Abstract

I study an extension to the dynamic model of trial-and-error search modeled by Steven Callander in which agents do not have complete knowledge of how choices are mapped into outcomes. In this model, the mapping is represented as the realized path of a Brownian motion, and agents learn about the mapping by observing the choices of earlier agents and the outcomes that are realized. I extend the model by considering the role of patent protection in this model's application to the product development cycle. The breadth of patent protection is operationalized as an interval in the product space, while the patent-induced rewards to experimentation are modeled as the cost of product imitation. This model predicts that a search will tend to stabilize to better outcomes when the breadth of protection is lower, the cost of imitation is greater, or both.

Contents

1 Introduction

Product development is a process that relies heavily on learning by trial and error. Uncertainty in the potential outcome of experimentation leads agents to estimate the returns to experimentation by observing the outcomes of previous products. This trial and error search for ideal outcomes is easily described algorithmically, but a new model addresses the search analytically from the perspective of rational optimizing agents. In this model, Callander (2011) models the uncertainty in trial-and-error search using Brownian motion. Each potential product is mapped into an outcome by the realized path of a Brownian motion.

Agents are aware of both the drift and the variance of the Brownian motion in Callander's search model. The variance serves as the source of the uncertainty in the search, while the agents' knowledge of the drift allows them to predict outcomes. In addition, agents are aware of all previous products and their outcomes, allowing the search to evolve over time through accumulative learning. Each agent's knowledge of previous products implies that knowledge in the Callander model is both non-rival and non-excludable. The production of new goods is, therefore, a public good *per se* in this search model.

The model is solved constructively, resulting in two distinct phases in the product development cycle. The first is the monotonic phase, in which search is increasing in one direction. The second phase is the triangulating phase, in which the search straddles the ideal product; the search then zeros in on the best outcome. The end result is stability, which can be reached from either the monotonic phase or the triangulating phase. When the search reaches stability, no experimentation occurs, and the most recent product is replicated into perpetuity. Section 2 provides a summary of this model.

In this paper, I extend the Callander model by considering patent protection. This paper models several dimensions of patent protection, including the breadth and strength of protection, and describes their implications on the search for good outcomes.

Motivation

Patents are typically designed with two main intentions in mind. The first is to encourage innovation by rewarding experimentation directly (Gallini, 1992). The second is to encourage the disclosure of inventions so that others can build upon past inventions (Mazzoleni and Nelson, 1998). Both of these goals aim to encourage economic growth through the development of new products and ideas (Porter, 1990). The enforcement of intellectual property rights (IPRs),[1] however, is also restrictive in nature; by preventing market entry and the consequent competition, patents inherently restrict experimentation to some extent. Therefore, a social planner who seeks to maximize the quantity and the quality of products developed through patent protection needs to consider the economic benefits associated with the increase in incentives to the individual entrepreneur on one hand, and the economic costs associated with the decrease in market entry on the other.

[1]Intellectual property rights can be divided into patents and utility models, which are easier to apply for. This paper does not differentiate between the two, and instead models a simple patent system as a generalization of intellectual property rights of any kind.

Patent protection can be categorized into several dimensions: the breadth, length, and strength. The length, or duration, of a patent has been studied as the subject of several influential papers in the context of optimizing from a social perspective.[2] I operationalize the strength of patent protection as the cost of imitation under a patent. This cost includes the fees an entrepreneur must pay to license a patented product (Shapiro, 1985) in addition to any other implicit costs of replication, such as the probability-adjusted cost of potential litigation. The breadth of protection can be thought of as the number of potential products that are covered by the patent; a patent with a large breadth of protection should prevent the entry of more products than the same patent with a smaller breadth of protection.

The economics of patent protection is evident in theoretical tradeoff mentioned above. Although patent protection may encourage experimentation by altering the incentive structure of the product development cycle (Grossman and Lai, 2004), it may also restrict experimentation if its breadth of protection is too great. If patent policy is too thorough in this regard, it may prevent the development of new products by covering too large of a range of potential products (Bessen and Maskin, 2009). Patent policy that is too restrictive stifles the spillover of knowledge by prematurely stopping the development cycle. While the strength and length of patent protection encourage experimentation by increasing the rewards to experimentation, the breadth of protection hinders it by restricting market entry. This paper attempts to model this tension through learning by doing.

[2]Gilbert and Shapiro (1990) suggest that longer patent lives are optimal. Though others (Judd, 1985) find that infinite patent duration may be suboptimal from a social perspective, this model assumes an infinite duration of protection for simplicity.

Literature Review

Both theoretical and empirical estimates of the effects of patent protection on economic growth have varied considerably over time and across studies (Kanwar and Evenson, 2003; Mazzoleni and Nelson, 1998). Current empirical results point to a differentiation in this effect between countries of varying developmental stages, concluding that enforcement of intellectual property rights contributes to growth in developed countries, but generally has a more negative contribution towards growth in developing countries (Allred and Park, 2007; Kim et al., 2012). While cross-country comparisons generally show that poor enforcement of intellectual property rights retards local innovation (Kanwar and Evenson, 2003), event studies like Lerner (2009) tend to find that patent-protection-enhancing shifts generally have a negative impact on patent filings. These contrasting findings are generally counter to economic theory and are, to my knowledge, not yet supported by a unified model of patent protection.

Furthermore, no work that I am aware of has modeled patent protection in the context of learning by doing. Learning models have only either considered patents to be an underlying assumption or have neglected the matter completely. In examining the role of patent protection in trial and error search, this paper represents the first model of patent policy with learning in the product market.

2 Summary of the Callander Model

A countable set of entrepreneurs chooses products sequentially in periods $t = 1, 2, 3, ...$ The set of potential products is the entire real line, $\mathbb{R}$. This set might represent a simple product with one dimension, or a single dimension of a more complicated product. On the market, a product yields an outcome according to the function, ψ, where the set of outcomes is also the real line: $\psi : \mathbb{R} \to \mathbb{R}$.

The true function ψ is determined and set as the realized path of a Brownian motion before play begins.[3] The Brownian motion has the following parameters: drift μ and variance σ^2. However, each entrepreneur knows only these two parameters of the Brownian motion in addition to the set of all the products and their outcomes chosen by previous entrepreneurs.

Each entrepreneur's utility is quadratic in the outcome of product $\psi(p)$, taking the following form for utility u_j of entrepreneur j:

$$u_j(p) = -\psi(p)^2 \tag{2.1}$$

From this equation, we have that each entrepreneur has a maximum utility of 0; the closer the outcome of a product is to zero, the better off he is.

Play begins with common knowledge of one product p_0^* and its outcome $\psi(p_0^*)$. This initial, or status-quo, product can be thought of as any product that opens up a new space. Callander notes that this product may be a new, groundbreaking product or an old product that is ripe for innovation. Figure 2.1 shows one possible realization of a Brownian path. The dashed line represents the first entrepreneur's best estimate of the mapping.

[3] The Brownian function remains constant over time so that entrepreneurs may learn from the trials of their predecessors.

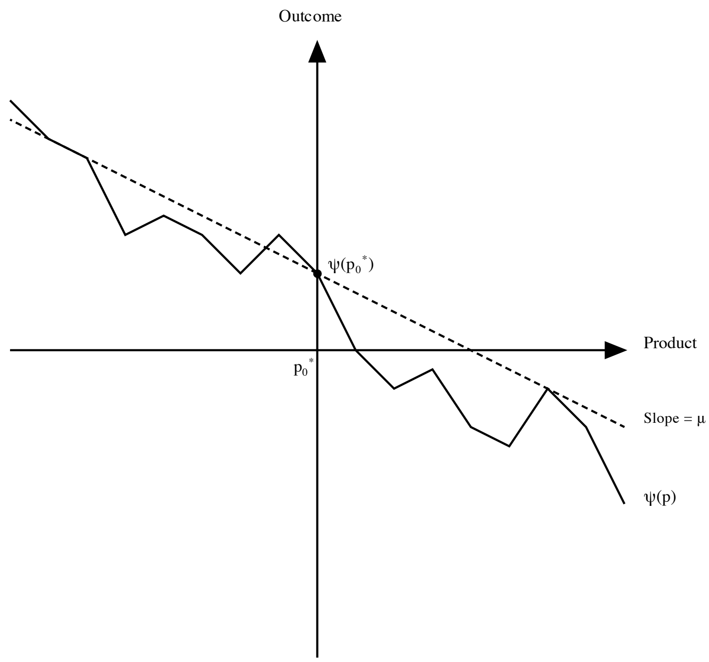

Figure 2.1: One possible Brownian mapping, $\psi(p)$. Modified from Figure 1 in Callander (2011).

Let $h^t = \{(p_0^*, \psi(p_0^*)), (p_1^*, \psi(p_1^*)), \ldots\}$ be the set of known points to entrepreneur t. This set contains all products produced over the duration of the search up to and including time $t-1$. Let l^t denote the left-most product in h^t and r^t denote the right-most product in h^t. By the Markov property of Brownian motion, beliefs about product p are determined only by the nearest points. Therefore, for any product to the right of r^t we have

$$\text{Expected outcome: } E\left(\psi(p)|h^t\right) = \psi(r^t) + \mu(p - r^t) \tag{2.2}$$

$$\text{Variance: } var\left(\psi(p)|h^t\right) = |p - r^t|\sigma^2 \tag{2.3}$$

All points to the left of l^t are analogous. For any product in between two known products in h^t, let q_1 denote the left neighbor and q_2 denote the right neighbor. Then for any product $p \in [q_1, q_2]$, we have

$$\text{Expected outcome: } E\left(\psi(p)|h^t\right) = \psi(q_1) + \frac{\psi(q_2) - \psi(q_1)}{q_2 - q_1}(p - q_1) \tag{2.4}$$

$$\text{Variance: } var\left(\psi(p)|h^t\right) = \frac{(p - q_1)(q_2 - p)}{q_2 - q_1}\sigma^2 \tag{2.5}$$

Additionally, entrepreneurs face mean-variance expected utility in the Callander model:

$$Eu_t(p|h) = -E\left(\psi(p)|h^t\right)^2 - var\left(\psi(p)|h^t\right)$$

which is simply the utility of the expected product, less the variance of the outcome.

Callander notes that this Brownian motion representation of uncertainty captures several key features of experimentation and learning in product markets that are consistent with empirical findings. Most importantly, it successfully describes the segmentation of product development into two distinct phases. In addition, it also incorporates many micro features of the product development cycle, such as path dependence of search and proportional invertibility.[4] Each of these properties makes this dynamic decision-making model ideal for studying the role of patent protection in the product development cycle.

[4] Callander describes proportional invertibility as the idea that "larger changes are associated with greater uncertainty" (2300).

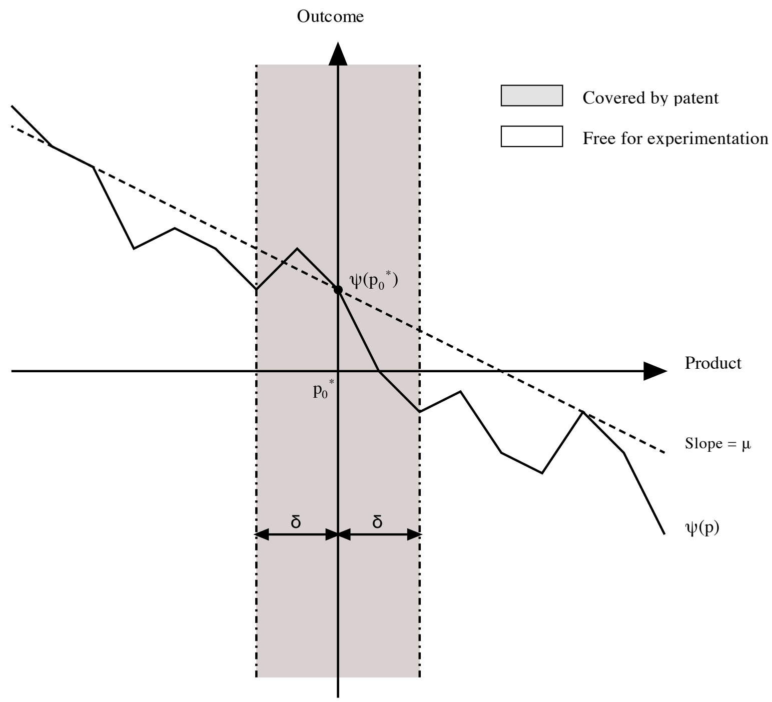

Figure 3.1: One possible Brownian mapping, $\psi(p)$, with patent protection

3 Basic Extension

Let δ denote the breadth of patent protection. Any product p is protected on both sides such that no new product may be developed on the interval $[p - \delta, p + \delta]$. Figure 3.1 depicts one possible realization of a Brownian path such that the status-quo product is covered by patent protection of breadth δ. Under this extension, patent protection is restrictive in that it limits the number of potential products that are available to be produced through experimentation in future periods.

In addition, I impose a utility function bounded by the best outcome to date. Define $\bar{\phi}$ as the price any entrepreneur must pay to reproduce a product, and define $\tau_j^* = \arg\min_{j'<j} |\psi(p_{j'}^*)|$ as the product with the most attractive outcome that has

been tried up to period j. For product p and outcome $\psi(p)$, the utility of entrepreneur j under this extension is:

$$U_j(p) = \begin{cases} -\psi(p)^2 & \text{if } |\psi(p)| \leq |\psi(\tau_j^*)| \\ \\ -\psi(\tau_j^*)^2 - \bar{\phi} & \text{if } |\psi(p)| > |\psi(\tau_j^*)| \end{cases} \tag{3.1}$$

In this model of patent protection, $\bar{\phi}$ captures not only the cost of licensing, but also any implicit costs of replication. As these costs increase, entrepreneur j is worse-off, as his lower bound on utility is now smaller: his utility is the same as before in the case that $|\psi(p)| \leq |\psi(\tau_t^*)|$, but his utility is less than before if $|\psi(p)| > |\psi(\tau_t^*)|$. This imitation cost can vary by the underlying market of the search but not over the span of a single search.

Although $\bar{\phi}$ is a cost by definition, it also serves as an implicit incentive to the individual entrepreneur. To see this, note that each entrepreneur must either imitate an old product or produce a new, experimental product. Increasing the cost of product imitation, $\bar{\phi}$, makes the alternative to imitation, namely experimentation, more desirable. Therefore, $\bar{\phi}$ represents the incentive to experiment imposed by the patent system.

Utility of this form implies that the market will support only the best product. If experimentation is successful, then $|\psi(p)| \leq |\psi(\tau_t^*)|$, and the experimenter will receive the utility according to his product. If, however, the experimentation is unsuccessful, then $|\psi(p)| > |\psi(\tau_t^*)|$, and the entrepreneur must produce product τ_t^* by paying the utility-penalty $\bar{\phi}$. An interpretation of this utility structure is that a single consumer finds the zero-outcome product to be ideal; as a result, this consumer is willing to pay more for a product closer to the ideal. Furthermore, the bounds on the entrepreneur's utility indicate that our consumer buys only the best product in the market.[5] This type of behavior is characteristic of monopsony in product markets, including the markets for military equipment, coal, and beef (Schroeter, 1988; Atkinson and Kerkvliet, 1989).

Let θ_t denote the perceived probability of a successful product at time t. The mean-variance expected utility of experimentation under this extension is:

$$Eu_t(p|h^t) = -\theta E\left[\psi(p)\right]^2 - (1-\theta)\left(\psi(\tau_t^*)^2 + \bar{\phi}\right) - var\left(\psi(p)|h^t\right) \tag{3.2}$$

Combined, the first two terms in Equation 3.2 capture the tradeoff between expected gains in outcome from experimentation and the cost of defaulting to the best-so-far product. The last term imposes an uncertainty-penalty on entrepreneur t from increasing uncertainty in product outcomes.

For the sake of tractability, I take θ to be constant in the experimental product p. This constant probability of success is denoted $\bar{\theta}$. By holding θ constant within a search, I allow equilibrium behavior to be described analytically without significant loss in detail. This might be a reasonable assumption in relatively complex markets, when an entrepreneur's predictions might be based only on the successes of his prede-

[5] If in period j, $|\psi(p*)| \leq |\psi(\tau_j^*)|$ then p_j^* is the new best product, and $\tau_{j+1}^* = p_j^*$. If, however, $|\psi(p)| > |\psi(\tau_j^*)|$, then our entrepreneur cannot find a buyer for his product, and will receive no utility from taking it to the market; he must instead sell product $\tau_j^* \neq p_j^*$ at the cost $\bar{\phi}$.

cessors.[6] Such markets would be characterized either by limited information or simple irrationality. Searches based on underlying markets with more optimistic outlooks, for example, would have a greater $\bar{\theta}$ than those based on more pessimistic markets. In Section 5 I relax this assumption by setting $\theta = \Pr(|\psi(p)| \leq |\psi(\tau_t^*)||h^t)$ to the true probability of a successful experiment.[7]

4 Equilibrium Behavior

Equilibrium is solved for constructively, starting with the first entrepreneur at $t = 1$. From the Callander paper, we have that product p_t^* is stable if $p_{t'}^* = p_t^*$ for any time $t' > t$. Therefore, once the first entrepreneur chooses not to experiment, then that product will be produced forever, as all future entrepreneurs will face the same decision as he.

The First Period

Define $\alpha = \sigma^2/2\bar{\theta}|\mu|$ as half of the adjusted search complexity in the underlying industry,[8] and $\lambda = \delta|\mu|$ as the outcome-breadth of a patent, or the range of outcomes the patent covers.

Proposition 1. *The equilibrium strategy of entrepreneur 1 is:*

[6] In this example, the estimated probability of successful experimentation is a function of $\psi(\tau_t^*)$. The results described in Section 4 hold as long as $\frac{\partial \theta}{\partial z} = 0$ for all $z \in \mathbb{R}$.

[7] An examination of Section 6 shows that both the constant-probability extension and the variable-probability extension yield similar qualitative patterns in equilibrium, which further justifies this paper's outline.

[8] Callander defines the search complexity as $\sigma^2/|\mu|$. I adjust this definition for the probability of successful experimentation by dividing by $\bar{\theta}$.

1. Stable at $p_1^* = p_0^*$ if $\psi(p_0^*) \in [-\alpha - \lambda, \alpha + \lambda]$.

2. Experimental if $\psi(p_0^*) \notin [-\alpha - \lambda, \alpha + \lambda]$, where

$$E\left(\psi(p_1^*|h^1)\right) = \psi(p_0^*) + \mu(p_1^* - p_0^*) = \begin{cases} \alpha & \text{if } \psi(p_0^*) > \alpha + \lambda \\ \\ -\alpha & \text{if } \psi(p_0^*) < -\alpha - \lambda \end{cases}$$

There are several unique features of this strategy. Firstly, the threshold for stability is less strict than in the Callander model. To see this, let $\alpha' = \sigma^2/2|\mu|$. In terms of the outcome space, the stable interval in the Callander model is $[-\alpha', \alpha']$, which is smaller than this extension's stable interval, $[-\alpha - \lambda, \alpha + \lambda]$, for nontrivial breadth of protection. The first reason for a wider interval in this extension is the change to the underlying utility structure; as the probability of successful experimentations decreases, the term $\alpha = \alpha'/\bar{\theta}$ increases, which widens the interval for stability in this model. Secondly, the restriction in the product space surrounding p_0^* increases the threshold for stability through the term λ; the greater the breadth δ is, the greater λ will be, and the wider the outcome-interval for stability will be.

Additionally, the cause of stability can be attributed to patent protection if

$$0 \leq \psi(p_0^*) - \alpha \leq \lambda \text{ or if } -\lambda \leq \psi(p_0^*) + \alpha \leq 0$$

In this case, which I call patent-induced stability, the entrepreneur is discouraged from experimentation by the presence of patent protection. In the absence of patents, the entrepreneur would have chosen to experiment such that his product would have been within λ units of the status-quo product in the outcome space. These equations imply that the threshold for patent-induced stability is greater for larger λ.

Two factors determine the size of λ. Firstly, λ is strictly increasing in the breadth of protection, δ. When the breadth of protection is greater, the patent covers more products so that there are fewer opportunities for future entrepreneurs to experiment. Similarly, λ is strictly increasing in the absolute value of the drift parameter, $|\mu|$. This is because a larger drift parameter implies that a patent of a given breadth will cover more potential outcomes, and is more likely to cover outcomes which future entrepreneurs would otherwise attempt to achieve through experimentation. Since α is decreasing in $|\mu|$, a larger magnitude of drift parameter implies that patent-induced stability increases at the expense of decreasing good-enough stability.

If the entrepreneur chooses to experiment, however, he still seeks the same product as he would with no breadth of protection, optimizing his expected utility by choosing the product that yields the expected result of either α or $-\alpha$. Put another way, if patent protection does not bind the entrepreneur to stability, then he is free to experiment according to the maximization of his expected utility.

Another intuitive result is that the expected outcome of experimentation, α, is decreasing (improving) in the expected probability of successful experimentation, $\bar{\theta}$. Recall that if the underlying market is one which is typically optimistic, then $\bar{\theta}$ will be closer to 1; as a result, the first entrepreneur will not only have a greater tolerance for experimentation, but also also act more aggressively in his experimentation.

The Second and Subsequent Periods

Assume, without loss of generality, that $\mu \leq 0$ for the following results. Recall that $\tau_t^* = \arg\min_{t'<t} |\psi(p_{t'}^*)|$ is the product with the most attractive outcome that has been tried up to period t. Definition 1 from the Callander paper states that a search is in the monotonic phase at time t if it has not yet stabilized, $p_0^* < p_1^* < \ldots < p_{t-1}^*$, and $\psi(p_0^*), \psi(p_1^*), \ldots, \psi(p_{t-1}^*) \geq 0$. The following proposition describes equilibrium behavior in the monotonic phase:

Proposition 2. *In the monotonic phase at period $t \geq 2$, the equilibrium strategy is:*

1. Stable at

$$p_t^* = \tau_t^* = p_{t-1}^* \text{ if } \psi(p_{t-1}^*) \in [0, \alpha + \lambda]$$

2. Stable at

$$p_t^* = \tau_t^* \neq p_{t-1}^* \text{ if } \psi(p_{t-1}^*) > \gamma_t = \frac{(2\bar{\theta} - 1)\alpha^2 + \psi(\tau_t^*)^2}{2\alpha}$$

3. Experimental if $\psi(p_{t-1}^*) \in [\alpha + \lambda, \gamma_t]$, where $p_t^* > p_{t-1}^*$, and

$$E\left(\psi(p_t^*|h^t)\right) = \psi(p_{t-1}^*) + \mu(p_t^* - p_{t-1}^*) = \alpha$$

In the monotonic phase there are two conditions for stability. Callander refers to the first condition as "good-enough stability," since stability here is achieved by setting the current product equal to the most recent product, which is the best so far; put another way, this stability occurs when the most recent product is "good enough" for stability to ensue. Again, the cause of stability can be attributed to patent protection if $0 \leq \psi(p_0^*) - \alpha \leq \lambda$. Condition 1 here is analogous to the first condition of Proposition 1 in logic and in results.

The second condition results from an unexpected outcome from the most recent product p_{t-1}^*, such that $\psi(p_{t-1}^*) > \psi(\tau_t^*)$. The decision to experiment occurs only if the expected utility of experimentation exceeds that of the given utility of that best-so-far product; if the utility of the best-so-far product exceeds the expected utility of experimentation, then the product search enters "bad-enough stability." Intuitively, the threshold for a search getting stuck in this type of stability is decreasing in the probability of success, $\bar{\theta}$. As with good-enough stability, the greater $\bar{\theta}$ is, the more attractive experimentation is, and the more is required to deter an entrepreneur from experimentation.

Interestingly, the threshold for bad-enough stability is not directly changed by the breadth of patent protection, since neither the utility of the best-so-far product nor the expected utility of experimentation immediately depend on the restriction in product space imposed by patent protection. This condition is, however, less likely to occur in a search, as the threshold for good-enough stability is higher in all time periods.

If neither of the two stability conditions hold in the monotonic phase, then it must be the case that the outcome of the previous product falls in the range $[\alpha + \lambda, \gamma_t]$. The current entrepreneur will continue to experiment to the right, in the same direction as his predecessors. The entrepreneur will again optimize his expected utility such that the expected outcome is equal to α.

If an experimental product from the monotonic phase of search crosses the $\psi(p) = 0$ line, then the search enters the triangulating phase. Using Definition 2 from the Callander paper, we have that a search is in the triangulating phase at period $t \geq t^{\Delta}$ if it has not yet stabilized, $p_0^* < p_1^* < \ldots < p_{t^\Delta-1}^*$, and $\psi(p_0^*), \psi(p_1^*), \ldots, \psi(p_{t^\Delta-2}^*) > 0 > \psi(p_{t^\Delta-1}^*)$.

Define $\alpha(\widehat{w \cdot z}) = \frac{\sigma^2}{2\theta}\left|\frac{z-w}{\psi(z)-\psi(w)}\right|$ and $\lambda(\widehat{w \cdot z}) = \delta\left|\frac{(\psi(z)-\psi(w))^2-\bar{\theta}\sigma^2(z-w)}{\bar{\theta}(\psi(z)-\psi(w))(z-w)}\right|$. Formally, $\alpha(\widehat{w \cdot z})$ generalizes α by replacing μ with the slope of the bridge, while $\lambda(\widehat{w \cdot z})$ generalizes λ by replacing μ with the probability-adjusted slope of the bridge less the variance as a fraction of the outcome-span of the bridge.

Proposition 3. *At period t^{Δ} in the triangulating phase, the equilibrium strategy is:*

1. Stable at $p_{t^\Delta}^* = p_{t^\Delta-1}^*$ if

$$|\psi(p_{t^\Delta-1}^*)| \leq \alpha\left(\widehat{p_{t^\Delta-2}^* \cdot p_{t^\Delta-1}^*}\right) + \lambda\left(\widehat{p_{t^\Delta-2}^* \cdot p_{t^\Delta-1}^*}\right)$$

2. Experimental otherwise, where $p_{t^\Delta}^* \in (p_{t^\Delta-2}^*, p_{t^\Delta-1}^*)$ solves

$$E\left(\psi(p_{t^\Delta}^*|h^t)\right) = \alpha\left(\widehat{p_{t^\Delta-2}^* \cdot p_{t^\Delta-1}^*}\right)\left[1 - 2\frac{p_{t^\Delta}^* - p_{t^\Delta-2}^*}{p_{t^\Delta-1}^* - p_{t^\Delta-2}^*}\right]$$

The first condition represents good-enough stability of the first period in the triangulating phase, t^{Δ}. As with the monotonic phase, the threshold for good-enough stability in the triangulating phase is increasing in the breadth of patent protection.

The cause is patent protection when

$$0 \leq |\psi(p^*_{t^\Delta-1})| - \alpha\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right) \leq \lambda\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right)$$

From this equation, it is clear that the threshold for patent-induced stability is greater when $\lambda(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}})$ is larger. Reducing $\lambda(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}})$ to

$$\left|\frac{\delta\left(\psi(p^*_{t^\Delta-1}) - \psi(p^*_{t^\Delta-2})\right)}{\bar{\theta}\left(p^*_{t^\Delta-1} - p^*_{t^\Delta-2}\right)} - \frac{\delta\sigma^2}{\psi(p^*_{t^\Delta-1}) - \psi(p^*_{t^\Delta-2})}\right|$$

we see that $\lambda\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right)$ is strictly increasing in δ.[9] As with the monotonic phase, a greater breadth of patent protection in the triangulating phase pushes the entrepreneur towards earlier stability. The threshold for patent-induced stability is similarly increasing in $\bar{\theta}$; when $\bar{\theta}$ increases, both $\alpha\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right)$ and $\lambda\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right)$ decrease, driving the search towards experimentation.

Another interesting feature of this model is that $\lambda\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right)$ is decreasing in $|p^*_{t^\Delta-1} - p^*_{t^\Delta-2}|$ and increasing in $|\psi(p^*_{t^\Delta-1}) - \psi(p^*_{t^\Delta-2})|$. A bridge that spans a wide range of outcomes will be more likely to result in patent-induced stability in the triangulating phase, as will a bridge that spans a smaller interval on the product space. This relationship implies that a steeper bridge span increases the chance of patent-induced stability at the expense of good-enough patent stability in the triangulating phase, as $\alpha\left(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}}\right)$ is decreasing in the slope of the bridge.

[9] First, pull δ out of the bracketed term. The proof for Proposition 3 in Appendix C shows that both terms inside the absolute values are of the same sign. Therefore, $\lambda(\widehat{p^*_{t^\Delta-2}\cdot p^*_{t^\Delta-1}})$ must be strictly increasing in δ.

In the context of the entire search, play is less likely to reach the triangulating phase under nontrivial patent protection as a result of the greater tendency for search to converge to patent-induced stability in the monotonic phase.[10] Therefore, the inclusion of patent protection has two competing effects on the total likelihood of patent-induced stability in the triangulating phase. The first is the decrease in probability that the search will make it to the triangulating phase, and the second is the increase in chance that, once search enters the triangulating phase, the search will result in patent-induced stability due to the breadth of protection.

For period $t > t^{\Delta}$, let p_l^* denote the left point of the spanning bridge,[11] and let p_r^* denote the right point of the spanning bridge. Proposition 4 characterizes equilibrium behavior after the first period of the triangulating phase.

Proposition 4. *At period $t > t^{\Delta}$ in the triangulating phase, the equilibrium strategy is:*

1. Stable at $p_t^* = p_{t-1}^* \in \{p_l^*, p_r^*\}$ if

$$\left|\psi(p_{t-1}^*)\right| \leq \alpha\left(\widehat{p_l^*, p_r^*}\right) + \lambda\left(\widehat{p_l^*, p_r^*}\right)$$

2. Stable at $p_t^* = \tau_t^* \notin \{p_l^*, p_r^*\}$ if

$$|\psi(\tau_t^*)| < \frac{\sigma}{2}\sqrt{|p_r^* - p_l^*|} \text{ and } \psi(p_l^*) \approx -\psi(p_r^*)$$

[10] See Tables 1 and 3 for evidence of this.

[11] Lemma 2 in the Callander paper shows that only one Brownian bridge is spanning.

3. Experimental otherwise, where $p_t^* \in (p_l^*, p_r^*)$ solves

$$E\left(\psi(p_t^*|h^t)\right) = \alpha\left(\widehat{p_l^*, p_r^*}\right)\left[1 - 2\frac{p_t^* - p_l^*}{p_r^* - p_l^*}\right]$$

Conditions 1 and 3 are analogous to period t^Δ. As with period t^Δ, the breadth of protection increases the threshold for good-enough stability for all periods $t > t^\Delta$, and the condition for patent-induced stability is also $0 \leq |\psi(p_{t-1}^*)| - \alpha\left(\widehat{p_l^*, p_r^*}\right) \leq \lambda\left(\widehat{p_l^*, p_r^*}\right)$. Condition 2, however, is unique to periods $t > t^\Delta$; it states that a search can get stuck if an outcome is moderately bad. The reasoning behind it is similar to the bad-enough stability of Proposition 2, even if the conditions for stability are different.[12]

Interestingly, a search that reaches the triangulating phase is more likely to achieve a better final outcome than a search that stabilizes in the monotonic phase. This is implied by Corollary 4 in the Callander model, which states that the boundary on good-enough stability is strictly converging in the triangulating phase.[13] Therefore, the breadth of protection encourages worse outcomes by loosening the conditions for good-enough stability in all time periods. This push towards good-enough stability pushes a search to converge prematurely, reducing time spent in the triangulating phase. Furthermore, this push towards early stability increases with the breadth of patent protection for each search stage. According to this extension, then, the breadth

[12] Callander notes that outcomes that are significantly bad increase the returns to experimentation, so only moderately bad outcomes lead to bad-enough stability in the triangulating phase. Moderately bad outcomes occur when $\psi(p_l^*) + \psi(p_r^*)$ is small, or $\psi(p_l^*) \approx -\psi(p_r^*)$.

[13] This is because in each period, the spanning bridge becomes steeper if it does not stabilize, thereby increasing the returns to experimentation.

of coverage offered by a patent policy is a deterrent of innovation, and will typically lead to outcomes further from the ideal zero-outcome. As patent policy loosens, innovation tends to continue for longer durations, resulting in superior products.

To the extent that the Callander model accurately describes product markets, patent policy may have significant effects on economic growth. To see this, note that in this model each successive entrepreneur builds upon the knowledge of his predecessors' products and their results, implying that experimentation has positive spillover effects onto future experimenters. Furthermore, the extent of the spillover effects in this extension can be thought of as the duration of experimentation. Applied to the Romer endogenous growth model (1987), a looser patent policy may contribute to an increase in the economic growth rate by increasing these spillover effects through an increase in the duration of experimentation.[14] If research and development are the source of economic growth, as the Romer model suggests, then political-economic systems that encourage innovation and experimentation through looser breadth of protection should see higher growth rates than those with a large breadth of protection in place.

5 Extension: Adjusting Expectations

Recall from Section 3 that the expected utility for entrepreneur t is dependent on the perceived probability of success:

[14] Another interesting feature of this extension is that the cost of replication does not factor into equilibrium results. By allowing entrepreneurs to know the true probability of successful experimentation in Sections 5 and 6, we find that the cost of replication plays a key role in determining equilibrium behavior.

$$Eu_t(p|h^t) = -\theta E\left[\psi(p)\right]^2 - (1-\theta)\left(\psi(\tau_t^*)^2 + \bar{\phi}\right) - var\left(\psi(p)|h^t\right)$$

for any product p. In this section, I more accurately characterize beliefs about the outcome of a product such that $\theta_t = \Pr(|\psi(p)| \leq |\psi(\tau_t^*)||h^t)$. Because beliefs about the distribution of the outcome of p are distributed normally,[15] we have

$$\theta_t = \left|\Phi\left(\frac{E\left[\psi(p)\right] + \psi(\tau)}{\sigma\sqrt{p - r^t}}\right) - \Phi\left(\frac{E\left[\psi(p)\right] - \psi(\tau)}{\sigma\sqrt{p - r^t}}\right)\right| \tag{5.1}$$

for open-ended beliefs, $p \geq r^t$, and

$$\theta_t = \left|\Phi\left(\frac{E\left[\psi(p)\right] + \psi(\tau)}{\sigma\sqrt{\frac{(p-q_1)(q_2-p)}{q_2-q_1}}}\right) - \Phi\left(\frac{E\left[\psi(p)\right] - \psi(\tau)}{\sigma\sqrt{\frac{(p-q_1)(q_2-p)}{q_2-q_1}}}\right)\right| \tag{5.2}$$

for experimentation on a Brownian bridge, $p \in [q_1, q_2]$. The function Φ denotes the standard normal cumulative distribution function. Substituting these into the equation for expected utility (Equation 3.2), we have the full specification of the entrepreneur's expectations.

This extension shares many properties in common with the constant-probability model in Section 3. Most importantly, because the stable product is necessarily the best product that a search produces, the outcome is monotonically increasing in the

[15]See Appendix A for the details on the distribution of beliefs under open-ended uncertainty, and Appendix B for the distribution of beliefs on a bridge.

number of experiments. Therefore, this extension retains the positive relationship between the number of products produced and the outcome in stability.

Furthermore, recall that as the number of iterations in the triangulating phase increases, the resulting final product must be no worse in outcome over many repeated searches. However, the boundary on stability does not converge as rapidly as in the triangulating phase, if at all.[16] It is clear, then, that time spent triangulating is a major driving factor in producing better outcomes. This result can be seen empirically in Section 6.

However, Appendix A and Appendix B show that equilibrium behavior under this extension cannot be described analytically in a tractable way due to the dependence of θ_t on the experimental product. Instead, equilibrium results are described computationally through Monte Carlo simulation in Section 6.

6 Simulations

The effect of patent protection on product outcomes in this search model can be better seen computationally, through Monte Carlo simulations.[17] For the following simulations, each data point represents the average of 10,000 iterations of searches, and can be thought of as describing the average market characterized by a specific set of exogenous variables. $\mu = -1$, $\sigma = 2$, and $p_0^* = 0$ are exogenously given for

[16] This is implied by Property II (found in the Appendix) of the Callander model. Consider two searches, the first of which is in the monotonic phase and the second is in the triangulating phase in period t^{Δ} such that product r_t in the first is equal to p_{t-2}^{Δ} in the second. The expected outcome for the next experimental product in the second search must be better in outcome than that of the first search because of the concavity of variance.

[17] The code for these simulations can be seen in Appendix E. The code for the constant-probability simulations is similar enough to the Callander model to render it unnecessary to include at all.

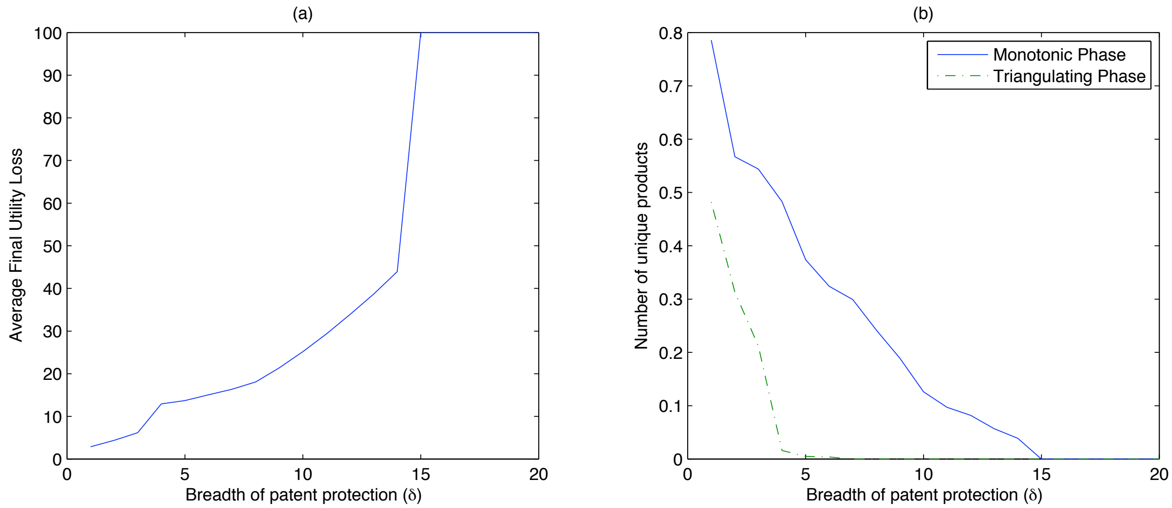

Figure 6.1: Average Utility Loss as a Function of δ

each search. $\delta = 1$, $\bar{\phi} = 3$, and $\psi(p_0^*) = 10$ unless otherwise noted. The results of the simulations performed under the assumption of a constant probability of success can be seen in Appendix D, while the more realistic, variable-probability-of-success simulations are discussed below.

Probability of success is a function of the experimental product

Figure 6.1 shows the average result of repeated searches for $\delta \in [1, 20]$. Each point on Figure 6.1.a represents the average squared final outcome of 10,000 searches,[18] and establishes that a greater breadth of patent protection leads to outcomes further from the ideal. Intuitively, this deterrent of experimentation is the result of the restrictive nature of the patent-breadth. The larger an interval that a single product commands in the product space, the fewer products can possibly be developed, according to the definition of δ established in Section 3. From the viewpoint of a so-

[18]The term utility "loss" is used here to indicate that less-than-ideal outcomes are achieved. The ideal product has no utility loss, while worse outcomes imply a forfeit of potential utility.

cial planner, a higher breadth of protection leads to generally inferior stable product outcomes, resulting in loss of utility to the consumer and future entrepreneurs.

It is interesting that the discontinuous jump in stable outcomes seen at $\delta \geq 15$ in Figure 6.1.a occurs at a greater breadth of protection than the same cutoff in the constant-probability equilibrium seen in Figure D.1. This jump signifies the level of δ after which no experimentation occurs, and the status-quo product is reproduced in stability. The deferred jump implies that entrepreneurs under accurate perceptions of successful experimentation have a greater tolerance for experimentation than those under the constant-probability model. The logic behind this difference can be found in quasi-concavity of θ in the experimental product z under the variable-probability model.[19] Furthermore, θ is immediately increasing in z in the first period,[20] implying that the first term in the equation for expected utility (Equation 3.2) increases with z until some $z > p_0^*$. This increasing weight on the expected outcome of an experiment across z intuitively causes the entrepreneur to value experimentation to a greater extent, implying that the variable probability of success is a driver of the aggressiveness in experimentation.

[19]See Figure A.2 to see an example of this graphically.

[20]To see this, note that $\tau_t^* = r^t = p_0^*$. Substituting into the equation for the $\partial\theta/\partial z$ in Appendix A, we have

$$\begin{aligned}
\frac{\partial \theta}{\partial z} &= \phi\left(\frac{2\psi(r^t) + \mu(z - r^t)}{\sigma\sqrt{z - r^t}}\right)\left[\frac{\mu}{\sigma\sqrt{z - r^t}} - \frac{2\psi(r^t) + \mu(z - r^t)}{2\sigma\left(z - r^t\right)^{3/2}}\right] \\
&\quad -\phi\left(\frac{\mu(z - r^t)}{\sigma\sqrt{z - r^t}}\right)\left[\frac{\mu}{\sigma\sqrt{z - r^t}} - \frac{\mu(z - r^t)}{2\sigma\left(z - r^t\right)^{3/2}}\right]
\end{aligned}$$

and $\lim_{z \to r^{t+}} \frac{\partial \theta}{\partial z} > 0$

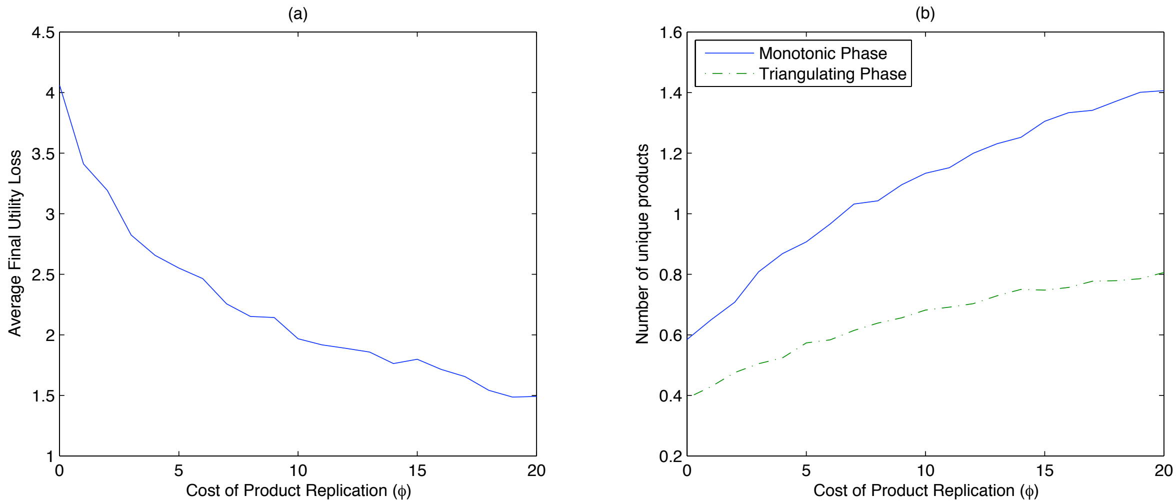

Figure 6.2: The effects of increasing cost of product imitation

Figure 6.1.b shows that the number of unique products produced in both the monotonic phase and the triangulating phase are decreasing in δ. The intuition behind this reduction is the same as with the constant-probability model. By definition, δ implies a restriction in the product space; as δ increases, the fewer products can possibly enter the search. As seen in Section 5, fewer products implies worse outcomes, other things equal. Therefore, we see that increases in the the breadth of protection negatively impact final product outcomes through the reduction in the number of experiments in the triangulating phase.

Another influence on triangulating time can be seen in Figure 6.2, which displays the result of varying the cost of product replication, $\bar{\phi}$. Figure 6.2.a displays the average utility loss in stability, while Figure 6.2.b displays the number of unique products produced in each phase. The effect of imitation costs on the stable product is unique to the variable-probability model, and is unambiguously negative. Other things equal, an increase in the cost of replication will push the search towards a

$\psi(p_0^*)$	2	5	7	10	15	20	30
Average in monotonic ($\delta = 1$)	0	0.56	0.72	0.80	0.81	0.82	0.83
Average in triangulating ($\delta = 1$)	0	0.15	0.33	0.50	0.66	0.76	0.86
Average in monotonic ($\delta = 2$)	0	0.35	0.54	0.60	0.65	0.67	0.70
Average in triangulating ($\delta = 2$)	0	0	0.17	0.29	0.41	0.51	0.62
Average in monotonic ($\delta = 3$)	0	0.24	0.43	0.52	0.55	0.60	0.63
Average in triangulating ($\delta = 3$)	0	0	0.01	0.20	0.30	0.39	0.49

Table 1: Number of unique products produced

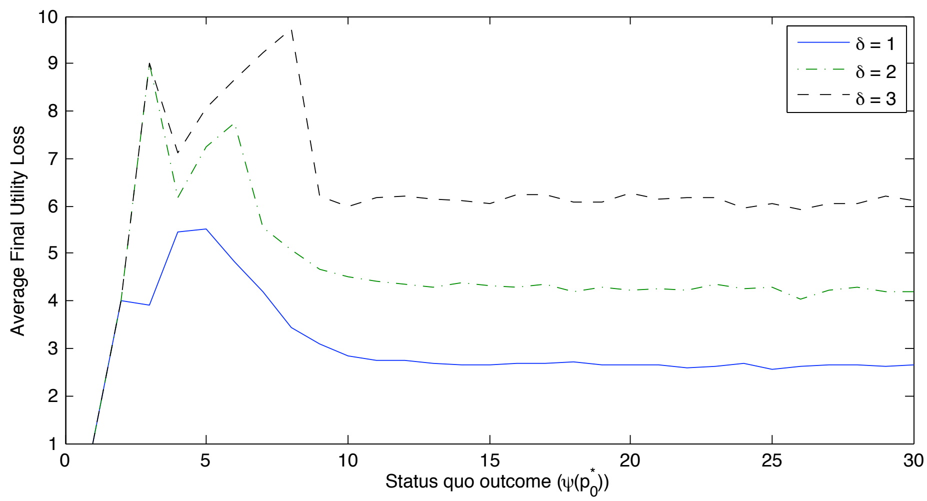

Figure 6.3: Utility loss as a function of status quo outcome

superior final product. This result stems from the fact that stability requires an entrepreneur to produce a product that has already been developed, with the additional cost $\bar{\phi}$. As a result, the cost of stability is greater, and each entrepreneur is more inclined to experiment, resulting in longer searches on average. Therefore, greater patent strength pushes a search towards better results by changing individual incentives in favor of experimentation. Again, this positive relationship between the quantity of experimental products and stable outcomes can be seen in Figure 6.2.b.

The relationship between the breadth of patent protection and the final outcome also depends on the complexity relative to the starting outcome, described mathematically by Callander as $\psi(p_0^*) - \alpha$. Figure 6.3 shows the results of varying this relative complexity through the status quo outcome.[21] Similar in shape to the constant-probability extension's Figure D.3, the average utility loss of a search is largely decreasing in the status-quo outcome $\psi(p_0^*)$. One cause of this can be found in Table 1, where we find that the time spent in the triangulating phase increases with the initial starting complexity. The logic behind this shift towards the triangulating phase is again straightforward. With more complex status quo products, a more aggressive first-period follows, frequently resulting in overshooting with triangulation to follow.

A striking feature of this variable-probability model can be seen in Table 1. It shows that better stable results are found under variable-probability than under constant-probability in spite of fewer experimental products produced before stability; not only does search stabilize to superior products in the variable-probability model, but it also takes less time to do so. This is because under accurate perceptions of success, searches spend drastically less time in the monotonic phase, and slightly more time in the triangulating phase. As discussed in Section 5, more time spent in triangulation implies better results. Entrepreneurs who fully understand the complexities of the market tend to act more aggressively, driving the search towards triangulation and better results.

[21] Callander (2011) shows that holding α fixed and letting $\psi(p_0^*)$ vary will yield the same quantitative patterns as holding $\psi(p_0^*)$ fixed and letting α vary.

7 Conclusion

This paper has introduced a discrete time model of sequential experimentation with explicit imitation costs and breadth of protection. It does so by extending the Callander (2011) model of searching by trial and error, using Brownian motion as the specification of uncertainty in experimental product outcomes. This model assumes an infinite duration of protection, but features robust predictions about the trade-off between the static incentives and dynamic restrictions imposed by patents: Experimental outcomes increase with cost of product imitation and decrease with the breadth of patent protection. In addition, this model predicts a significant positive relationship between product outcomes and the quantity of experimental products.

The equilibrium results in Section 4 show that the constant-probability-of-success extension is a generalization of the Callander model. Although the underlying utility structure in Section 3 is different, the results of the Callander model can be derived from Section 4 by setting $\delta = 0$ and $\bar{\theta} = 1$. Patent protection in this extension is unambiguously negative; the incentives imposed by imitation costs do not factor into equilibrium under constant-probability, while the breadth of protection acts to push the search towards earlier stability, deterring experimentation and reducing product outcomes in stability.

In the variable-probability-of-success extension, the overall effect of patent protection on experimentation is mixed. The effect of the breadth of protection is similarly negative in this extension; however, under variable-probability, searches yield superior results following increases in cost of product imitation. Therefore, this

		Cost of Imitation, $\bar{\phi}$									
		1	2	3	4	5	6	7	8	9	10
Breadth, δ	1	3.51	3.12	2.82	2.585	2.59	2.32	2.30	2.12	2.09	1.96
	2	4.95	4.68	4.46	4.17	4.00	3.93	3.69	3.71	3.64	3.46
	3	6.41	6.16	6.08	5.70	5.82	5.60	5.34	5.44	5.28	5.05
	4	13.14	13.30	12.74	13.00	12.54	12.49	12.44	12.56	12.18	12.37
	5	14.39	14.12	14.54	13.93	13.89	14.16	13.44	13.63	13.43	13.03

Table 2: Utility loss as a function of δ and $\bar{\phi}$

variable-probability model successfully captures both the economic benefits and costs of patent protection. Table 2 shows that increasing the cost of product imitation and decreasing the breadth of protection will positively contribute to both the quantity and the quality of experimentation. It suggests that innovation-oriented policy should push patent protection in the northeast direction: larger imitation costs but smaller breadth of protection.[22]

Empirical Implications

Many studies have attempted to estimate the effects of patent protection on innovation. The results of these studies, however, are mixed; several studies suggest that overall patent strength is responsible for higher rates of innovation, while other studies suggest that the effect on innovation is negligible or even negative (Allred and Park, 2007). This model contributes to the literature in its specification of the level of patent protection which yields unique results; while the breadth of protection is restrictive in its impact on innovation, the imitation costs imposed by patent protec-

[22]What this looks like in practice is more difficult to say. In order to achieve high imitation costs, a sovereign might attempt to increase the penalties for patent infringement. At the same time, however, that same sovereign must attempt to limit these penalties to a narrow enough range of product.

tion are positive in their impact. This finding corresponds well with the conflicting empirical studies on the economic impact of patent protection, as the overall index of patent protection has been defined differently across studies. Studies that define the overall strength of protection with a larger weight on the cost of imitation should find that patent protection has a positive effect on innovation, while studies that place a heavier emphasis on the breadth of protection should find that patent protection has a negative effect on innovation.

One theme that can be seen throughout this paper is that better search outcomes result from increased experimentation. More specifically, better product outcomes can be explained largely through the number of unique products produced in the triangulating phase. Put another way, experimentation quantity proxies product quality in this model. This relationship is cited frequently in empirical studies examining innovative activity, where innovative activity is operationalized as the quantity of patents cited (Kortum and Lerner, 1999). Economists such as Lerner (2009), however, note that the mapping from experimentation quantity to innovative activity may not be exact, as this model suggests. Because this mapping is so frequently cited, it is important to verify empirically that experimentation quantity corresponds with product outcomes.

Lastly, empirical studies suggest that optimal patent policy varies with the characteristics of the underlying market (Grossman and Lai, 2004). In the model developed in this paper, this variation across markets is represented by a change in the parameters of the underlying Brownian motion. The results in Section 4 show that changes in these parameters significantly affect search outcomes under the constant-probability extension, while Appendix A and B show that this is true under the variable-probability extension.

Future Research

This extension investigates only two of the three dimensions of patent protection; while this model characterizes innovation in the face of changing strength and breadth of patent protection, it neglects analysis of the duration of protection. Future studies might investigate the effects of finite duration of patent protection on experimental outcomes, either game theoretically or computationally. This research would result in a more complete understanding of the constituents of patent protection, and shed further light on optimal patent policy.

Additionally, searches in this model were analyzed largely in terms of the average of many samples for each set of market characteristics. Analysis of the risk involved with each search is outside the scope of this paper, but should be investigated further. A preliminary examination shows that the variance in outcomes for each sample did not vary significantly with the exogenous variables. However, the actual distribution of search outcomes for any given market is important from a social perspective, and should be described more fully.

Finally, the results predicted by this model should be tested empirically. The empirical implications described above are straightforward, and could be tested by indexing patent protection in terms of both breadth and strength. Utilizing this model's specification of protection, studies should find that the breadth of protection negatively affects innovation and imitation costs positively affect innovation, both in terms of quantity and quality.

A Open-Ended Uncertainty

Consider all products $z > r_t$ such that $\psi(r_t) > 0$. Expected utility for an arbitrary entrepreneur is

$$Eu_t(z|h^t) = -\theta E\left[\psi(z)\right]^2 - (1-\theta)\left(\psi(\tau)^2 + \bar{\phi}\right) - (z - r^t)\sigma^2$$

for

$$E\left[\psi(z)\right] = \psi(r^t) + \mu(z - r^t)$$

Probability of success is constant in the experimental product

Holding θ constant for all entrepreneurs at $\bar{\theta}$, entrepreneur t faces the following problem:

$$\max_z Eu_t(z|h^t) = -\bar{\theta}E\left[\psi(r^t) + \mu(z - r^t)\right]^2 - (1-\bar{\theta})\left(\psi(\tau)^2 + \bar{\phi}\right) - (z - r^t)\sigma^2$$

Differentiating,

$$\begin{aligned} \frac{\partial Eu(z)}{\partial z} &= -2\mu\bar{\theta}E\left[\psi(z)\right] - \sigma^2 \\ &= -2\mu\bar{\theta}\left[\psi(r^t) + \mu(z - r^t)\right] - \sigma^2 \\ \frac{\partial^2 Eu(z)}{\partial z^2} &= -2\mu^2\theta \leq 0 \end{aligned}$$

The strictly negative second derivative ensures a unique maximum. The corner solution $z = r_t + \delta$ is optimal if and only if $\frac{-\sigma^2}{2\mu\bar{\theta}} \geq \psi(r^t) + \mu\delta$. Solving the first-order condition for an internal solution gives

$$\psi(r^t) + \mu(z - r^t) = E\left[\psi(z)\right] = \frac{-\sigma^2}{2\mu\bar{\theta}}$$

Probability of success is a function of the experimental product

Beliefs about the outcome of z are distributed normally:

$$\frac{\psi(z) - E\left[\psi(z)\right]}{\sigma\sqrt{z - r^t}} \sim \mathcal{N}(0,1)$$

Therefore, the probability of success is

$$\theta_t = \Pr(|\psi(z)| \leq |\psi(\tau_t^*)||h^t) = \left|\Phi\left(\frac{E\left[\psi(p)\right] + \psi(\tau)}{\sigma\sqrt{p - r^t}}\right) - \Phi\left(\frac{E\left[\psi(p)\right] - \psi(\tau)}{\sigma\sqrt{p - r^t}}\right)\right|$$

and entrepreneur t faces the following problem:

$$\max_{z} Eu_t(z|h^t) = -\theta E\left[\psi(z)\right]^2 - (1-\theta)\left(\psi(\tau)^2 + \bar{\phi}\right) - (z - r^t)\sigma^2$$

Differentiating,

$$\begin{aligned}
\frac{\partial Eu(z)}{\partial z} &= -\frac{\partial\theta}{\partial z}E\left[\psi(z)\right]^2 - 2\mu\theta E\left[\psi(z)\right] + \frac{\partial\theta}{\partial z}\left(\psi(\tau)^2 + \bar{\phi}\right) - \sigma^2 \\
\frac{\partial^2 Eu(z)}{\partial z^2} &= -\frac{\partial^2\theta}{\partial z^2}E\left[\psi(z)\right]^2 - 4\mu E\left[\psi(z)\right]\frac{\partial\theta}{\partial z} - 2\mu^2\theta + \frac{\partial^2\theta}{\partial z^2}\left(\psi(\tau)^2 + \bar{\phi}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial\theta}{\partial z} &= \phi\left(\frac{E\left[\psi(z)\right] + \psi(\tau_t^*)}{\sigma\sqrt{z - r^t}}\right)\left[\frac{\mu}{\sigma\sqrt{z - r^t}} - \frac{E\left[\psi(z)\right] + \psi(\tau_t^*)}{2\sigma\left(z - r^t\right)^{3/2}}\right] \\
&\quad -\phi\left(\frac{E\left[\psi(z)\right] - \psi(\tau_t^*)}{\sigma\sqrt{z - r^t}}\right)\left[\frac{\mu}{\sigma\sqrt{z - r^t}} - \frac{E\left[\psi(z)\right] - \psi(\tau_t^*)}{2\sigma\left(z - r^t\right)^{3/2}}\right]
\end{aligned}$$

where ϕ denotes the normal PDF.

Several problems arise in solving for a z that maximizes expected utility. Mainly, the second derivative is neither strictly positive nor strictly negative, so a unique maximum is not guaranteed. As a result, I solve for equilibrium computationally, noting that the solution is in the set $r^t \cup z^*$, where $z^* > r^t + \delta$ solves

$$E\left[\psi(z^*)\right]^2 + \frac{2\mu\theta}{\partial\theta/\partial z}E\left[\psi(z^*)\right] = \psi(\tau)^2 + \bar{\phi} + \sigma^2$$

rearranging:

$$E\left[\psi(z^*)\right] = \frac{-\mu\theta}{\partial\theta/\partial z} \pm \sqrt{\psi(\tau)^2 + \bar{\phi} + \sigma^2 + \left(\frac{\mu\theta}{\partial\theta/\partial z}\right)^2}$$

B Brownian Bridge

Consider a spanning bridge $\widehat{p_l p_r}$, where $\psi(p_r) < 0 < \psi(p_l)$ and suppose $|\psi(p_r)| \geq |\psi(p_l)|$. Expected utility for an arbitrary entrepreneur for $z \in [p_l, p_r]$ is

$$Eu_t(z|h^t) = -\theta E\left[\psi(z)\right]^2 - (1-\theta)\left(\psi(\tau)^2 + \bar{\phi}\right) - \frac{(z - p_l)(p_r - z)}{p_r - p_l}\sigma^2$$

for

$$E\left[\psi(z)\right] = \psi(p_l) + \frac{z - p_l}{p_r - p_l}\left(\psi(p_r) - \psi(p_l)\right)$$

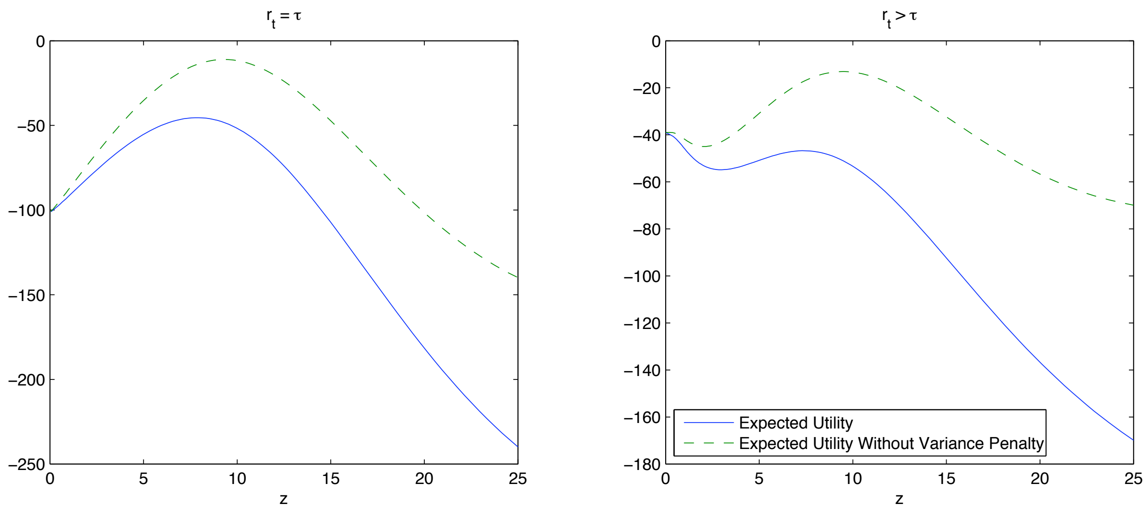

Figure A.1: Expected utility in the monotonic phase for $r_t = 0$, $\psi(r_t) = 10$, $\mu = -1$, $\sigma = 2$, $\bar{\phi} = 3$, and $\delta = 1$. $\psi(\tau_t^*) = \psi(r_t)$ on the left and $\psi(\tau_t^*) = 6$ on the right.

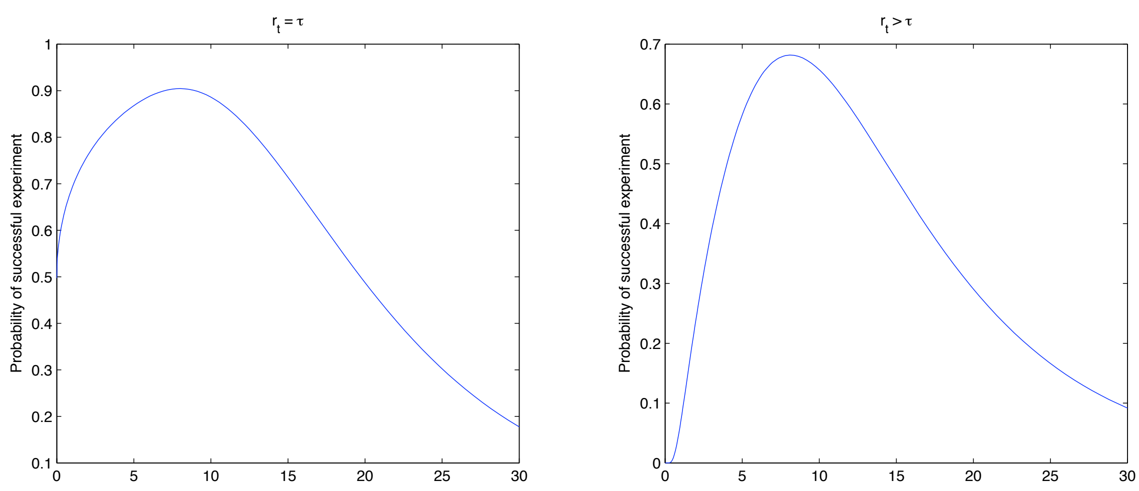

Figure A.2: Probability of success in the monotonic phase for $r_t = 0$, $\psi(r_t) = 10$, $\mu = -1$, $\sigma = 2$, $\bar{\phi} = 3$, and $\delta = 1$

Probability of success is constant in the experimental product

The maximization problem faced by entrepreneur t in the triangulating phase is:

$$\begin{aligned}
\max_z Eu_t(z|h^t) &= -\bar{\theta}\left[\psi(p_l) + \frac{z-p_l}{p_r-p_l}\left(\psi(p_r)-\psi(p_l)\right)\right]^2 \\
&\quad -(1-\bar{\theta})\left(\psi(\tau)^2+\bar{\phi}\right) - \frac{(z-p_l)(p_r-z)}{p_r-p_l}\sigma^2
\end{aligned}$$

Differentiating,

$$\begin{aligned}
\frac{\partial Eu(z)}{\partial z} &= -2\bar{\theta}\frac{\psi(p_r)-\psi(p_l)}{p_r-p_l}\left[\psi(p_l)+\frac{z-p_l}{p_r-p_l}\left(\psi(p_r)-\psi(p_l)\right)\right] - \frac{p_r+p_l-2z}{p_r-p_l}\sigma^2 \\
\frac{\partial^2 Eu(z)}{\partial z^2} &= -2\bar{\theta}\left[\frac{\psi(p_r)-\psi(p_l)}{p_r-p_l}\right]^2 + \frac{2}{p_r-p_l}\sigma^2
\end{aligned}$$

The second derivative is independent of z, so we find that p_l is the optimal product when $\partial Eu(z)/\partial z \leq 0$ evaluated at $z = p_l + \delta$, regardless of the sign of $\partial Eu(z)^2/\partial^2 z$. This is true if and only if $\psi(p_l) \leq \alpha\left(\widehat{p_l \cdot p_r}\right) + \lambda\left(\widehat{p_l \cdot p_r}\right)$. An experimental product is optimal otherwise, and is found by rearranging the internal solution $\partial Eu(z)/\partial z = 0$.

Probability of success is a function of the experimental product

Beliefs about the outcome of z are again distributed normally:

$$\frac{\psi(\tau_t^*) - E\left[\psi(z)\right]}{\sigma\sqrt{\frac{(z-p_l)(p_r-z)}{p_r-p_l}}} \sim \mathcal{N}(0,1)$$

Therefore, the probability of success is

$$\theta_t = \Pr(|\psi(z)| \leq |\psi(\tau_t^*)||h^t) = \left|\Phi\left(\frac{E\left[\psi(z)\right]+\psi(\tau)}{\sigma\sqrt{\frac{(z-p_l)(p_r-z)}{p_r-p_l}}}\right) - \Phi\left(\frac{E\left[\psi(z)\right]-\psi(\tau)}{\sigma\sqrt{\frac{(z-p_l)(p_r-z)}{p_r-p_l}}}\right)\right|$$

and entrepreneur t faces the following problem:

$$\max_z Eu_t(z|h^t) = -\theta E\left[\psi(z)\right]^2 - (1-\theta)\left(\psi(\tau)^2+\bar{\phi}\right) - \frac{(z-p_l)(p_r-z)}{p_r-p_l}\sigma^2$$

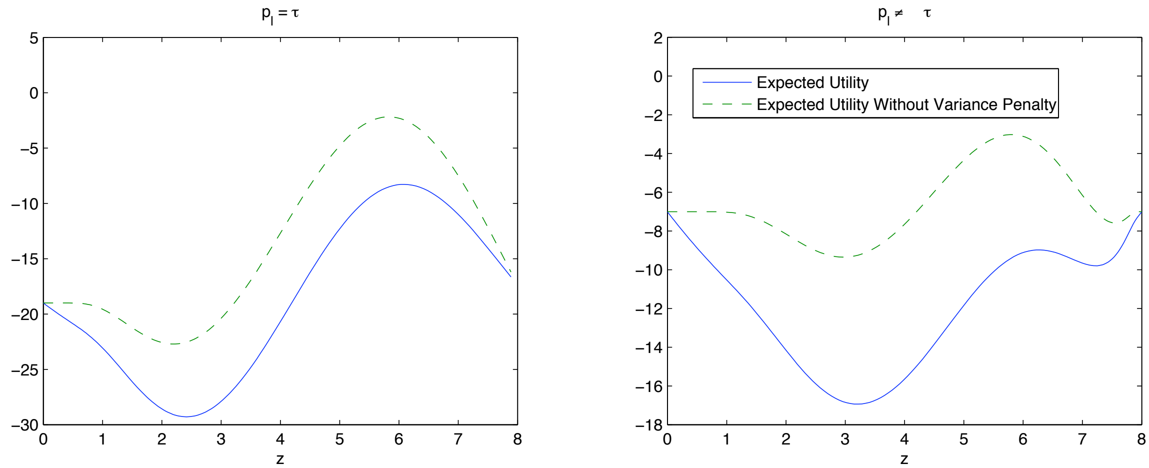

Figure B.1: Expected utility in the triangulating phase for $p_l = 0$, $\psi(p_l) = 10$, $p_r = 8$, $\psi(p_r) = -4$, $\mu = -1$, $\sigma = 2$, $\bar{\phi} = 3$, and $\delta = 1$. $\psi(\tau_t^*) = \psi(p_r)$ on the left and $\psi(\tau_t^*) = 2$ on the right.

Differentiating,

$$\begin{aligned}
\frac{\partial Eu(z)}{\partial z} &= -\frac{\partial \theta}{\partial z} E\left[\psi(z)\right]^2 - 2\theta \frac{\psi(p_r) - \psi(p_l)}{p_r - p_l} E\left[\psi(z)\right] + \frac{\partial \theta}{\partial z}\left(\psi(\tau)^2 + \bar{\phi}\right) \\
&\quad - \frac{p_r + p_l - 2z}{p_r - p_l}\sigma^2 \\
\frac{\partial^2 Eu(z)}{\partial z^2} &= -\frac{\partial^2 \theta}{\partial z^2} E\left[\psi(z)\right]^2 - 4\theta \frac{\psi(p_r) - \psi(p_l)}{p_r - p_l} E\left[\psi(z)\right] \frac{\partial \theta}{\partial z} \\
&\quad - 2\theta \left[\frac{\psi(p_r) - \psi(p_l)}{p_r - p_l}\right]^2 + \frac{\partial^2 \theta}{\partial z^2}\left(\psi(\tau)^2 + \bar{\phi}\right)
\end{aligned}$$

and

$$\begin{aligned}
\frac{\partial \theta}{\partial z} &= \phi\left(\frac{E\left[\psi(z)\right] + \psi(\tau_t^*)}{\sigma\sqrt{\frac{(z-p_l)(p_r-z)}{p_r-p_l}}}\right)\left[\frac{\frac{\psi(p_r)-\psi(p_l)}{\sqrt{p_r-p_l}}}{\sigma(z-p_l)(p_r-z)} - \frac{E\left[\psi(z)\right] + \psi(\tau_t^*)}{2\sigma\frac{((z-p_l)(p_r-z))^{3/2}}{\sqrt{p_r-p_l}}}(p_r + p_l - 2z)\right] \\
&\quad - \phi\left(\frac{E\left[\psi(z)\right] - \psi(\tau_t^*)}{\sigma\sqrt{\frac{(z-p_l)(p_r-z)}{p_r-p_l}}}\right)\left[\frac{\frac{\psi(p_r)-\psi(p_l)}{\sqrt{p_r-p_l}}}{\sigma(z-p_l)(p_r-z)} - \frac{E\left[\psi(z)\right] - \psi(\tau_t^*)}{2\sigma\frac{((z-p_l)(p_r-z))^{3/2}}{\sqrt{p_r-p_l}}}(p_r + p_l - 2z)\right]
\end{aligned}$$

As was the case for open-ended uncertainty, I solve for equilibrium computationally, noting that the solution is in the set $\{p_l, p_r\} \cup z^*$, where $z^* \in (p_l + \delta, p_r - \delta)$ solves

$$E\left[\psi(z)\right]^2 + \frac{2\theta\frac{\psi(p_r)-\psi(p_l)}{p_r-p_l}}{\partial\theta/\partial z} E\left[\psi(z)\right] = \psi(\tau)^2 + \bar{\phi} + \sigma^2$$

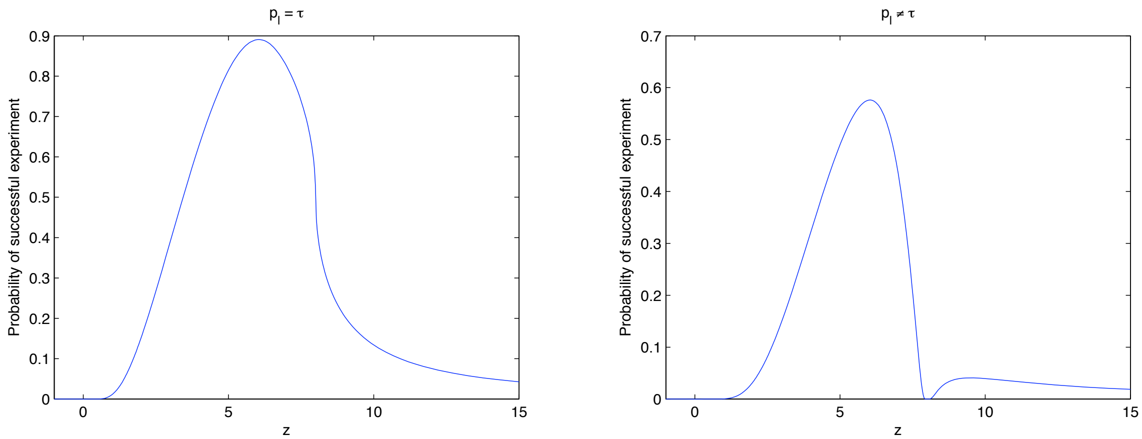

Figure B.2: Probability of success in the triangulating phase for $p_l = 0$, $\psi(p_l) = 10$, $p_r = 8$, $\psi(p_r) = -4$, $\mu = -1$, $\sigma = 2$, $\bar{\phi} = 3$, and $\delta = 1$. $\psi(\tau_t^*) = \psi(p_r)$ on the left and $\psi(\tau_t^*) = 2$ on the right.

C Proofs of Propositions

Assume without loss of generality that $\mu \leq 0$ and $\psi(p_0^*) \geq 0$. Recall that r^t and l^t are the right-most and left-most points in h^t.

Proof of Proposition 1

Under patent protection, r_t dominates all $z > r^t - \delta$ if $\psi(r^t) \leq 0$, and likewise l^t dominates all $z < l^t + \delta$ if $\psi(l^t) \geq 0$. Consider, then, $\psi(r^t) > 0$ and all products $z \geq r^t + \delta$; the case $\psi(l^t) \leq 0$ and $z < l^t - \delta$ is analogous. The first-order condition is

$$\psi(r^t) + \mu(z - r^t) = \frac{-\sigma^2}{2\mu\bar{\theta}}$$

Then $z = r_t + \delta$ is optimal if and only if $\frac{-\sigma^2}{2\mu\bar{\theta}} \geq \psi(r^t) + \mu\delta$. This implies that the product is stable for the general case when

$$|\psi(p_0^*)| \leq \frac{\sigma^2}{2\bar{\theta}|\mu|} + \delta|\mu|$$

As by construction, $r_t < z < r^t + \delta$ implies that patents are the cause of stability. Callander shows that without patent protection, the analogous corner solution is optimal if and only if $|\psi(p_0^*)| \leq \sigma^2/2\bar{\theta}|\mu|$. Therefore, the cause of stability can be attributed to patent protection when

$$\frac{\sigma^2}{2\bar{\theta}|\mu|} < |\psi(p_0^*)| \leq \frac{\sigma^2}{2\bar{\theta}|\mu|} + \delta|\mu|$$

If $\mu = 0$, then $\frac{\partial Eu(z)}{\partial z} < 0$ for all z and r^t is optimal.

Proof of Proposition 2

If the most recent product is better than any product before it, $p^*_{t-1} = \tau^*_t$, then the problem reduces to the same problem as the first period. Consider the case when $p^*_{t-1} \neq \tau^*_t$. Expected utility from the optimal experimental product z^* is

$$\begin{aligned} Eu(z^*) &= -\left[\frac{\sigma^2}{-2\mu\bar{\theta}}\right]^2 - \frac{\psi(p^*_{t-1}) - \frac{\sigma^2}{-2\mu\bar{\theta}}}{-\mu}\sigma^2 \\ &= \frac{\sigma^2}{2\mu\bar{\theta}}\left[\frac{(2\theta-1)\sigma^2}{2\mu\bar{\theta}} + 2\bar{\theta}\psi(p^*_{t-1})\right] \end{aligned}$$

The result for Condition 2 follows by setting $Eu(z^*) = -\left[\psi(\tau^*_t)\right]^2$.

Proof of Proposition 3

As in the Callander model, assume that $\psi(p_l) > 0 > \psi(p_r)$ and $|\psi(p_l)| \leq |\psi(p_r)|$. The case $\psi(p_r) > 0 > \psi(p_l)$ is analogous. If $p_l + \delta \geq p_r - \delta$, then p_l is the optimal product. Consider, then, the case $p_l + \delta < p_r - \delta$. Looking at the second-order condition in the constant-probability section of Appendix B, we see that $\frac{\partial^2 Eu(z)}{\partial z^2}$ is constant in z . Therefore, p_l is the optimal product if $\frac{\partial\theta}{\partial z} \leq 0$ at $z = p_l$. This is true when

$$-\psi(p_l) \geq \frac{(p_r - p_l)\sigma^2}{2\bar{\theta}\left[\psi(p_r) - \psi(p_l)\right]} + \delta\left[\frac{(\psi(p_r) - \psi(p_l))^2 - \bar{\theta}\sigma^2(p_r - p_l)}{\bar{\theta}\left(\psi(p_r) - \psi(p_l)\right)(p_r - p_l)}\right]$$

The second term on the right-hand side of the inequality is negative when $(\psi(p_r) - \psi(p_l))^2 - \bar{\theta}\sigma^2(p_r - p_l) > 0$, which reduces to $\psi(p_r) - \psi(p_l) > \frac{\bar{\theta}\sigma^2(p_r - p_l)}{\psi(p_r) - \psi(p_l)}$. This inequality always holds in the triangulating phase (otherwise, the search would reach stability). Therefore, this second term is the same sign as the first term (both are negative in this example), so the generalization of this result

$$|\psi(p_l)| \geq \left|\frac{(p_r - p_l)\sigma^2}{2\bar{\theta}\left[\psi(p_r) - \psi(p_l)\right]} + \delta\left[\frac{(\psi(p_r) - \psi(p_l))^2 - \bar{\theta}\sigma^2(p_r - p_l)}{\bar{\theta}\left(\psi(p_r) - \psi(p_l)\right)(p_r - p_l)}\right]\right|$$

reduces to

$$|\psi(p_l)| \geq \left|\frac{(p_r - p_l)\sigma^2}{2\bar{\theta}\left[\psi(p_r) - \psi(p_l)\right]}\right| + \delta\left|\frac{(\psi(p_r) - \psi(p_l))^2 - \bar{\theta}\sigma^2(p_r - p_l)}{\bar{\theta}\left(\psi(p_r) - \psi(p_l)\right)(p_r - p_l)}\right|$$

Experimentation is optimal otherwise, and is found by solving the first-order condition. Callander's Proof of Proposition 3 shows that this product dominates all others.

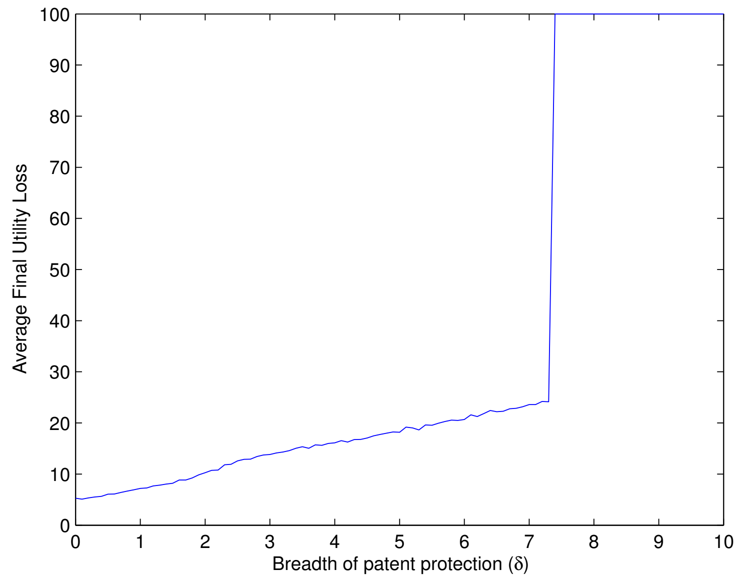

Figure D.1: Average utility loss as a function of δ

Without patent protection, Callander shows that stability occurs when $|\psi(p_l)| \geq \left|\frac{(p_r-p_l)\sigma^2}{2\bar{\theta}[\psi(p_r)-\psi(p_l)]}\right|$. Therefore, the cause is patent protection when

$$\left|\frac{(p_r - p_l)\sigma^2}{2\bar{\theta}\left[\psi(p_r) - \psi(p_l)\right]}\right| < |\psi(p_l)| \leq \left|\frac{(p_r - p_l)\sigma^2}{2\bar{\theta}\left[\psi(p_r) - \psi(p_l)\right]}\right| + \delta \left|\frac{\left(\psi(p_r) - \psi(p_l)\right)^2 - \bar{\theta}\sigma^2(p_r - p_l)}{\bar{\theta}\left(\psi(p_r) - \psi(p_l)\right)(p_r - p_l)}\right|$$

Proof of Proposition 4

This proof is the same as Callander's Proof of Proposition 4.

D Simulations under constant-probability of success

For each of the constant-probability simulations, the probability of successful experimentation is exogenously given at $\bar{\theta} = 0.75$. $\mu = -1$, $\sigma = 2$, $\bar{\phi}$=3, and $p_0^* = 0$ are exogenously given for each search. $\delta = 1$ and $\psi(p_0^*) = 10$ unless otherwise noted.

Figure D.1 shows that the average utility loss is monotonically increasing in the breadth of patent protection. The jump at $\delta = 8 = \frac{\psi(p_0^*)-\alpha}{|\mu|}$ occurs when the status-quo product covers the entire experimental space that would persist under no patent protection; in this scenario, play never begins, and the outcome of the search is the same as the status-quo outcome.

Figure D.2 displays the frequencies of patent-induced stability for the same simulation. Patent-induced stability in the monotonic phase increases with the breadth

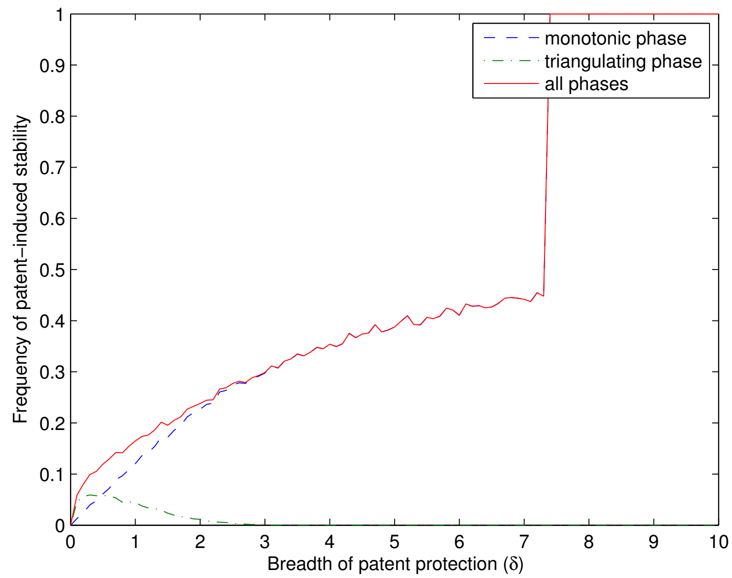

Figure D.2: Frequency of patent-induced stability

$\psi(p_0^*)$	5	7	10	15	20	30
Average in monotonic ($\delta = 1$)	0	0.24	0.46	0.53	0.58	0.62
Average in triangulating ($\delta = 1$)	0	0.01	0.10	0.23	0.31	0.44
Average in monotonic ($\delta = 2$)	0	0.22	0.45	0.53	0.57	0.60
Average in triangulating ($\delta = 2$)	0	0	0.01	0.09	0.16	0.26
Average in monotonic ($\delta = 3$)	0	0.12	0.34	0.45	0.49	0.52
Average in triangulating ($\delta = 3$)	0	0	0	0.03	0.08	0.17

Table 3: Number of products produced until stability (includes first period of stability)

of patent protection, while patent-induced stability in the first triangulating phase is quasi-concave in the breadth of patent protection. Patent-induced stability in the triangulating phase is decreasing in the frequency of patent-induced stability in the preceding monotonic phase of the search. However, at the same time, triangulating patent-induced stability is increasing in the breadth of protection, conditional on arrival to the triangulating phase. The result is the immediate increase in triangulating patent-induced stability followed by a drop as the frequency of monotonic patent-induced stability rises.

Figure D.3 shows the results of varying this relative complexity by varying the status quo outcome, holding everything else constant at the same values as above. When $\psi(p_0^*)$ is low, the stable outcome rises linearly with complexity and peaks at α. This is because when the status quo outcome is below α, experimentation will converge immediately as stability is reached in the first period. When the the search reaches the second period, $\psi(p_0^*)-\alpha-\delta > 0$, and the average stable outcome decreases

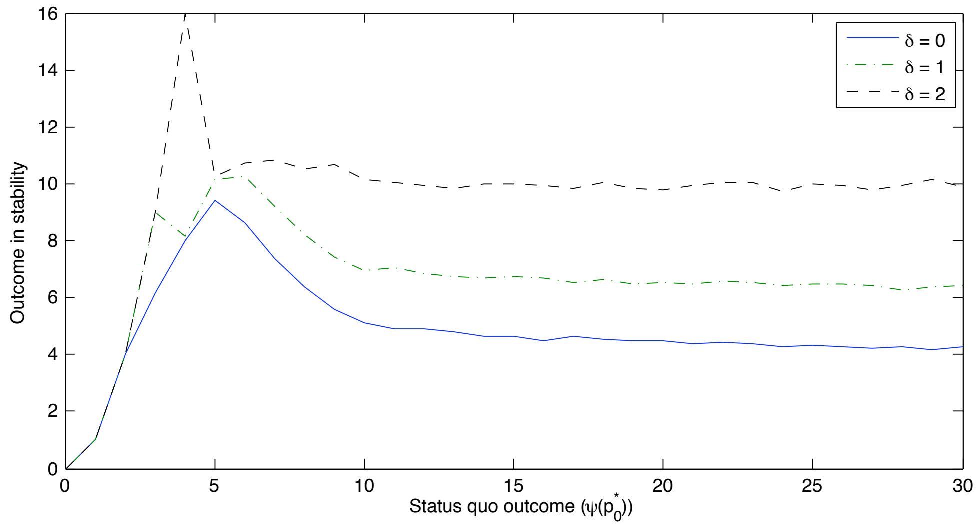

Figure D.3: Stable outcome as a function of status quo outcome

with the status quo outcome. Examination of Table 3 shows that this is because search is more likely to overshoot the $E\left(\psi(p_t^*|h^t)\right) = \alpha$ mark and reach the triangulating phase, which results in better outcomes.

E Matlab Code

Monotonic Phase Experiment

```
function [xb oxb stable stuck] = monotonic(delta,mu,sigma,fee,x,ox);

% Return vector:
%           1 xb — the next product in the search
%           2 oxb — outcome of the next product in the search
%           3 stable — indicator for stablility
%           4 stuck — indicator for getting stuck

n = max(size(x)); % number of iterations to date
[~, i] = min(abs(ox)); % i is the index of the best—so—far
tau = x(i); % best—so—far product
oTau = ox(i); % outcome of the best—so—far
rt = x(n); % last point tried = rightmost point
ort = ox(n); %outcome of rt
stable = 0;  % indicator for stability
stuck = 0;   % indicator for getting stuck

% helper functions
feZ = @(ort, mu, z, rt) ort + mu*(z — rt);
fy1 = @(oTau, sigma, z, rt) (feZ(ort, mu, z, rt) — oTau)/...
    sigma/abs(z — rt)^.5;
```

```
fy2 = @(oTau, sigma, z, rt) (feZ(ort, mu, z, rt) + oTau)/...
    sigma/abs(z - rt)^.5;
fphi = @(x, mu, sigma) .5 - 0.5*erf((mu - x)/2^.5/sigma);
ez = @(z) feZ(ort, mu, z, rt);
y1 = @(z) fy1(oTau, sigma, z, rt);
y2 = @(z) fy2(oTau, sigma, z, rt);
phi = @(x) fphi(x, 0, 1);

% describe the expected utility accros z > rt
prob = @(z) abs(-phi(y1(z)) + phi(y2(z)));
negProb = @(z) -prob(z);
E = @(z) -(prob(z))*ez(z)^2 - (1 -prob(z))*(oTau^2 + fee) - ...
    abs(z - rt)*sigma^2;
negE = @(z) -E(z);

% find the optimal product
[xb exb] = fminbnd(negE, rt + delta, 2*(rt+delta) - 2*ort/mu);
exb = -exb;
if (E(rt + delta) > exb)
    xb = rt + delta;
    exb = E(rt + delta);
end

if (exb < -oTau^2 - fee) % no experimentation
    xb = tau;
    oxb = oTau;
    stable = 1;
    if (tau ~= rt) stuck = 1;
    end
else % Random draw for oxb
    oxb = normrnd(ez(xb), sqrt(xb - rt)*sigma);
end

end
```

Triangulating Phase Experiment

```
function [xb oxb stable stuck] = triangulating(delta,mu,sigma,fee,x,ox);

% Return vector:
%           1 xb - the next product in the search
%           2 oxb - outcome of the next product in the search
%           3 stable - indicator for stablility
%           4 stuck - indicator for getting stuck

n = max(size(x)); % number of iterations to date
[~, i] = min(abs(ox)); % i is the index of the best-so-far
tau = x(i); % best-so-far product
oTau = ox(i); % outcome of the best-so-far
stable = 0;  % indicator for stability
stuck = 0;   % indicator for getting stuck
```

```
%find spanning bridge
[xs i] = sort(x);
oxs = ox(i);
for i = n:-1:2
    if (oxs(i)*oxs(i-1) < 0)
        pl = xs(i - 1);
        opl = oxs(i - 1);
        pr = xs(i);
        opr = oxs(i);
    end
end


% helper functions
feZ = @(mu, z, pl, pr, opl, opr) opl + (z - pl)/(pr - pl)*(opr - opl);
fy1 = @(oTau, sigma, z, pl, pr, opl, opr) feZ(mu,z,pl,pr,opl,opr) -...
    oTau)/sigma/abs((z - pl)*(pr - z)/(pr - pl))^.5;
fy2 = @(oTau, sigma, z, pl, pr, opl, opr) feZ(mu,z,pl,pr,opl,opr) +...
    oTau)/sigma/abs((z - pl)*(pr - z)/(pr - pl))^.5;
fphi = @(x, mu, sigma) .5 - 0.5*erf((mu - x)/2^.5/sigma);
ez = @(z) feZ(mu, z, pl, pr, opl, opr);
y1 = @(z) fy1(oTau, sigma, z, pl, pr, opl, opr);
y2 = @(z) fy2(oTau, sigma, z, pl, pr, opl, opr);
phi = @(x) fphi(x, 0, 1);

% describe the expected utility accros z \in [pl, pr]
prob = @(z) abs(-phi(y1(z)) + phi(y2(z)));
negProb = @(z) -prob(z);
E = @(z) -(prob(z))*ez(z)^2 - (1 -prob(z))*(oTau^2 + fee) -...
    abs((z - pl)*(pr - z)/(pr - pl))*sigma^2;
negE = @(z) -E(z);

% find the optimal product
if (pl + delta >= pr - delta)
    xb = pl;
    exb = -opl^2 - fee;
elseif (pr - pl - 2*delta > 2)
    % split in half to make it easier on fminbnd
    % not necessary to split recursively for small simulations
    mid = pl + (pr - pl)/2;
    [xb1 exb1] = fminbnd(negE, pl + delta, mid);
    [xb2 exb2] = fminbnd(negE, mid, pr - delta);
    if (exb1 < exb2)
        exb = exb1;
        xb = xb1;
    else
        exb = exb2;
        xb = xb2;
    end
    exb = -exb;
else
    [xb exb] = fminbnd(negE, pl + delta, pr - delta);
    exb = -exb;
end
```

```
if (exb <= -oTau^2 - fee) % no experimentation
    xb = tau;
    oxb = oTau;
    stable = 1;
    if (tau ~= x(n)) stuck = 1;
    end
else % Random draw for oxb
    oxb = normrnd(ez(xb),...
        sqrt(abs((xb - pl)*(pr - xb)/(pr - pl)))*sigma);
end

end
```

Search

```
function [xl oxl oxb2 n j k l m p q x ox varox] =...
    search(sq, osq, delta, mu, sigma, fee);

% Return vector:
%           1 xl - the last product (convergence point)
%           2 oxl - outcome of the last product
%           3 oxb2 - square of the outcome of the last product
%           4 n - number of iterations until convergence
%           5 j - number of iterations in the monotonic phase
%           6 k - number of iterations in the triangulating phase
%
%  What induces stability?
%           7 l monotonic phase
%           8 m stuck monotonic
%           9 p triangulating phase
%           10 q stuck triangulating

x=sq;                          %Vector of products attempted
ox=osq;                        %Outcomes of products attempted
xb=sq;                         %Best product so far
oxb=osq;                       %Outcome of best product so far

% Housekeeping:
j=0;k=0;n=0;m=0;l=0;
stable=0;p=0;q=0;

%First Period%%%%%%%%%%%%%%%
i=1;
[xb oxb stable ~] = monotonic(delta, mu, sigma, fee, x, ox);
x = [x xb];
ox = [ox oxb];

%The Second and Subsequent Periods%%%%%%%%%%%%%%

if (stable) %accounting
    j = 1;
```

```
end

% The Monotonic Phase
while (~stable && oxb > 0)
    i = i + 1;
    j = j + 1;
    [xb oxb stable m] = monotonic(delta, mu, sigma, fee, x, ox);
    x = [x xb];
    ox = [ox oxb];
end

% Indicate that stability happens from the monotonic phase
if (stable)
    l = 1;
    j = j - 1;
end

% The Triangulating Phase
while (~stable)
    i = i + 1;
    k = k + 1;
    [xb oxb stable q] = triangulating(delta,mu,sigma,fee,x,ox);
    x = [x xb];
    ox = [ox oxb];
end

% Indicate that stability happens from the triangulating phase
if (~l)
    p = 1;
    k = k - 1;
end

% Final calculations
n = i;
oxb2 =oxb^2;
xl=x(i+1);
oxl=ox(i+1);
varox = var(ox);
```

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