF plots of each exchange's first-differenced series.

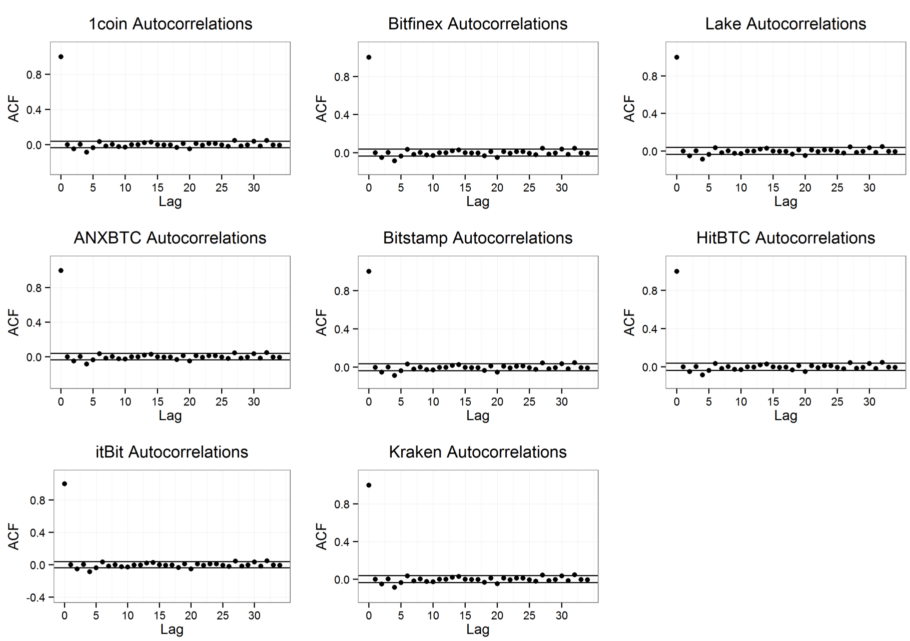

Fig. 4: Seasonal-Trend LOESS Decomposition of differenced Kraken series.

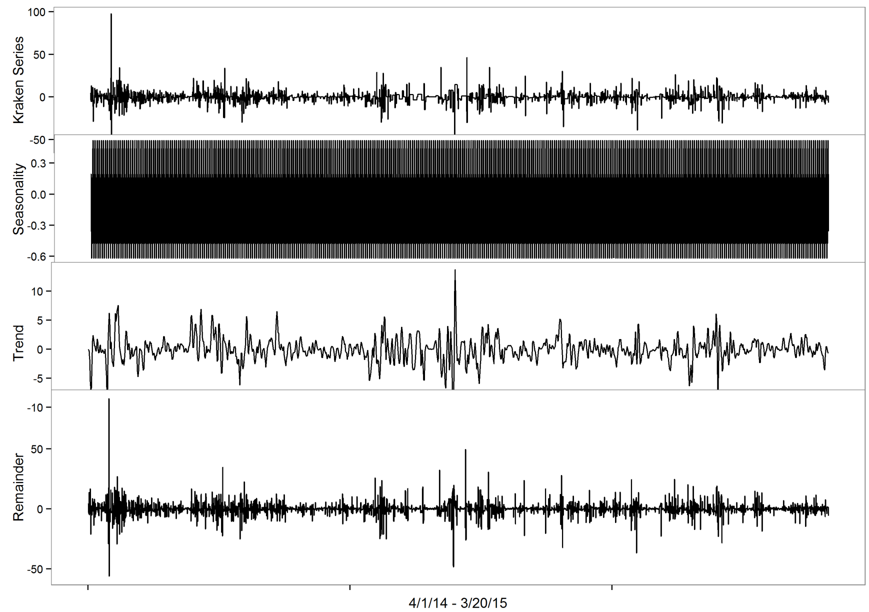

Fig. 5: The VAR(1) residual series for each exchange.

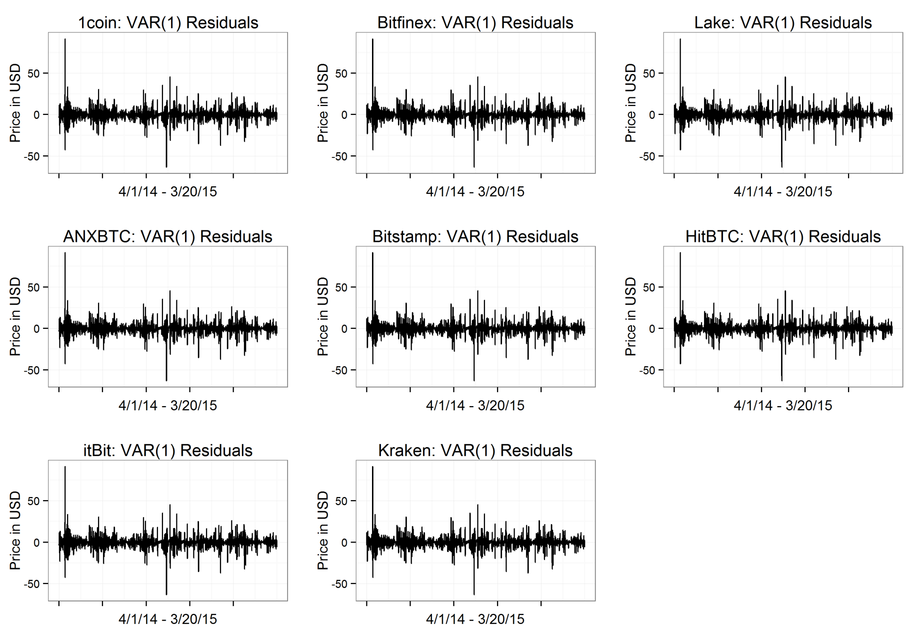

Fig. 6: P-values calculated by Mcleod & Li's (1983) test for ARCH effects.

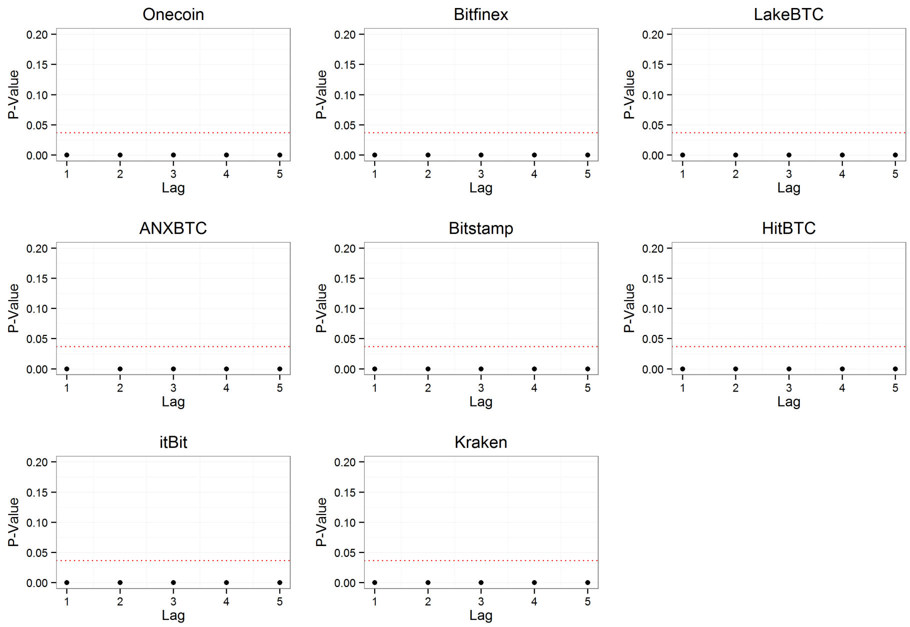

Fig. 7: Plots of the time-variant correlations estimated by DCC-GARCH. Each exchange is compared to the two large Bitstamp and Bitfinex exchanges.

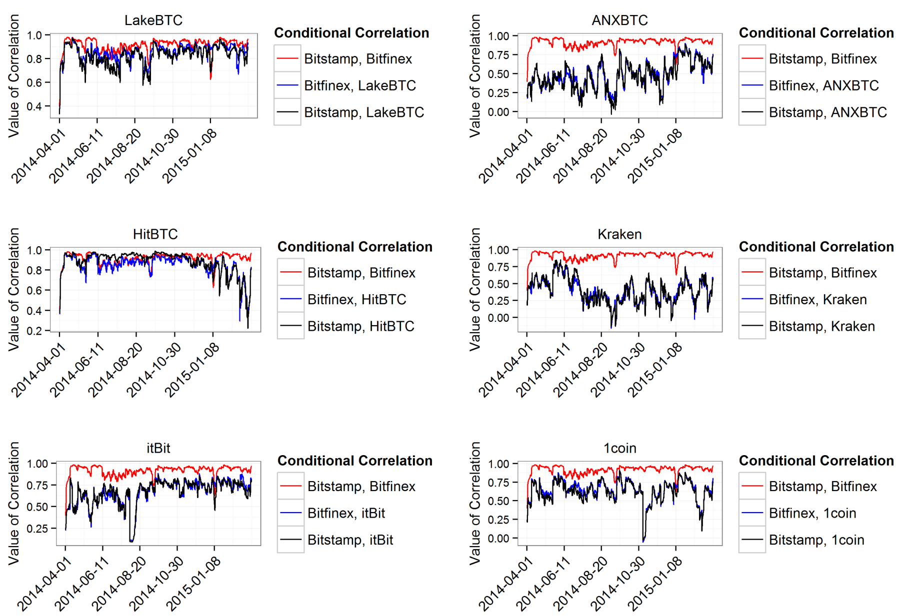

Table 5: VAR(1) coefficients. Regressors are on the columns.

	$\epsilon_{1coin,t-1}$	$\epsilon_{Bitfinex,t-1}$	$\epsilon_{LakeBTC,t-1}$	$\epsilon_{ANXBTC,t-1}$	$\epsilon_{Bitstamp,t-1}$	$\epsilon_{HitBTC,t-1}$	$\epsilon_{itBit,t-1}$	$\epsilon_{Kraken,t-1}$	Constant
$\epsilon_{1coin,t}$	0.031	0.532	-0.187	-0.001	0.051	-0.107	-0.001	0.026	-0.042
$\epsilon_{Bitfinex,t}$	0.028	0.315	-0.212	-0.002	0.094	-0.130	-0.012	0.037	-0.067
$\epsilon_{LakeBTC,t}$	0.020	0.556	-0.332	-0.006	0.123	-0.146	0.001	0.025	0.050
$\epsilon_{ANXBTC,t}$	0.034	0.391	0.087	-0.377	0.069	-0.070	0.022	0.054	-0.067
$\epsilon_{Bitstamp,t}$	0.029	0.552	-0.230	0.009	-0.117	-0.112	-0.014	0.039	-0.064
$\epsilon_{HitBTC,t}$	0.029	0.552	-0.230	0.009	-0.117	-0.112	-0.014	0.039	-0.064
$\epsilon_{itBit,t}$	0.037	0.533	-0.268	0.009	0.231	-0.035	-0.466	0.066	-0.061
$\epsilon_{Kraken,t}$	0.003	0.336	-0.101	-0.003	0.097	-0.025	-0.017	-0.106	-0.070

Table 6: VAR(1) p-values for the coefficients.

	$\epsilon_{1coin,t-1}$	$\epsilon_{Bitfinex,t-1}$	$\epsilon_{LakeBTC,t-1}$	$\epsilon_{ANXBTC,t-1}$	$\epsilon_{Bitstamp,t-1}$	$\epsilon_{HitBTC,t-1}$	$\epsilon_{itBit,t-1}$	$\epsilon_{Kraken,t-1}$	Constant
$\epsilon_{1coin,t}$	1.718e-01	2.159e-15	1.189e-03	9.411e-01	5.036e-01	9.426e-02	9.602e-01	1.456e-01	6.779e-01
$\epsilon_{Bitfinex,t}$	1.399e-01	2.591e-08	1.349e-05	8.738e-01	1.460e-01	1.700e-02	3.678e-01	1.483e-02	4.401e-01
$\epsilon_{LakeBTC,t}$	2.126e-01	6.648e-31	9.609e-16	6.547e-01	2.381e-02	1.396e-03	9.252e-01	4.985e-02	4.890e-01
$\epsilon_{ANXBTC,t}$	1.770e-01	1.271e-07	1.747e-01	1.044e-78	4.162e-01	3.210e-01	2.005e-01	6.458e-03	5.517e-01
$\epsilon_{Bitstamp,t}$	1.264e-01	9.104e-23	1.682e-06	5.179e-01	6.771e-02	3.598e-02	2.467e-01	9.523e-03	4.524e-01
$\epsilon_{HitBTC,t}$	1.450e-01	1.118e-24	7.037e-10	5.376e-01	2.719e-02	6.667e-07	1.743e-01	3.680e-02	5.506e-01
$\epsilon_{itBit,t}$	1.993e-01	3.151e-10	2.349e-04	6.572e-01	1.699e-02	6.648e-01	1.523e-115	3.504e-03	6.356e-01
$\epsilon_{Kraken,t}$	9.030e-01	2.080e-05	1.370e-01	8.760e-01	2.856e-01	7.441e-01	3.500e-01	6.145e-07	5.795e-01

Table 7: Correlations estimated by CCC-GARCH.

Pair	Coef.	StdErr.	z	p	$Left_{95\%}$	$Right_{95\%}$
$corr$(ANXBTC, Kraken)	0.247	0.018	13.7	0	0.212	0.282
$corr$(itBit, Kraken)	0.254	0.018	13.9	0	0.218	0.290
$corr$(1coin, Kraken)	0.268	0.018	15.1	0	0.233	0.302
$corr$(1coin, ANXBTC)	0.290	0.018	16.4	0	0.256	0.325
$corr$(1coin, itBit)	0.349	0.017	20.4	0	0.316	0.383
$corr$(ANXBTC, itBit)	0.372	0.017	22.0	0	0.339	0.405
$corr$(LakeBTC, Kraken)	0.385	0.016	23.4	0	0.352	0.417
$corr$(HitBTC, Kraken)	0.400	0.016	24.5	0	0.368	0.432
$corr$(Bitstamp, Kraken)	0.419	0.016	26.2	0	0.388	0.451
$corr$(Bitfinex, Kraken)	0.420	0.016	26.3	0	0.389	0.451
$corr$(ANXBTC, HitBTC)	0.480	0.015	32.4	0	0.451	0.509
$corr$(LakeBTC, ANXBTC)	0.509	0.014	35.5	0	0.481	0.537
$corr$(ANXBTC, Bitstamp)	0.513	0.014	36.0	0	0.485	0.541
$corr$(1coin, HitBTC)	0.519	0.014	37.0	0	0.491	0.546
$corr$(Bitfinex, ANXBTC)	0.525	0.014	37.7	0	0.498	0.553
$corr$(1coin, Bitstamp)	0.535	0.014	39.0	0	0.508	0.562
$corr$(1coin, Bitfinex)	0.548	0.013	40.8	0	0.522	0.575
$corr$(HitBTC, itBit)	0.569	0.013	42.9	0	0.543	0.595
$corr$(LakeBTC, itBit)	0.582	0.013	44.7	0	0.556	0.607
$corr$(1coin, LakeBTC)	0.593	0.012	47.5	0	0.568	0.617
$corr$(Bitstamp, itBit)	0.625	0.012	51.8	0	0.601	0.649
$corr$(Bitfinex, itBit)	0.626	0.012	51.9	0	0.602	0.649
$corr$(LakeBTC, HitBTC)	0.830	0.006	138.9	0	0.819	0.842
$corr$(LakeBTC, Bitstamp)	0.851	0.005	159.4	0	0.840	0.861
$corr$(Bitfinex, HitBTC)	0.868	0.005	182.2	0	0.859	0.878
$corr$(Bitfinex, LakeBTC)	0.874	0.005	191.9	0	0.865	0.883
$corr$(Bitstamp, HitBTC)	0.901	0.004	245.0	0	0.894	0.908
$corr$(Bitfinex, Bitstamp)	0.929	0.003	350.6	0	0.924	0.934

Table 8: Quasicorrelations, adjustment parameters estimated by DCC-GARCH.

Pair	Coef.	StdErr.	z	p	$Left_{95\%}$	$Right_{95\%}$
$corr$(Bitfinex, Bitstamp)	0.913	0.010	87.30	0	0.893	0.934
$corr$(Bitstamp, HitBTC)	0.892	0.012	72.17	0	0.868	0.917
$corr$(Bitfinex, LakeBTC)	0.850	0.018	46.46	0	0.814	0.886
$corr$(Bitfinex, HitBTC)	0.846	0.018	47.77	0	0.811	0.881
$corr$(LakeBTC, Bitstamp)	0.823	0.022	38.03	0	0.780	0.865
$corr$(LakeBTC, HitBTC)	0.806	0.023	35.26	0	0.761	0.851
$corr$(Bitfinex, ANXBTC)	0.573	0.045	12.75	0	0.485	0.661
$corr$(ANXBTC, Bitstamp)	0.572	0.046	12.48	0	0.482	0.662
$corr$(LakeBTC, ANXBTC)	0.570	0.046	12.48	0	0.481	0.660
$corr$(ANXBTC, HitBTC)	0.557	0.047	11.96	0	0.465	0.648
$corr$(Bitfinex, itBit)	0.487	0.048	10.15	0	0.393	0.582
$corr$(LakeBTC, itBit)	0.458	0.048	9.44	0	0.363	0.553
$corr$(Bitfinex, Kraken)	0.378	0.058	6.54	0	0.265	0.491
$corr$(Bitstamp, Kraken)	0.376	0.059	6.41	0	0.261	0.491
$corr$(Bitstamp, itBit)	0.366	0.057	6.41	0	0.254	0.478
$corr$(HitBTC, Kraken)	0.351	0.060	5.87	0	0.234	0.468
$corr$(LakeBTC, Kraken)	0.341	0.060	5.71	0	0.224	0.458
$corr$(1coin, Bitfinex)	0.318	0.058	5.48	0	0.204	0.432
$corr$(HitBTC, itBit)	0.309	0.060	5.13	0	0.191	0.427
$corr$(ANXBTC, itBit)	0.290	0.055	5.28	0	0.183	0.398
$corr$(ANXBTC, Kraken)	0.247	0.060	4.11	0	0.130	0.365
$corr$(1coin, LakeBTC)	0.229	0.069	3.30	0.001	0.093	0.364
$corr$(1coin, Bitstamp)	0.223	0.064	3.46	0.001	0.097	0.349
$corr$(1coin, HitBTC)	0.211	0.065	3.26	0.001	0.084	0.338
$corr$(itBit, Kraken)	0.204	0.056	3.63	0	0.094	0.314
$corr$(1coin, itBit)	0.199	0.046	4.29	0	0.108	0.290
$corr$(1coin, Kraken)	0.097	0.058	1.66	0.096	0.017	0.210
$corr$(1coin, ANXBTC)	0.016	0.058	0.28	0.78	0.098	0.130
λ	0.034	0.002	22.24	0	0.031	0.037
λ_2	0.958	0.002	431.79	0	0.953	0.962

Table 9: Coefficients from the EDCC-GARCH ARCH matrix.

	$\epsilon^2_{1coin,t-1}$	$\epsilon^2_{Bitfinex,t-1}$	$\epsilon^2_{LakeBTC,t-1}$	$\epsilon^2_{ANXBTC,t-1}$	$\epsilon^2_{Bitstamp,t-1}$	$\epsilon^2_{HitBTC,t-1}$	$\epsilon^2_{itBit,t-1}$	$\epsilon^2_{Kraken,t-1}$
$h_{1coin,t}$	0.064	0.037	0.000	0.000	0.000	0.000	0.164	0.002
$h_{Bitfinex,t}$	0.000	0.038	0.011	0.000	0.004	0.008	0.088	0.040
$h_{LakeBTC,t}$	0.000	0.067	0.015	0.000	0.000	0.000	0.102	0.061
$h_{ANXBTC,t}$	0.000	0.022	0.015	0.006	0.096	0.000	0.003	0.000
$h_{Bitstamp,t}$	0.000	0.049	0.014	0.000	0.000	0.016	0.083	0.098
$h_{HitBTC,t}$	0.00	0.008	0.008	0.000	0.015	0.080	0.097	0.079
$h_{itBit,t}$	0.110	1.278	0.0100	0.000	0.000	0.009	0.032	0.008
$h_{Kraken,t}$	0.001	0.042	0.000	0.000	0.084	0.078	0.141	0.850

Table 10: Coefficients from the EDCC-GARCH GARCH matrix.

	$h_{1coin,t-1}$	$h_{Bitfinex,t-1}$	$h_{LakeBTC,t-1}$	$h_{ANXBTC,t-1}$	$h_{Bitstamp,t-1}$	$h_{HitBTC,t-1}$	$h_{itBit,t-1}$	$h_{Kraken,t-1}$
$h_{1coin,t}$	0.000	0.000	0.000	0.000	0.000	0.000	0.666	0.000
$h_{Bitfinex,t}$	0.000	0.249	0.051	0.289	0.022	0.020	0.000	0.000
$h_{LakeBTC,t}$	0.000	0.105	0.015	0.147	0.004	0.007	0.000	0.000
$h_{ANXBTC,t}$	0.000	0.155	0.008	0.793	0.006	0.005	0.000	0.000
$h_{Bitstamp,t}$	0.000	0.158	0.071	0.264	0.023	0.051	0.000	0.000
$h_{HitBTC,t}$	0.000	0.144	0.052	0.20	0.015	0.018	0.000	0.000
$h_{itBit,t}$	0.083	0.108	0.171	0.017	0.048	0.081	0.000	0.009
$h_{Kraken,t}$	0.000	0.111	0.134	0.231	0.068	0.080	0.000	0.001