METHODOLOGY

ANCHORING

A two-stage regression model for detecting anchoring is specified in Beggs & Graddy (2009) who themselves cite Genesove & Mayer (2001). The same model is used to detect anchoring effects in later papers such as Hong et al. (2015), and in general, may be estimated for goods that exhibit unchanging hedonic quality over time – a key assumption of their work. Intuitively, their model identifies anchoring by looking at two sales of an item, say a painting, at different points in time. By controlling for hedonic characteristics (artist, medium, etc.) and unobserved inputs into the past price (bidding behavior), the difference between past price and hedonic quality can be isolated, and identified as the anchoring effect on current price.

Hedonic regressions are commonly used to estimate demand for highly heterogeneous items such as art, wine, and real estate as a function of their constituent attributes[1] [2]. For example, the value of a painting may depend on its dimensions and authenticity, while a bottle of wine may be appraised based on its age and where it was grown. In the first stage of the model, Beggs & Graddy (2009) regress the sale prices $P_t \in$

[1] Edmonds, Radcliffe G. "A theoretical basis for hedonic regression: A research primer." *Real Estate Economics* 12.1 (1984): 72-85.

[2] Costanigro, Marco, Jill J. McCluskey, and Ron C. Mittelhammer. "Segmenting the wine market based on price: hedonic regression when different prices mean different products." *Journal of agricultural Economics* 58.3 (2007): 454-466.

$R^{[n,1]}$ of n resold paintings[3] on their k hedonic and temporal variables $\boldsymbol{X} \in R^{[n,(k+1)]}$[4], while also controlling for temporal effects $\delta_t \in R^{[n,1]}$. This yields a hedonic price prediction $\pi_t \in R^{[n,1]}$ for each observation of a painting sale. For my replication work, I use the same variables that Beggs & Graddy use on the Impressionist and Contemporary datasets, respectively. For Impressionist art this includes painting date, length, width, medium of the artwork, indicators of authenticity (signed, monogrammed, stamped), and artist. For Contemporary art this includes painting date, length, width, medium, and artist. The temporal effects are modelled by half-year time dummies.

$$\pi_t = \boldsymbol{XB} + \delta_t$$

In the same vein as Beggs & Graddy, I use the natural log of prices and hedonic price predictions, which allows us to interpret the regression results as relative effects. For unsold items, we proxy value with 80% of the low estimate as they do. It is important to note that multiple hedonic price predictions at different times $\pi_t, t \in \{1, 2, 3 \dots\}$ may differ for the same painting, since these are estimated based on the price index $P_t, t \in$

[3] The data here consists of all sale observations that correspond to the set of paintings that have been resold multiple times. Beggs & Graddy have painstakingly verified each observation against presale catalogs. Because those are not available, in my replication analysis I make the assumption that duplicate observations in their Impressionist and Contemporary data refer to multiple sales of the same item.
[4] Each sale observation includes the auction date, hence the $k + 1$ dimensions in the data.

$\{1, 2, 3 \ldots\}$. The price index reflects demand for art, which varies over time. The k hedonic variables, however, are assumed to remain constant across sales.

In the second stage of the model, Beggs & Graddy specify the following regression in order to separate out anchoring from other effects. They for each unique painting.

$$\omega = a_1\pi_t + a_2(P_{t-1} - \pi_t) + a_3(P_{t-1} - \pi_{t-1})$$

Above, P_{t-1} is the previous hammer price of a painting at time $t - 1$ and P_t is the currents sale at time t. Beggs and Graddy fit several regressions where the response ω represents either the hammer price, an indicator for whether the item sells (which involves a probit transformation), or the presale estimate. The anchoring effect is captured in the term $(P_{t-1} - \pi_t)$, which specifies how information from the past price (the anchor) P_{t-1} differs the later hedonic price prediction π_t and thus the dependent variable ω. The last term $P_{t-1} - \pi_{t-1}$ controls for unobservable non-hedonic effects on price. For example, if the past price was not only a function of the painting's hedonic characteristics, but was also a function of bidding activity at the time, this will be controlled for in the $P_{t-1} - \pi_{t-1}$ term. Otherwise, $P_{t-1} - \pi_t$ not only reflects the impact by past price on the later hedonic prediction, but also past bidding activity and other non-hedonic factors inputted into P_{t-1}. In the case of the dependent variable P_t (for a regression for hammer price), we see that those non-hedonic inputs, usually captured by $P_t - \pi_t$, would instead be contained in the residuals. One should also note that because hedonic prices may vary over time, $P_{t-1} - \pi_t$ is distinct from $P_{t-1} - \pi_{t-1}$.

ANCHORING AND SUBSTITUTION

As we discussed earlier and as Beggs & Graddy (2009) note, it is extremely difficult to track down multiple sales of the same item, to the extent that even auction house specialists formulate estimates from researching sales of related goods (substitutes) instead. The same art piece can become a drastically different hedonic object within its lifetime. And, many years or decades may elapse between sales of the same art piece – far too long to reliably measure anchoring biases.

It is reasonable to believe that buyers (and specialists), when bidding on an artwork, make judgments based not only on that artwork's past sales, but also what similar pieces went for as well. This allows for a much more versatile approach to identifying anchoring effects, or if between different goods, cross-effects – given that we control adequately for hedonic differences. Here, we build on the two-stage regression model presented earlier.

Suppose, as before, we have our same design matrix $\boldsymbol{X} \in R^{[n,k+1]}$ and our hammer prices $P_t \in R^{[n,1]}$. We run the first hedonic regression as before, except that we are not concerned specifically with resale and simply treat auction date as another explanatory variable.

$$\pi = \boldsymbol{XB} + \delta_t$$

We next depart from the original model. Denote the sale observation of our current good as x_c and the observation of a single substitute as x_s, such that the hedonic predictions estimated above are π_c, π_s, and P_c, P_s are the respective hammer prices[5]. Then our second regression is:

$$\omega_c = b_1\pi_c + b_2(P_s - \pi_c) + b_3(P_s - \pi_s) + b_4 Q$$

Here, the subscripts for the past and current sales $t-1$ and t are replaced by subscripts for the substitute s and current good c. The previous regression model assumed that there was no unobserved quality changes in the painting across sales, such that $\boldsymbol{X}_t = \boldsymbol{X_{t-1}} = \boldsymbol{X}$ (though hedonic prices π_t could still change). However, in this generalized framework, we assume that characteristics do differ across goods, that is $\boldsymbol{X_c} \neq \boldsymbol{X_s}$. Thus, we need to control for those hedonic differences by including a measure of substitution Q in our regression model, which may be constructed from either π or $\boldsymbol{X}$. This allows us identify anchoring effects in $(P_s - \pi_c)$, as before.

[5] As with resale, we add the temporal constraint that the sale of a substitute must occur before the sale of the current good – in this context, one can only anchor on the past.

What if a painting has multiple substitutes – the multivariate case? Let a given good x_c have a vector of substitutes $X_s = \{x_{s1}, x_{s2}, \dots x_{sd}\}$. We can write:

$$w_c = b_1\pi_c + b_{i2}(P_{RS} - \pi_c) + b_{i3}(P_{RS} - \pi_{RS}) + b_3 Q$$

$$P_{RS} = \frac{1}{d}\sum_{i=1}^{d} P_{si} \qquad \pi_{RS} = \frac{1}{d}\sum_{i=1}^{d} \pi_{si}$$

Here, P_{RS} and π_{RS} are price and hedonic prediction for a representative substitute. Two goods c_1 and c_2 may have different numbers of substitutes d_1, d_2, which is why for our regression model it is necessary to aggregate them via a function such as the mean or maximum (I use the former). Hence, this multivariate regression tests whether there exists anchoring effects for the sale of the current good with respective to the "average" substitutive – a conglomerate of all substitutes together. The marginal effect of Q on w_c, then, represents how strongly the dependent variable (such as price) is affected by our quality of substitution. As before, the measure of substitution Q may be calculated from the multivariate π or $\boldsymbol{X}$.

MEASURING SUBSTITUTION (SIMILARITY) ACROSS ART PIECES

One of our interviewees stated that no two art pieces are the same. Even prints, an artistic medium where a batch of 100 or 150 copies (editions) of the same piece are produced, can vary dramatically in quality and price. In this research, we experiment with two simple measures of substitution Q_1, Q_2 between art pieces. The first is derived from the hedonic predictions, and represents unobserved quality differences. The second is formulated from our interviews with art experts and specialists. These do not and cannot perfectly capture differences between artworks, but do provide a starting point for quantitatively measuring art similarity.

MEASURE #1: SECOND MOMENT OF HEDONIC PRICE DIFFERENCES

For a current good x_c and other art pieces $X_s = \{x_{s1}, x_{s2}, \dots x_{sd}\}$, which are aggregated into an "average substitute," one way we can measure substitution is by examining differences between the hedonic price predictions. These correspond to unobserved quality differences. We use the following measure, which is essentially a second moment estimator about the current good's hedonic prediction[6]:

$$Q_1 = -\log \frac{1}{n} \sum_{i=1}^{d} (\pi_c - \pi_{si})^2$$

As described before, we work in logs for relative effects, and the negative sign allows a higher Q_1 (smaller hedonic differences) to correspond to higher substitutability. The squared term is used instead of absolute value so that the estimator also captures the

[6] We do not subtract the other term $(E[X = |\pi_c - \pi_s|])^2$ typically used in calculating variance $V[X] = E[X^2] - E[X]^2$, since that (squared) first moment term reflects absolute hedonic differences $X = |\pi_c - \pi_s|$ between pieces which we still wish to account for. Hence, our measure captures both variance and mean.

variability of hedonic differences. This is important because substitutability may differ drastically across goods. As the above measure describes how accurate our average substitute is, we note higher variability in the differences also corresponds to lower substitutability, since it is preferable to have uniformly substitutable goods rather than a motley mix of good and bad ones.

MEASURE #2: DOMAIN KNOWLEDGE

For our second measure of substitution, we draw upon domain knowledge from our expert interviews. We found some of the most commonly mentioned and important determinants of artwork similarity (substitutability) are artist, medium, signs of authenticity, size of the artwork, and how recently the artwork was auctioned. The opinions of our interviewees on more complex factors, such subject matter and artistic style, seemed to be mixed: some said these were key to measuring similarity between pieces, while others looked more to the factors above[7]. One thing we were surprised to learn about size in particular was that its importance in determining similarity varies at different price points. For the lower and middle price ranges, people usually purchase art as a decoration, and tend to purchase pieces of similar sizes to display next to each other. As price increases, people tend to value artwork more as an investment, and so the importance of size in determining similarity decreases.

[7] For further discussion: http://www.jstor.org/stable/pdf/20715780.pdf?acceptTC=true

To capture these anecdotal observations about art similarity, we present a second measure of substitution between a current piece x_c and a substitute piece x_s, here formulated as two sale observations at different points in time. This measure of substitution depends on size S_i, hedonic price π_i, and auction date t_i. Artist, authenticity, medium are categorical variables and thus used primarily to filter for substitutes, as we describe later.

$$Q_2 = -\log\frac{1}{n}\sum_{i=1}^{n}\left[\frac{(S_c - S_s)^2}{1 + (\pi_c + \pi_s)} + \Delta days(t_c, t_s)\right]$$

Greater differences in size between the two goods correspond to decreased similarity and thus substitutability[8]. However, this effect decreases as the hedonic values of the pieces rise. Consistent with the anchoring literature discussed earlier, the farther the anchor (the substitute here) is in the past, the weaker the anchoring effect is. Note that we use hedonic prices to indicate increasing value. This is because P_c, P_s can reflect not only π_s but also non-hedonic determinants of price, and furthermore, P_c is the dependent variable to be determined in our main anchoring regression. No possible past anchoring effects are considered with the hedonic prices here: we assume buyers are myopic, as captured in the time difference effect, and assess similarity primarily based on hedonic factors.

[8] We add one in the denominator of the first term to protect against results exploding toward infinity. Empirically, however, this is negligible compared to the magnitude of our hedonic prices.

The measures Q_1 and Q_2 are similar, given that both draw upon hedonic features, and often yield similar results (as we find later). However, they are distinct because Q_2 includes temporal effects (which do not enter into the hedonic regressions), and accounts for the relative differences in size between works, which are not explicitly captured in the individual π terms.

HOW TO EVALUATE MEASURES OF SUBSTITUTION

We are also interested in how accurate these measures of substitution Q_1, Q_2 are, independently of our anchoring regressions. < do all this stuff after full draft. You CAN already talk about how measures of substitution are helpful/not helpful for understanding anchoring, but independently of that, might be nice to have regression. Maybe regress real prices for pieces on measures of substitution to assess their accuracy.
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OTHER PRACTICAL CONSIDERATIONS

It is intractable to calculate substitution measures between a current good and all other goods, many of which may be irrelevant. Thus, to find substitutes for the sale of a current art piece, we search through our data for past sales of other pieces with the same artist, medium, and signs of authenticity. I also omit observations where no substitutes were found. This allows us to run our regressions for anchoring cross-effects with the two substitution measures described above.

I begin by replicating Beggs & Graddy's original anchoring regression for their two Impressionist and Contemporary datasets, then apply it to my new dataset of assorted art sales. Then, I run my anchoring cross-effects regression on all three datasets. I find significant evidence of anchoring effects and cross-effects.