From the collections of the Princeton University Archives, Princeton, NJ

Statement on Copyright Restrictions

This senior thesis can only be used for education and research (among other purposes consistent with "Fair use") as per U.S. Copyright law (text below). By accessing this file, all users agree that their use falls within fair use as defined by the copyright law. They further agree to request permission of the Princeton University Library (and pay any fees, if applicable) if they plan to publish, broadcast, or otherwise disseminate this material. This includes all forms of electronic distribution.

U.S. Copyright Law (Title 17, United States Code)

The copyright law of the United States governs the making of photocopies or other reproductions of copyrighted material. Under certain conditions specified in the law, libraries and archives are authorized to furnish a photocopy or other reproduction. One of these specified conditions is that the photocopy or other reproduction is not to be "used for any purpose other than private study, scholarship or research." If a user makes a request for, or later uses, a photocopy or other reproduction for purposes in excess of "fair use," that user may be liable for copyright infringement.

Inquiries should be directed to:

Princeton University Archives
Seeley G. Mudd Manuscript Library
65 Olden Street
Princeton, NJ 08540
609-258-6345
mudd@princeton.edu

Debt Composition and the Determinants of Default:
An Examination of the Optimal Waiver Policy upon a Covenant Violation

Zachary Savidge

Submitted to Princeton University
Department of Economics
In Partial Fulfillment of the Requirements for the A.B. Degree
April 15th, 2015

This paper represents my own work in accordance with University regulations.

_____ _____

Zachary Savidge Date

Acknowledgements

I thank my advisor, Harrison Hong, for his advice and guidance, and my friends, for their help revising.

Abstract

I offer a model of the optimal creditor and firm owner decisions upon a covenant violation. The model implies that the share of private debt in a firm's capital structure is more predictive of foreclosure than the share of total debt and that asset volatility is not predictive of foreclosure. I find the empirical evidence supports these hypotheses. These finding suggest novel conclusions about the determinants of creditor induced default and re-emphasize the need to treat the default threshold as endogenous in credit risk and pricing applications.

Table of Contents

I. Introduction **1**

II. Literature Review **4**
2.1 The Basic Structural Credit Model …… 4
2.2 Covenant Design…….. 5
2.3 Endogenous Default Boundaries……... 6
2.4 The Effects of Covenant Violations……... 8

III. Model **11**
3.1 Model Firm Structure…… 12
3.2 Value of the Claims Upon Foreclosure…….. 13
3.3 Value of the Continuing Loan, Note and Equity Claim…….. 13
3.4 Optimal Bank Choice…… 20

IV. Comparative Statistics and Testable Hypotheses **23**
4.1 Comparative Statistics: Waiver Fee and Default Threshold…… 23
4.2 The Breakeven Wavier Fee…… 26
4.3 Comparative Statistics: Asset Characteristics…….. 28
4.4 Comparative Statistics: Capital Structure Characteristics…… 34
4.5 Testable Hypotheses…….. 41

V. Empirical Methodology and Data **42**
5.1 Data……... 42
5.2 Dependent Variable Construction…….. 46
5.3 Independent Variable Construction……... 47
5.4 Hypothesized Coefficient Values……... 51

VI. Empirical Results **53**
6.1 Bank Debt Share……..……..... 53
6.2 Asset Volatility……...……..... 56
6.3 Bank Loan Spread……...……..... 58
6.4 Subordinated Debt Spread…….…… 59
6.5 Auxiliary Variables……. 62

VII. Conclusion **63**

Appendix **65**

Bibliography **77**

I. Introduction

The 2008 financial crisis emphasized both the importance of measuring credit risk – not only for the return profile or stability of a single portfolio or institution, but for the broader economic welfare of society – and the difficulty we have in doing so. Though the 2008 meltdown may have been sparked by a poor appraisal of the risks embedded in securitized mortgages specifically, it reflects the insufficiencies in models not only of securitized mortgage credit, but of credit risk broadly. This work aims to further the understanding of corporate credit risk, in particular, by shedding light on the determinants of default in corporate debt instruments.

The theoretically robust means of modeling the credit risk of an individual corporate debt instrument is referred to as the "structural" approach to credit modeling. In the structural approach, the value of a debt instrument is treated as a function of the underlying asset value evolution and a threshold asset value at which the firm defaults. The standard formulation of the structural credit model produces forecasts of key parameters that do not match the empirical data observed across several dimensions, however. Though several modifications, some more plausible than others, have been suggested to better reconcile the model with the data, there are still substantial inconsistencies that have not been satisfactorily addressed by the literature.

This work is motivated by one of these key inconsistencies – namely, the portion of the difference between the forecasted and observed recovery rate that can be attributed to a misspecified threshold asset value (or "default boundary") used to proxy the point at which default occurs. The default boundary is traditionally formulated as a linear function of total debt principal outstanding. Moody's KMV CreditMonitor – a widely used quantitative credit

risk tool – estimates the default boundary as the short-term debt principal outstanding plus one half of the long-term debt principal outstanding in its model[1]. Though such estimates are valued for their easy implementation, they misrepresent the default boundary because they over simplify the factors that affect it. Indeed, treating the default boundary as a fixed percentage of the total principal outstanding fails to capture the firm-specific variables influencing the complex creditor/debtor dynamic that is primarily responsible for default.

A firm can default either because it is forced to do so by a creditor or because its owner pro-actively chooses to initiate default; though empirical evidence, presented in Zhang (2009) and Carey and Gordy (2008), suggest that creditors often trigger default in practice. That being said, creditors cannot foreclose simply because they perceive the monies owed to them (i.e. their "claim") to be at risk. Only the violation of specified covenants, which outline financial and operating metrics that the firm must adhere to and are stipulated in the indenture of each debt instrument, enable creditors to take action against the company. When a firm does violate a covenant, the respective creditor must decide whether to exercise his right to foreclose or to formally excuse ("waive") the violation. Dichev and Skinner (2002) – in conjunction with a number of empirical studies – establish that covenants are set "tightly" or, in other words, written such that violations occur often even when the violator is not in financial distress. Hence debt covenants are both frequently violated and frequently waived. Thus, the important determinant of the threshold at which a firm defaults is not the threshold at which it violates a covenant. Rather, a firm defaults whens the value of its assets have sufficiently deteriorated such that, in conjunction with case specific characteristics, a creditor is induced to foreclose upon a covenant violation.

---

[1] Moody's KMV is a division of Moody's Analytics Enterprise Risk Solutions. Its CreditMonitor and EDFCalc products are the industry standard quantitative credit risk tools for measuring credit risk in real time using market data. Statistic from Crosbie and Bohn (2003).

To aid in the formulation of a more accurate default boundary, I focus on the determinants of the waiver/foreclosure decision around a covenant violation. In particular, I aim to provide evidence that private creditors endogenously select the default boundary without regard to other claimants because they are the sole creditors with the capability to force a default (for reasons expanded upon in Section 2). To do so, I develop a model of the waiver/foreclosure decision upon a violation that values the claims in a structural framework and treats the decision as a simultaneous game with symmetric information between two agents: the firm owner and the private creditor. I show that predictions of this model are reflected in the data, bolstering the notion that deriving the default boundary as a function of multiple asset and debt characteristics increases the accuracy of structural pricing models. The data particularly support the two key hypotheses of the model. First, that private creditors dictate the waiver/foreclosure decision, suggesting that the share of private debt in the capital structure – as opposed to simply the share of total debt (i.e. total principal outstanding) – is a key determinant of the default boundary. Second, that asset volatility has an indeterminate effect on the waiver/foreclosure decision, suggesting that traditional metrics, such as distance to default, are poor predictors of foreclosure.

This work proceeds as follows: Section 2 reviews the existing literature relevant to the topic, Section 3 develops a model of the waiver/foreclosure decision, Section 4 examines the implications of the model and outlines a series of stylized hypotheses, Section 5 outlines the empirical approach for testing these hypotheses, Section 6 presents the results of the empirical analysis and Section 7 concludes.

II. Literature Review

This work relates to three significant bodies of work in the literature: the literature focused on modeling credit risk, the literature on the role of financial covenants and, peripherally, the literature on optimal contract design. The existing literature outlines a rich and theoretically robust framework for modeling credit risk. It does not, however, adequately address the point at which default occurs in this framework. The literature broadly accepts that covenant thresholds and violations influence this point of default. A growing body of research, however, demonstrates that creditors predominantly use covenants as a means of enacting change on the behavior of the debtor rather than as a means of forcing default. Thus, there is a need, currently un-explored in the literature, to understand the conditions in which creditors opt to use a covenant violation to foreclose on their claim and force default. My contribution is to elucidate the determinants of the waiver/foreclosure decision upon a covenant violation. In particular, I demonstrate several novel determinants by accounting for their impact on the actions of both the creditor and equity holder in a company with a multi-tier capital structure.

I begin by reviewing the basics of structural credit models as they relate to this work. Next, I discuss the design of covenants and their role in enabling creditors to force default. I then review the attempts to endogenousize the default boundary to date. I focus, in particular, on the work of Carey and Gordy (2008). Finally, I review the impact of covenant violations on the behavior of creditors and the firm.

2.1 The Basic Structural Credit Model

Structural models, as pioneered by Merton (1974), employ the principles of modern option pricing theory to value corporate debt. By recognizing that the value of a debt claim is derived from the value of the assets that underlie it, structural credit models enable the

valuation of debt under a risk neutral framework with only basic assumptions about the characteristics of the underlying asset evolution and the point of default. Merton (1974) prices zero-coupon debt by treating the asset evolution as a continuous geometric Brownian motion with a specified mean α and volatility σ and assuming that default can only occur when the outstanding debt reaches maturity. Thus, the asset evolution is captured by equation (2.1) and the payout to the bondholder is the minimum of the asset value and the principal outstanding at maturity.

$$dV = (\alpha V - C)dt + \sigma V dz \tag{2.1}$$

Black and Cox (1976) extend Merton's work by creating a "first passage of time" model that allows for coupon payments, and, more importantly, for default to occur before maturity. They do so by imposing an exogenous asset threshold, subject to certain constraints, at which default spontaneously occurs. Each new iteration of the structural credit model has built upon the base form outlined by the work of Merton (1974) and Black and Cox (1976). The structural model I develop as part of my waiver/foreclosure model is a generalized version of the Black and Cox first passage of time model that allows for multiple tranches of debt and does not constrain the default boundary.

2.2 Covenant Design

Financial covenants come in one of two forms: maintenance or incurrence. Maintenance covenants stipulate that a firm must maintain certain financial metrics whereas incurrence covenants only stipulate that a firm must comply with financial metrics in order to take specified actions – such as pay dividends. The violation of either covenant type enables the respective creditor to foreclose on his claim. Because a firm can avoid violating an incurrence covenant simply by refusing to take action regardless of its financial condition, however, only maintenance covenants typically present creditors with an opportunity to

foreclose before a firm misses a payment. While private debt indentures traditionally include maintenance covenants, bond debt indentures only include incurrence covenants (Sansone and Taylor 2007). Thus, only private debt tranches – which are, almost universally, the most senior debt instrument in the capital structure – provide creditors with the ability to force default before a company misses a payment.

Dichev and Skinner (2002), in an exhaustive study of over 30,000 private lending agreements, suggest that covenants are set tightly relative to the underlying metrics they monitor. They note that, because of these strict limits, covenant violations are common – occurring in approximately 30% of the loans in their sample. Dichev and Skinner conclude that covenants are used as "trip wires" that enable lenders to involve themselves in the operations of the companies to which they lend. Hence, violations do not necessarily indicate that the violators are in financial distress. In fact, Nini, Sufi and Smith (2012) estimate that between 10-20% of publicly listed U.S. firms with financial data available through Compustat violate a covenant annually. These finding emphasizes the need for a more complex manner of endogenously determining the creditor induced default threshold, as they demonstrate the weak linkage between a covenant threshold and the point of default. Indeed, they suggest that structural models which use a tangible net worth covenant to proxy the default boundary in an attempt to estimate the asset threshold at which creditors will force default, such as Leland (1994) and Brennan and Schwartz (1978), systemically misestimate the default boundary.

2.3 Endogenous Default Boundaries

Beginning with Leland (1994), there have been a series of attempts in the literature to endogenousize the default boundary in order to improve the predictions of Merton (1974) and Black and Cox (1976). In Leland's paper, the equity holder endogenously selects the

bankruptcy threshold in a first passage of time style model by setting it equal to the asset value at which he can no longer issue new equity to fund coupon payments. Given Leland's set-up, this corresponds to the point at which the equity is worthless. Anderson and Sundaresan (1996) devise a similar default boundary in discrete time by modeling default as a game comprised of a series of subgames – with each subgame corresponding to a contractually obligated payment to the creditor. Because the equity holder specifies the payment made in each subgame, the firm defaults only when it is optimal for it to do so from the standpoint of the equity holder.

By treating the default boundary as the level at which an equity holder opts to default, neither Leland (1994) nor Anderson and Sundaresan (1996) allow for creditor-initiated bankruptcies. To fill this void, Carey and Gordy (2008) formulate a model in which creditors endogenously set the default threshold. In a first passage of time model, they allow the senior creditor in a firm with two classes of debt to initiate foreclosure. Because the senior creditor selects the threshold without regard for junior claimants, Carey et al's model implies that debt composition is a primary determinant of the default threshold. As such, they hypothesize that, if their model is representative, the firm level recovery rate of a bankrupt firm should be correlated with the share of private debt in its capital structure. Their empirical findings support this hypothesis, validating the concept that private creditors strongly affect the asset value threshold at which default occurs.

My analysis provides additional evidence that the private creditor is the key actor in determining the default threshold by directly examining the creditor's role upon a covenant violation – the point at which the creditor can actually initiate default. Hence I further clarify the need to model the default threshold as an endogenous variable while providing a new

framework in which is it endogenously selected upon a covenant violation through an interplay between the senior creditor and the equity holder.

2.4 The Effects of Covenant Violations

Traditionally, corporate creditors were thought to exert minimal control on a firm unless it defaulted in association with its failure to make a contractual payment. This view was, in large part, a product of the absence of empirical evidence documenting the influence of creditors. Because creditors exert much of their influence behind the scenes, it is difficult to correctly capture the changes they are affecting on a firm. Recently, however, this thought paradigm is changing as a result of empirical work that has demonstrated the numerous and significant firm policies shaped by creditors pre-default.

Chava and Roberts (2008) use a regression discontinuity analysis on a sample of approximately 3,000 loans to show that capital investment declines sharply following a financial covenant violation. They attribute this to creditor intervention and identify financial covenants as the channel through which creditors intervene in the management of the firm in order to protect their interests. Nini, Sufi and Smith (2012) expand upon this conclusion. By analyzing a sample of nearly 8,000 covenant violations, Nini et al find that violations are immediately followed by a decline in capital investment, a reduction in leverage, a reduction in payments made to shareholders, and an increase in CEO turnover. Though the changes in financing and operating behaviors are likely a consequence of contractual amendments to debt covenants, the increase in management turnover suggests creditors also exercise influence on firm policies outside of those in their contractual jurisdiction. Moreover, the magnitude of these behavioral changes is substantial: debt issuance comes to a halt, balance sheet debt declines by ~10%, and balance sheet assets shrink by more than 2% as firms sell assets to increase liquidity.

Because of the control rights transferred to creditors upon a covenant violation, violations are costly to the debtor. Beneish and Press (1993) estimate that the technical violation of a financial covenant costs the debtor, on average, 1.2 – 2% of its market equity value or, alternatively, 4.4% – 7.3% of the respective debt instrument's principal outstanding. To avoid these costs, firms manage accounting based financial figures to meet financial covenant stipulations. DeFond and Jiambalvo (1991) evidence that managers manipulate accounting accruals as a means to meet covenant thresholds and Sweeney (1992) demonstrates that the expected default costs imposed by creditors are an important determinant of the degree to which management implements methods to increase accounting figures.

Though the body of literature demonstrating the effects of a covenant violation on the debtor is growing, Chen and Wei's (1993) paper is the only study to date that examines a covenant violation from the perspective of the creditor. They model the waiver/foreclosure decision of the creditor using a structural credit framework with a single class of discount bond. In Chen and Wei's framework, the creditor decides between waiving and foreclosing by comparing the value of his claim in bankruptcy and the value of his claim if the firm continues as a going concern and then selecting the option which correspond to the greater of the two without consideration for waiver fees or other waiver incentives. Using a logit regression, they find that the creditor's decision to foreclose is positively correlated with total leverage and negatively correlated with the ratio of net income to total assets.

My analysis builds on the work of Chen and Wei by employing a more complex framework to better capture the waiver/foreclosure decision. First, my framework accounts for the payments and concessions made by the equity holder, in the form of interest rate step-ups and waiver fees, to induce a waiver. Hence my model more accurately depicts the

waiver/foreclosure decision by considering the effects of a given characteristic not only on the value of the creditor claim, but also on the value of the equity claim, and, thus, on the concessions the equity holder will make to secure a waiver. Second, my framework allows for multiple tranches of debt and treats the waiver/foreclosure decision as the sole dominion of the private creditor. This treatment is more realistic given the type of covenants included in each debt's tranche – as discussed in Section 2.2 – and the empirical conclusions of Carey and Gordy (2008) – discussed in Section 2.3. This framework enables me to outline a more complete set of determinants driving the waiver/foreclosure decision and suggests that the correlation that Chen and Wei find between total leverage and foreclosure is, in fact, the result of a confounding correlation rather than purely causal, as they conclude.

III. The Model

I model the optimal creditor and equity holder decisions upon a financial covenant violation. A financial covenant violation gives the relevant lenders the option to demand immediate repayment, or foreclose, on the relevant debt principal outstanding. Hence, upon a violation, the creditors decide whether they will exercise this right or whether they will waive the violation. The equity holder, meanwhile, decides whether to incentivize a wavier, and the degree to which he will incentivize a waiver, by choosing whether to offer the creditors a payment (a "waiver fee"), and by choosing the magnitude of this payment, in return for a waiver. Throughout the model, I refer to the debt tranche protected by covenants as the senior bank loan and the creditor holding this tranche the bank. This terminology is used solely for simplicity, however, and the implications of the model apply to any covenant protected debt regardless of the instrument, the number of tranches or the creditor in question. The aim of the model is to shed light on the determinants of the waiver/foreclosure decision and the magnitude of the waiver fee paid, with particular focus on the effect of debt composition.

The bank's decision is a function of the value of the loan upon a waiver, the value of any incentives paid to the bank in return for a waiver and the value received upon foreclosure. The equity holder's decision is a function of the value of the equity upon foreclosure and the value of the equity claim after a waiver fee is paid upon a waiver. To capture these decision processes, I first independently assess the value of each of these components and then utilize them to create a model of the optimal bank and equity holder decisions. Section 3.1 sets up the structure and overarching assumptions used throughout the succeeding subsections, Section 3.2 develops a model to assess the value of the claims upon foreclosure, Section 3.3 develops a first passage of time model in the spirit of Black

and Cox (1976) to assess the continuing value of the claims upon a waiver and Section 3.4 utilizes these prior subsections to construct a model of the optimal decisions processes upon a covenant violation.

3.1 Model Firm Structure

I begin by imitating the set up used in Carey and Gordy (2008). I assume the firm is capitalized by three claims, listed in order of seniority: a senior bank loan, a subordinated note and common equity. Without compromising generality, I assume that the senior bank loan face value is ℓ and the subordinated note face value is $1 - \ell$ – and, thus, the total debt principal outstanding is equal to 1. I assume that the bank loan and subordinated note are perpetual and receive continuous coupons of c and μ, respectively. Moreover, I assume that the equity holder receives a continuous dividend of $\delta + \rho V_t$ (where V_t is the asset value of the firm at time t). This formulaic dividend can be interpreted as a covenant that protects the creditors from asset appropriation by the equity holder. For simplicity, I assume there is no interest tax shield. To emphasize the term structure of credit risk, I assume that the risk-free rate is fixed at a constant r and that the relationship between the coupon rates, the dividend and the risk-free rate is governed by the inequality $0 \leq p < r \leq c$. Lastly, I assume that the value of the firm's assets, V_t, is exogenously determined and evolves continuously in accord with a geometric Brownian motion with variance σ^2 and mean α. Thus, the value of the assets can be expressed by the differential equation:

$$dV = [V(\alpha - \rho) - c\ell - \mu(1 - \ell) - \delta]dt + \sigma V dz \qquad (3.1)$$

where dz is the standard Gauss-Wiener process.

3.2 Value of the Claims upon Foreclosure

Given the firm structure outlined in Section 2.1, I formulate a basic model for the value of the claims upon a foreclosure. Upon a foreclosure, I assume the asset value is spontaneously altered by a shock of magnitude $(1 - \Psi)$. This can be interpreted as the cost of bankruptcy when foreclosure forces the firm into bankruptcy – but can also represent the indirect costs of distress incurred when the firm is able to avoid bankruptcy through the sale of assets or transfer of collateral. Assume absolute priority holds and that bankruptcy resolution is immediate.

The value of the bank loan upon foreclosure is then:

$$B_{\ell}(V) = \min\,(\ell, \Psi \mathrm{V}) \tag{3.2}$$

Similarly, the value of the subordinated debt and the value of the equity, respectively, are:

$$B_S(V) = \min\,(1 - \ell, \max(0, \Psi \mathrm{V} - \ell)) \tag{3.3}$$

$$B_E(V) = \max\,(0, \Psi \mathrm{V} - 1) \tag{3.4}$$

3.3 Value of the Continuing Loan, Note and Equity Claim

Utilizing the methods first presented by Merton (1974) and further developed by Black and Cox (1976) and the assumptions about the firm structure outlined in Section 3.1, I next form a baseline model for the value of the continuing claims. The model is a generalized version of Black and Cox (1976), in which I assume an arbitrary default boundary, $\bar{V}$, at which the firm enters bankruptcy. The model reduces to that presented in Black and Cox (1976) when there is no subordinated debt principal outstanding and this boundary is set to 0.

Assume the market value of the loan, L, can be expressed as a function of the firm value, $L = F(V, t)$, at any point in time. Given the assumptions presented, the dynamics of the loan's value can formally be expressed as:

$$dL = (\alpha_L L - c\ell)dt + \sigma_L L dz_L \tag{3.5}$$

where σ_L is the volatility of the loan, α_L is the expected return on the loan and dz_L is a standard Gauss-Wiener process.

By Ito's lemma, equation (3.5) can be re-written as:

$$dL = (\tfrac{1}{2}\sigma^2 V^2 F_{vv} + F_v(V(\alpha - p) - C) + F_t)dt + \sigma V F_v dz_L \tag{3.6}$$

where I let $C = c\ell - \mu(1-\ell) - \delta$ for convenience.

By construct, $F_t = 0$ (due to the assumption of perpetual debt). This enables us to express the loan value as a function, $F(V)$, independent of time. Additionally, by definition, $dz_L = dz$.

By comparing equations (3.5) and (3.6) and integrating these observations, I write:

$$\alpha_L = (\tfrac{1}{2}\sigma^2 V^2 F'' + F'(V(\alpha - p) - C) + c\ell)/F \tag{3.7a}$$

$$\sigma_L = \sigma V F'/F \tag{3.7b}$$

To find the ordinary differential equation that captures the evolution of the loan value, I imitate the methodology presented by Merton (1974, eq. 4) and construct a self-financing portfolio comprised the firm assets, the loan and a risk-free bond. Let P represent the investment in the firm assets, P_L the investment in the loan, P_r the investment in the riskless debt and *dp* the portfolio's instantaneous return. The portfolio's instantaneous return can then be expressed as a sum of its components instantaneous returns weighted by their share:

$$dp = P(\alpha dt + \sigma dz) + P_L(\alpha_L dt + \sigma_L dz_L) + P_r r dt \tag{3.8a}$$

$$dp = [P(\alpha - r) + P_L(\alpha_L - r)]dt + [P\sigma + P_L\sigma_L]dz \tag{3.8b}$$

where (3.8b) follows because the portfolio is self financing (i.e. $P + P_L \equiv -P_r$) and because $dz_L \equiv dz$ by definition.

By selecting a P and P_L such that $[P\sigma + P_L\sigma_L] = 0$, I create a deterministic portfolio. Under no arbitrage assumptions, this portfolio must have no return because it has no risk and requires no capital investment. Hence, $[P(\alpha - r) + P_L(\alpha_L - r)] = 0$. Combining these expressions in a system of equations, I find:

$$\frac{\alpha - r}{\sigma} = \frac{\alpha_L - r}{\sigma_L} \tag{3.9}$$

Substituting (3.7a) and (3.7b), I solve for the desired ordinary differential equation:

$$\frac{\alpha - r}{\sigma} = \frac{\frac{\left[\frac{1}{2}\sigma^2 V^2 F'' + F'(V(\alpha - p) - C) + c\ell)\right]}{F} - r}{\frac{\sigma V F'}{F}}$$

$$\frac{1}{2}\sigma^2 V^2 F'' + F'[V(r - p) - C] + c\ell - Fr = 0 \tag{3.10}$$

To emphasize, ordinary differential equation in equation (3.10) describes the dynamics of the loan value regardless of its expected returns, volatility or the preferences of investors.

The general solution to this equation is:

$$F(V) = Q_1 * \left(\frac{2C}{V\sigma^2}\right)^{-\Omega} \times {}_1F_1\left(-\beta + \frac{r - p}{\sigma^2} - \frac{1}{2}; 1 - 2\beta; \frac{-2C}{V\sigma^2}\right) + Q_2 * \left(\frac{2C}{V\sigma^2}\right)^{\Omega} \times {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{V\sigma^2}\right) + Q_3 \tag{3.11}$$

where $\Omega = \frac{\sqrt{8r\sigma^2 + (\sigma^2 + 2p - 2r)^2} - 2p + 2r - \sigma^2}{2\sigma^2}$, $\beta = \frac{\sqrt{8r\sigma^2 + (\sigma^2 + 2p - 2r)^2}}{2\sigma^2}$, ${}_1F_1$ is the Kummer confluent hypergeometric function and Q_1, Q_2 and Q_3 are arbitrary constants defined by the boundary conditions.

Before examining the boundary conditions, I consider Ω. By expanding the expression, I write:

$$\Omega - \frac{r-p}{\sigma^2} = \frac{\sqrt{\sigma^4+4p^2+4r^2-8pr+4r\sigma^2}-\sigma^2}{2\sigma^2} \tag{3.12}$$

Note that $4p^2 + 4r^2 \geq 8pr$. Hence $\sqrt{\sigma^4 + 4p^2 + 4r^2 - 8pr + 4r\sigma^2} - \sigma^2 \geq 0$. This allows me to conclude $\Omega \geq \frac{r-p}{\sigma^2} > 0$ given the previously stated assumption that $r > p$.

Next, I consider the boundary conditions. The upper and lower boundary, respectively, are:

$$F(\infty) = \frac{c\ell}{r} \tag{3.13a}$$

$$F(\bar{V}) = B_\ell(\bar{V}) = \min(\ell, \Psi\bar{V}). \tag{3.13b}$$

The upper boundary of the loan is the value of a riskless perpetuity of face value ℓ paying a continuous coupon c because, as the asset value approaches infinity, the probability of default approaches zero. To clarify the value of the constants in the general equation, I take the limit of each term in the general equation as the asset value approaches infinity. First, I examine at the limit of the term by which Q_1 is multiplied:

$$\lim_{V\to\infty}[(\frac{2C}{V\sigma^2})^{-\Omega} \times {}_1F_1\left(-\beta + \frac{r-p}{\sigma^2} - \frac{1}{2}; 1-2\beta; \frac{-2C}{V\sigma^2}\right)] =$$

$$\lim_{V\to\infty}[(\frac{V\sigma^2}{2C})^{\Omega} \times \sum_{n=0}^{\infty} \frac{\left(-\beta + \frac{r-p}{\sigma^2} - \frac{1}{2}\right)_n \left(\frac{-2C}{V\sigma^2}\right)^n}{(1-2\beta)_n n!}] = \infty \tag{3.14}$$

where $(\cdot)_n$ are Pochhammer symbols.

As $V \to \infty$, the term approaches infinity because Ω is conclusively positive, as I demonstrated above, and because the confluent hypergeometric equation approaches 1. Because the upper limit of the particular solution is $\frac{c\ell}{r}$, this term cannot increase without bound in the particular solution, however. Hence, I conclude $Q_1 = 0$.

Next, I examine the limit of the term by which Q_2 is multiplied:

$$\lim_{V\to\infty}[(\frac{2C}{V\sigma^2})^{\Omega}\times {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{V\sigma^2}\right)] =$$

$$\lim_{V\to\infty}[\frac{2C}{V\sigma^2})^{\Omega}\times\sum_{n=0}^{\infty}\frac{(\Omega)_n\left(\frac{-2C}{V\sigma^2}\right)^n}{(1+2\beta)_n n!}] = 0 \tag{3.15}$$

This limit approaches zero for the same reasons, concerning the convergence of the hypergeometric function and the sign of Ω, as the limit in equation (3.14). Though this does not provide evidence about the value of Q_2, by considering the value of Q_1, the value of the limit in equation (3.15) and the upper bound, I can conclude that $Q_3 = \frac{c\ell}{r}$.

The lower boundary of the loan follows from the assumption that the firm spontaneously defaults upon hitting an arbitrary boundary $\bar{V}$ and that the senior lender receives the minimum of the bank loan face value and the asset value upon default. By plugging the values for Q_1 and Q_3 into the general equation when $V = \bar{V}$, I can write:

$$Q_2 * (\frac{2C}{\bar{V}\sigma^2})^{\Omega}\times {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)+\frac{c\ell}{r} = \min(\ell,\Psi\bar{V})$$

and by re-arranging, I solve for Q_2:

$$Q_2 = \left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)\frac{(\frac{\bar{V}\sigma^2}{2C})^{\Omega}}{{}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)} \tag{3.16}$$

Finally, replacing Q_1, Q_2 and Q_3 in the general solution with the constants values I determined from the boundary solutions, I write the desired particular solution for the loan value as a function of the asset value:

$$F(V) = \left(\min(\ell, \Psi\bar{V}) - \frac{c\ell}{r}\right) \frac{(\frac{2C}{V\sigma^2})^{\Omega}\ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega}\ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} + \frac{c\ell}{r} \tag{3.17}$$

Using the same techniques, it is easily shown that the general equation for the value of the subordinated note as a function of the underlying assets, $S = G(V)$, is:

$$\begin{gathered} G(V) = \zeta_1 * (\frac{2C}{V\sigma^2})^{-\Omega} \times {}_1F_1\left(-\beta + \frac{r-p}{\sigma^2} - \frac{1}{2}; 1-2\beta; \frac{-2C}{V\sigma^2}\right) + \\ \zeta_2 * (\frac{2C}{V\sigma^2})^{\Omega} \times {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V\sigma^2}\right) + \zeta_3 \end{gathered} \tag{3.18}$$

where ζ_1, ζ_2 and ζ_3, like Q_1, Q_2 and Q_3, are constants defined by the boundary conditions. The upper and lower boundary, respectively, are:

$$G(\infty) = \frac{\mu(1-\ell)}{r} \tag{3.19a}$$

$$G(\bar{V}) = \min(1-\ell, \max(0, \Psi\bar{V} - \ell)). \tag{3.19b}$$

By replicating the logic used to examine the upper bound of the loan function, I find ζ_1=0 and $\zeta_3 = \frac{\mu(1-\ell)}{r}$. Similarly, by plugging these values back into the general equation, equation (3.18) , at the lower bound and solving for ζ_2, I find:

$$\zeta_2 = \left(\min(1-\ell, \max(0, \Psi\bar{V} - \ell)) - \frac{\mu(1-\ell)}{r}\right) \times \frac{(\frac{\bar{V}\sigma^2}{2C})^{\Omega}}{{}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{3.20}$$

Thus, the particular solution for the subordinated note value as a function of the firm assets is described by:

$$G(V) = \left(\min(1-\ell, \max(0, \Psi\bar{V} - \ell)) - \frac{\mu(1-\ell)}{r}\right) \times$$
$$\frac{(\frac{2C}{V\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} + \frac{\mu(1-\ell)}{r} \tag{3.21}$$

The equity value cannot be directly computed with the same methodology because it has no upper bound. Because there are three claims on the assets – namely the senior bank loan, the subordinated note and the equity – the equity value, E, is equal to the residual value of the assets after subtracting the loan and note values and accounting for default costs, however.

To calculate the ex-ante default costs, consider a security, $\mathbb{C}(V)$, that pays no coupon but does pays a fixed amount $(1-\Psi)\bar{V}$, equal the cost of default, upon default. This value of this fictitious security is then equal the ex-ante cost of default. Moreover, because this security is a function of the underlying asset value, its value dynamics are captured by a general solution of the same form as the general solutions capturing the value dynamics of the loan and the bond – namely, equations (3.11) and (3.18). The upper and lower bounds, respectively, of this general solution are:

$$\mathbb{C}(\infty) = 0 \tag{3.22a}$$

$$\mathbb{C}(\bar{V}) = (1-\Psi)\bar{V} \tag{3.22b}$$

Let $\wp_1$, $\wp_2$ and $\wp_3$ denote the constants analogous to Q_1, Q_2 and Q_3 in equation (3.11). From the upper bound, I conclude $\wp_1 = 0$ and $\wp_3 = 0$. From the lower bound, I conclude:

$$\wp_2 = \frac{(1-\Psi)\bar{V}}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{3.23}$$

Substituting these values into the general equation, I find the particular solution for the security:

$$\mathbb{C}(V) = (1-\Psi)\bar{V}\frac{(\frac{2C}{V\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{3.24}$$

Finally, I calculate the value of the equity by treating it as the asset residual:

$$E(V) = V - G(V) - F(V) - \mathbb{C}(V) \tag{3.25}$$

3.4 Optimal Bank Choice

In this section, I use the value of the claims as formulated in Sections 3.2 and 3.3 to create a model of the optimal bank and equity holder decisions upon a covenant violation. I assume that the senior bank loan, and only the senior bank loan, has a covenant that, when violated, enables the bank lender to foreclose on its loan. For simplicity, I assume that if the bank waives this right to foreclose, a new, pre-specified default boundary, $\bar{V}$, that is strictly less than the outstanding value of the assets, V, is enacted. This new default boundary can be interpreted as the new threshold established by the bank covenant after the bank waiver has relaxed the covenant requirements. Moreover, I assume that the firm cannot access the capital market upon a violation; hence its capital structure at the time of violation is both exogenous and fixed. This assumption can be interpreted as a reflection of the prohibitively high costs of refinancing while in financial distress. Because the firm's capital structure is an endogenous choice of the firm's equity holder at an earlier date, however, the effects of capital structure on the optimal decision may justifiably be interpreted as a reflection of the firm characteristics at the time at which the capital structure was imposed. Lastly, I assume that both the equity holder and bank lender are rational actors whose sole aim is to maximize the value of their respective claims.

Upon a covenant violation, the equity holder may pay the bank a one-time payment (henceforth referred to as a "waiver fee") from the company's balance sheet as compensation for waiving its right to foreclose. Thus, the value of the bank claim upon a covenant violation is a function of a vector of all available information, including the waiver fee. I denote this information vector as $\mathbb{I}_B$ and denote the value of the bank claim upon a violation as $\mathrm{M}(\mathbb{I}_B)$. Because the value of the bank claim upon a covenant violation is equal to either the value of the bank claim upon foreclosure or the value of the bank claim upon a waiver (henceforth referred to as the "continuing bank claim") plus the value of the waiver fee, I can write $\mathrm{M}(\mathbb{I}_B)$ as a piecewise function of these two expressions. For an exogenously specified value of the waiver fee, the bank's claim is the maximum of these two expressions because the bank selects the waiver/foreclosure option in order to maximize the value of its claim:

$$\mathrm{M}\big((\mathbb{I}_B|W)\big) = \max\,(B_\ell(V), F(V_\omega - W) + W) \tag{3.26}$$

where W is the discrete, one-time waiver fee, ω is an increase in the continuously compounded loan coupon rate and V_ω is the value of the assets given a loan coupon rate of $c + \omega$.

Similarly, the value of the equity claim upon a covenant violation can be written as a piecewise function of the value of the equity claim upon a waiver (henceforth referred to as the "continuing equity claim") and the value of the equity claim upon foreclosure. I denote the vector of all information available to the equity holder as $\mathbb{I}_E$ and the value of the equity claim as $\Pi(\mathbb{I}_E)$. Then, for a specified waiver fee, I can write:

$$\Pi(\mathbb{I}_E) = B_E(V) \times \mathbb{i} + E(V_\omega - W) \times (1 - \mathbb{i}) \tag{3.27}$$

where $\mathbb{i}$ is an indicator variable equal to 1 if $B_\ell(V) > (F(V_\omega - W) + W)$ and 0 otherwise.

The equity holder specifies the value of the waiver fee offered to the bank in order to maximize the value of their claim. Because the value of the equity holder's continuing claim is decreasing in the waiver fee, the equity holder will only specify a non-zero waiver fee if the benefit of receiving a waiver is greater than the cost of paying the fee. Moreover, he will specify the smallest waiver fee necessary to induce the bank to waive. Together, the equity and bank claimant decisions form a sequential game, in which the equity holder first specifies the waiver fee and the bank then decides whether to foreclose given the magnitude of this fee. In order to render this game mathematically tractable, I assume there is complete and symmetric information between the bank lender and equity holder. Hence, I model the optimal decision as a result of a sequential game with symmetric information.

I rewrite equation (3.26), the value of the bank's claim, with the indicator variable, $\mathbb{i}$:

$$M(F(V), \omega, W) = B(V) \times \mathbb{i} + [F(V_\omega - W) + W)] \times (1 - \mathbb{i}) \qquad (3.28)$$

The equity holder's decision is to specify W given the firm's characteristics, the bank's value function and their own value function. The bank's decision is to specify $\mathbb{i}$ given the firm's characteristics, the value of W and its own value function. Mathematically:

$$\max_W [\,\mathrm{E}(V), M(V), \omega)] \qquad (3.29a)$$

$$\max_X [\,M(V), \omega, W)] \qquad (3.29b)$$

The covenant will therefore be waived if there exists a ϕ that satisfies:

$$E(V_\omega - \phi) > B_E(V) \; and \;\; F(V_\omega - \phi) + \phi > B_\ell(V) \qquad (3.30)$$

and the waiver fee paid to the bank will be the minimum non-negative ϕ that satisfies both these inequalities.

IV. Comparative Statistics and Testable Hypotheses

The model developed in Section 3 estimates the optimal waiver fee, the optimal bank decision and the value of the claims upon a covenant violation for given parameter values. My aim in creating this model, however, was to shed light on the determinants of these optimal decisions. In particular, to shed light on the loan, asset and capital structure characteristics which influence the equity holder and bank decisions. To do so, I view the bank decision through the lens of the minimum waiver fee necessary to induce the bank to waive the covenant and the equity holder's decision through the lens of the maximum waiver fee at which it is still optimal for the equity holder to receive the waiver. I form stylized hypotheses by examining the waiver fees at which the bank and the equity holder are indifferent between waiving the covenant and calling the loan. Section 4.1 examines the effect of the waiver fee and the new default threshold – the variables which must be negotiated upon a covenant violation – on the values of the claims, Section 4.2 formally develops the concept of a breakeven waiver fee, which will be used to quantify the effect of varying the input parameters on the optimal waiver/foreclosure decision and waiver fee, Section 4.3 examines the effect of the asset characteristics on the optimal decision and wavier fee, Section 4.4 examines the effect of the capital structure on the optimal decision and waiver fee and Section 4.5 summarizes these finding and outlines a set of testable hypotheses.

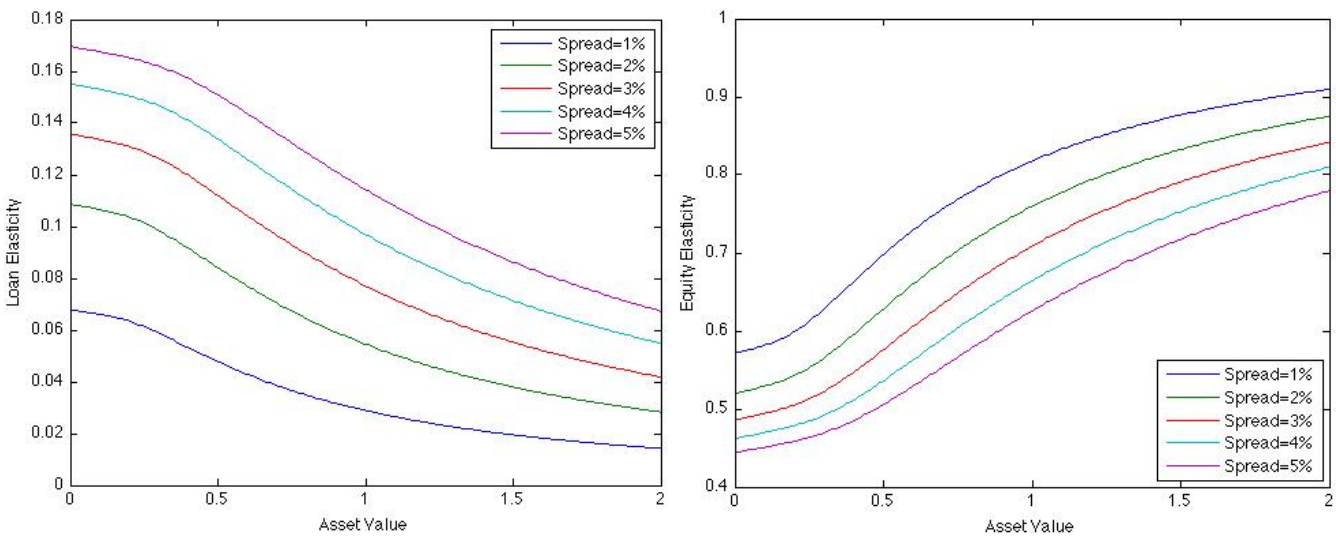
4.1 Comparative Statistics: Waiver Fee and Default Threshold

Basic intuition suggests that the value of the loan claim upon a waiver ("the continuing loan claim") plus the waiver fee is increasing in the value of the waiver fee and that the value of the equity claim upon a waiver ("the continuing equity claim") is decreasing.

Because the waiver fee effectively reduces the value of the assets, examining the elasticity of the continuing claims, or their derivatives with respect to the underlying asset value, illustrates the effect of the waiver fee on the claim values. Due to the analytical complexity of these derivatives, I examine their values graphically for parameter values fixed at relevant levels using numerical methods, as presented in Figures 4.1 and 4.2.

The elasticity of the continuing loan with respect to the asset value, though positive, is less than one globally. Thus, the value of the continuing loan claim plus the waiver fee must be increasing in the waiver fee. To illustrate this, consider a one-unit increase in the

Figures 4.1 and 4.2: Elasticity of the Bank Loan (Left) and Equity Claim (Right) at Varying Spreads



Spread $= c - r$. Parameters: $r = .02,\ \mu =. c + .01,\ \rho = .01,\ d = .01,\ \ell = .5, w = 0, \bar{V} = .3\ and\ \sigma = .3$

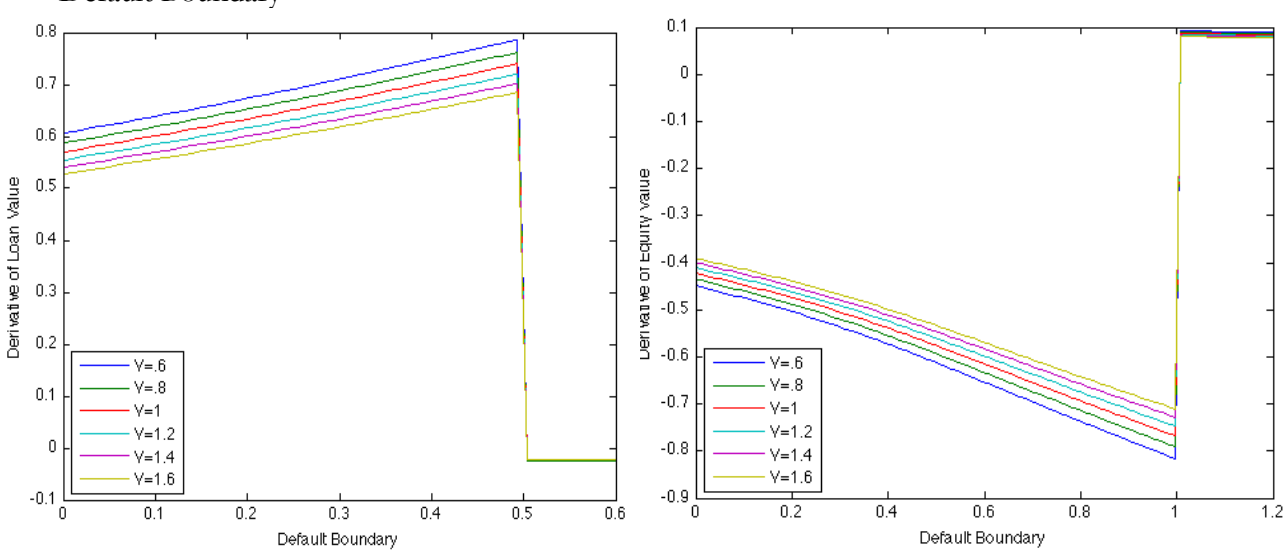
waiver fee. The value of the waiver will increase by one unit, obviously, but the value of the continuing claim will decline by less than one unit – hence the value of the claim upon a waiver, net, will increase. The value accretion to the bank is greatest when the asset value is substantially greater than the loan principal outstanding because the reduction of the asset value due to the payment of the waiver fee only marginally increases the probability of default. Notably, the value of the continuing loan claim plus the waiver fee is increasing at a

substantial rate even when the asset value is close to the loan principal outstanding, however. This implies that receiving a waiver fee can be of substantial value to the bank regardless of the distance to default.

Similarly, the elasticity of the equity claim is positive globally, and hence the equity value is globally decreasing in the waiver fee. The elasticity of the equity declines as the asset value approaches the default threshold because the debt claims absorb an increasing proportion of the change in asset value. Thus, a unit of waiver fee paid to the bank becomes less costly to the equity holder as the asset value approaches the default threshold.

The new default threshold set in conjunction with a waiver also has a significant effect on the value of the claims. The effect of a small revision to the default boundary is highly dependent on the absolute level of the default boundary. As illustrated in Figures 4.3 and 4.4, the first derivatives of the claims' values with respect to the default boundary are

Figures 4.3 and 4.4: Derivative of Loan Value (Right) and Equity Value (Left) with Respect to the Default Boundary



Parameters: $r = .02,\ c = .025\ \mu = .035,\ \rho = .01,\ d = .05,\ \ell = .5, W = 0, \Psi = 1\ and\ \sigma = .3$

piecewise functions. Below the loan principal outstanding, the value of the loan claim is rapidly increasing in the default boundary because each unit increase in the default boundary

directly corresponds to a unit increase in the recovery of the loan. For values above the loan principal outstanding plus the cost of bankruptcy, the value of the loan claim is gradually decreasing in the default boundary. To make economic sense of this, consider the risk of the loan when the default boundary is greater than the loan principal plus the default cost. The value recovered by the bank will always be equal to the loan principal; hence the loan has no risk. I assume that the loan coupon rate is greater than the risk-free rate, however. The loan therefore earns a rate that is greater than the risk free rate for a riskless product until default – hence its value is increasing in time to default. Because a higher default boundary shortens the time till default for a given asset value, it decreases the value of the riskless loan. Moving forward, the analysis shall only focus on instances in which the new default boundary is less than the loan principal outstanding plus the cost of default because the bank will always optimally waive its covenant when the default boundary is greater than the loan principal outstanding and the coupon rate is greater than the risk-free rate.

The equity claim value, on the other hand, is decreasing in the default boundary for all values of the default threshold less than the total debt principal outstanding. The economic sense mirrors that discussed for the loan claim: a higher default boundary shortens the time till default, and, since the equity holder receives no recovery upon default when the debt principal outstanding is greater than the default threshold, the value of the equity claim is unambiguously increasing in the time to default.

4.2 The Breakeven Wavier Fee

To examine the optimal bank decision upon a covenant violation, I develop the concept of a breakeven waiver fee. As outlined in Section 4.1, the value of the debt claim is increasing in the waiver fee and the value of the equity claim is decreasing. Thus, the bank wants to receive the maximal waiver fee possible and the equity holder want the company to

pay the minimal wavier fee possible to induce the bank to waive. The breakeven bank waiver fee is defined as the waiver fee at which the bank is indifferent between waiving and foreclosing and the breakeven equity wavier fee is defined as the waiver fee at which the equity holder is indifferent between allowing the company to pay the fee to receive a waiver and allowing the bank to foreclose on the loan. Hence, the breakeven bank fee is the minimal fee the bank will accept to waive the covenant and the breakeven equity fee is the maximal fee the equity holder will pay to receive a waiver. We denote the breakeven bank waiver fee as a function $\mathbb{W}_B(\cdot)$ and the breakeven equity waiver fee as a function $\mathbb{W}_E(\cdot)$ of the input parameters. Mathematically, these breakeven fees are the waiver fees that set the value of the continuing claims equal to the value of the claims upon foreclosure:

$$E([V_\omega - \mathbb{W}_E(V, \mathbb{Q})], \mathbb{Q}) = B_E(V, \Psi) \tag{4.1}$$

$$F([V_\omega - \mathbb{W}_B(V, \mathbb{Q})], \mathbb{Q}) + \mathbb{W}_B(V, \mathbb{Q}) = B_\ell(V, \Psi) \tag{4.2}$$

where $\mathbb{Q} = [\bar{V}, \Psi, C, \rho, \sigma, \ell]$ is a specific subset of all parameters.

If the breakeven equity waiver fee is greater than the breakeven bank waiver fee for a given set of parameters, the bank will optimally waive the covenant

The concept of a breakeven wavier fee allows me to, first, articulate the optimal waiver/foreclosure decision for given inputs in a simple manner and, second, determine the directional impact of varying a parameter on the likelihood of foreclosure. This second use is particularly helpful. Plotting the breakeven waiver fee as a function of a parameter illustrates whether the waiver fee necessary to induce a waiver is increasing in the parameter and, hence, whether increasing a particular parameter should, all else equal, decrease (increase) the value of the bank (equity) claim upon a waiver relative to the claim's foreclosure value. A positive correlation between a parameter and the bank breakeven fee implies that the relative value of the continuing loan claim is decreasing in the parameter and the slope of this

correlation represents the magnitude of this effect. Likewise, a positive correlation between a parameter and the equity breakeven waiver fee implies that the relative value of the continuing equity claim is increasing in the parameter.

. I speak of the breakeven waiver fee that the equity holder is willing to pay throughout the remainder of this section. I do so for brevity – the equity holder does not directly pay the fee, but, instead the equity holder specifies the fee and the company pays it from its balance sheet. My terminology should be understood as implying such.

4.3 Comparative Statistics: Asset Characteristics

The two defining asset-characteristic parameters in the model are asset volatility and the cost of default. Asset volatility demonstrates a positive correlation with the breakeven equity and bank fees, as illustrated in Figures 4.5 and 4.6, and therefore the waiver fee paid upon a waiver is increasing in volatility. By examining volatility as it approaches infinity, I help to characterize its effect on the probability of a waiver.

Proposition 1

$\lim_{\sigma\to\infty} \mathbb{W}_E(\sigma) = \mathrm{V} - \min(1-\ell, \max(0, \Psi\bar{V} - \ell)) - \min(\ell, \Psi\bar{V}) - (1-\Psi)\bar{V} - \max(\Psi\mathrm{V} - 1, 0)$

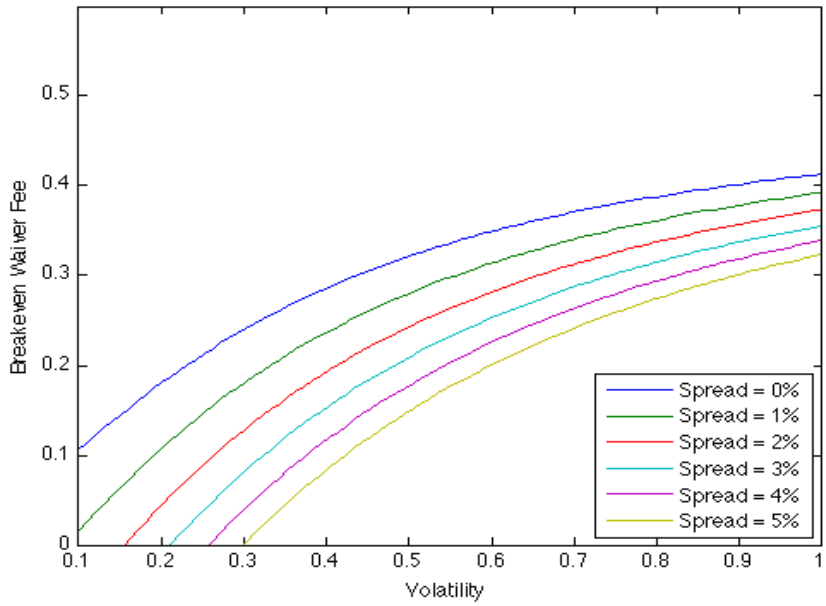
Proposition 2 $\lim_{\sigma\to\infty} \mathbb{W}_B(\sigma) = \min(\ell, \Psi V) - \min(\ell, \Psi\bar{V})$.

When $\Psi V > 1 > \ell > \Psi\bar{V}$, these terms simplify to: $\lim_{\sigma\to\infty} \mathbb{W}_E(\sigma) = 1 - \Psi\bar{V}$ and $\lim_{\sigma\to\infty} \mathbb{W}_B(\sigma) = \ell - \Psi\bar{V}$, respectively. Proofs for these propositions are outlined in the Appendix.

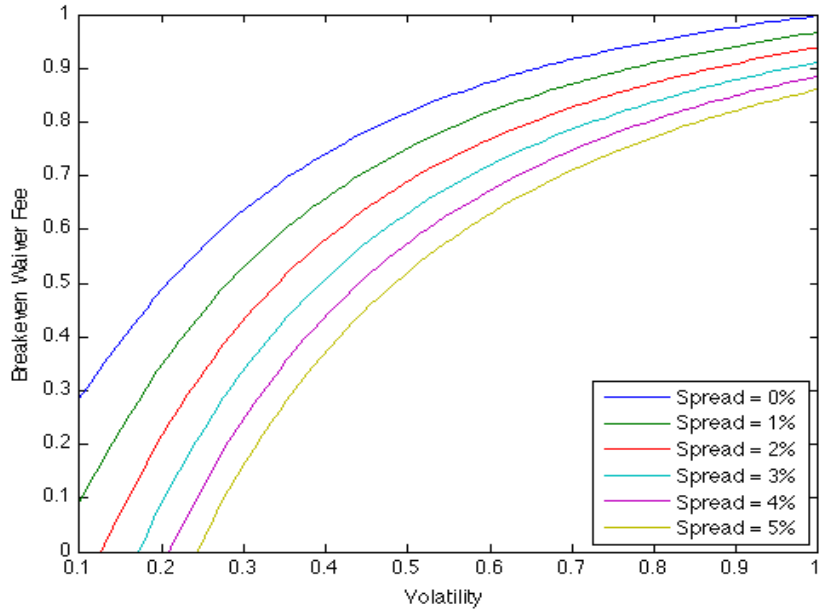
These limits demonstrate that increasing the volatility sufficiently will ensure that a waiver is optimal if the asset value is greater than debt principal outstanding and the company has subordinated debt outstanding. This is counterintuitive, as increasing asset volatility reduces a debt claim's value. As illustrated Figures 4.5 and 4.6, the slope of the

breakeven equity waiver fee is greater than the slope of the breakeven bank waiver in volatility for large values of the volatility, however. Thus, when subordinated debt exists and volatility grows very large, the equity claimant extracts enough value from high volatility that

Figures 4.5 and 4.6: The Effect of Volatility on the Breakeven Bank Waiver Fee (above) and Equity Waiver Fee (below) at Varying Spreads



Spread $\equiv c - r$. Parameters: $r = .03,\ \mu =. c + .01,\ \rho = .005,\ d = 0,\ \ell = .5,, \bar{V} = 0\ and\ \Psi = .9$



Spread $\equiv c - r$. Parameters: $r = .03,\ \mu =. c + .01,\ \rho = .005,\ d = 0,\ \ell = .5,, \bar{V} = 0\ and\ \Psi = .9$

he is willing to more than sufficiently compensate the bank for the reduced value of the continuing loan claim. At more realistic values of the volatility, a change in volatility should not affect probability of a waiver, however, because both the equity and breakeven waiver fees increase in lockstep.

Default cost has a more complex relationship with the breakeven waiver fees than does volatility. Proposition 3 demonstrates that, as the cost of default approaches the value of the assets (i.e. $\Psi \to 0$), the value of the breakeven bank waiver fee approaches a negative boundary regardless of the other parameter values. Though not economically feasible – a bank owns the option to waive, it never has to pay for the right and hence the waiver fee can be zero at the least – a mathematically negative breakeven waiver fee implies that a waiver is strongly optimal regardless of the equity breakeven waiver fee. Though the result appears to be an oddity at first, the value of the bank claim upon foreclosure is zero when $\Psi = 0$ and,

Proposition 3[2] $\mathbb{W}_B(\Psi = 0) \leq -\left(\frac{c\ell}{r}\right)\left(1 - \frac{(\frac{2C}{V\sigma^2})^{\Omega_*}\ {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{V\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega_*}\ {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)}\right) \leq 0$

hence, waiving the covenant is optimal whenever the continuing claim has any residual value. This conclusion suggests that, paradoxically, companies with abnormally high default costs will receive waivers and pay minimal waiver fees. The proofs for Propositions 3 and 4 are given in the Appendix.

Proposition 4[2] $\lim_{\Psi \to \frac{1}{V}} \mathbb{W}_E(\Psi) = V - \bar{V}$

The breakeven equity waiver fee as a function of the cost of default approaches a fixed value as the cost of bankruptcy increases, as formalized in Proposition 4. This fixed value, $\frac{V-1}{V}$, is the level of default cost that renders the equity claim worthless upon a foreclosure.

[2] The cost of default is (1- Ψ). Hence these propositions refer to the limits of the recovery rate as a proportion of the total assets.

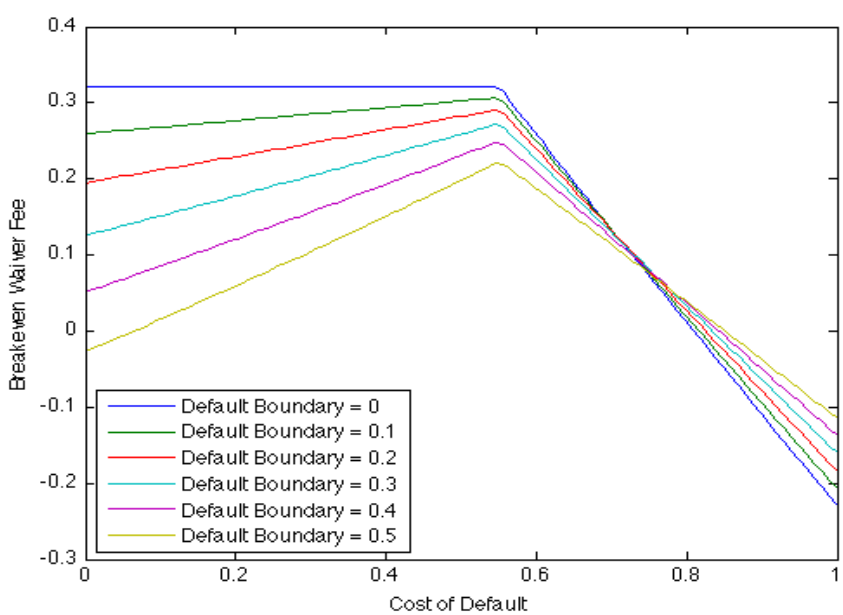
Because the equity claim is worthless upon a foreclosure when default costs are sufficient to wipe out any recovery for the equity holder, the equity holder logically is willing to pay out the maximal amount possible without triggering an automatic default, i.e. $V - \bar{V}$, to induce the bank to waive the covenant.

Proposition 5[3] $\mathbb{W}_B{}'(\Psi) > 0$ if $\Psi < \frac{\ell}{V}$, $\mathbb{W}_B{}'(\Psi) < 0$ if $\frac{\ell}{\bar{V}} > \Psi > \frac{\ell}{V}$ and $\mathbb{W}_B{}'(\Psi) = 0$ if $\Psi > \frac{\ell}{\bar{V}}$

Proposition 6[3] $\mathbb{W}_E{}'(\Psi) < 0$ if $\Psi > \frac{1}{V}$ and $\mathbb{W}_E{}'(\Psi) = 0$ if $\Psi \leq \frac{1}{V}$

As illustrated in Figures 4.7 and 4.8, the breakeven bank fee is a piecewise function of the default cost. Proposition 5 formalizes the threshold at which the breakeven fee is increasing as opposed to decreasing in the default cost. When the default cost does not impair the value of the loan claim upon foreclosure but does impair the value of the loan

Figures 4.7: The Effect of Default Costs on the Breakeven Bank Waiver Fee at Varying Default Boundaries

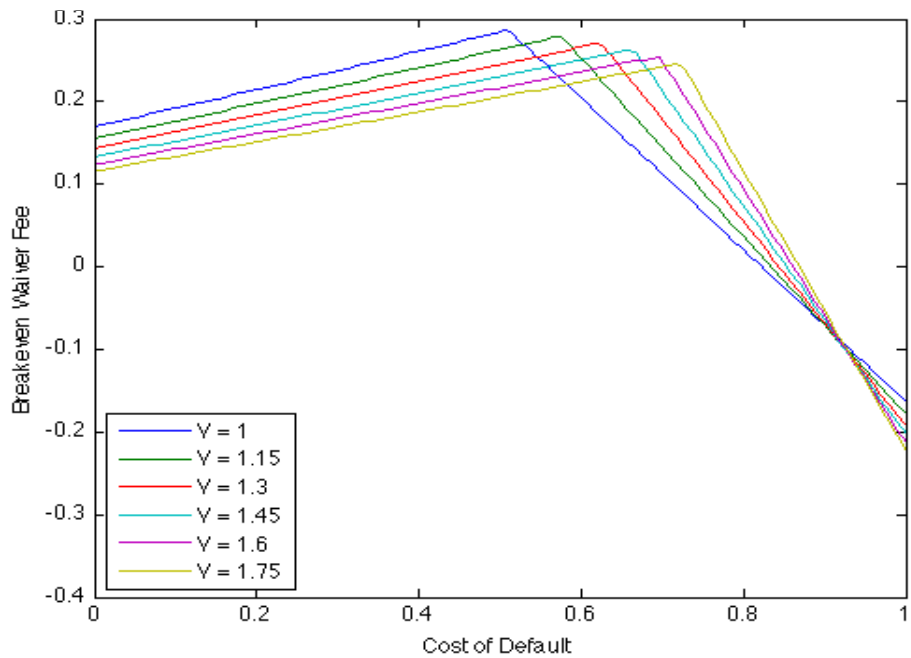


Parameters: $r = .03, c = .04, \mu = .05, \rho = .01, d = .02, \sigma = .4, \ell = .5, V = 1.5 \; and \; w = 0$

[3] The cost of default is the cost of default is (1- Ψ). Hence these propositions refer to the derivatives of the recovery rate as a proportion of the asset value.

claim upon a future default, the breakeven bank waiver fee is increasing in the default cost because a further increase in the cost of default reduces the value of the continuing loan claim. Once the cost of default is great enough to impair the value of the loan claim upon foreclosure, however, the breakeven waiver fee is decreasing in the default cost as the value of the loan upon foreclosure is declining more quickly than the value of the continuing claim.

Figure 4.8: The Effect of Default Costs on the Breakeven Bank Waiver Fee at Varying Asset Values

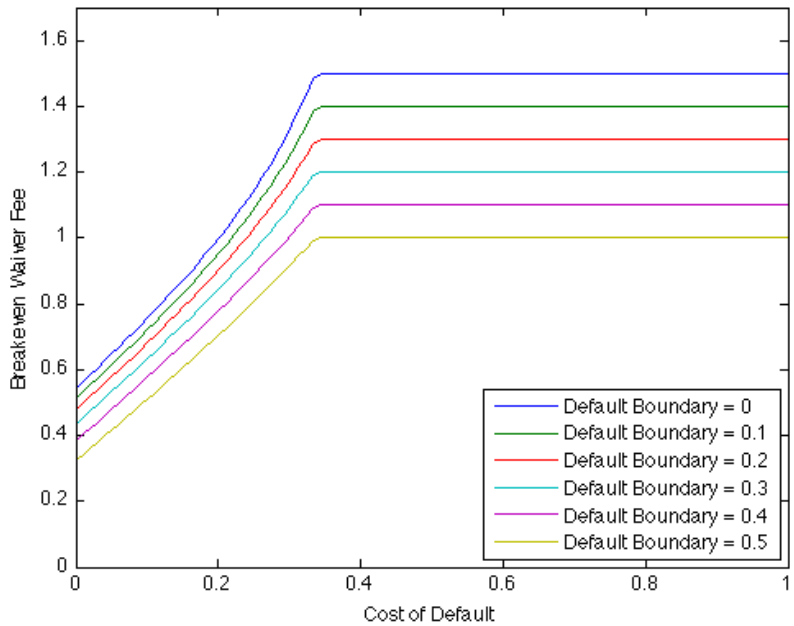


Parameters: $r = .03,\ c = .04, \mu = .05,\ \rho = .01, d = .02\ \ \sigma = .4,\ \ell = .5,\ \bar{V} = .25, and\ w = 0$

The breakeven equity fee as a function of the default cost is similarly piecewise. Proposition 6 proves that, for levels of the default cost at which the equity holder still receives a payout upon foreclosure, the breakeven equity waiver fee is increasing in the cost of default. When the cost of default is low enough that the equity claim has value upon foreclosure, increasing the default cost reduces the value of the equity upon foreclosure more rapidly than it reduces the value of the continuing equity claim – even if the equity

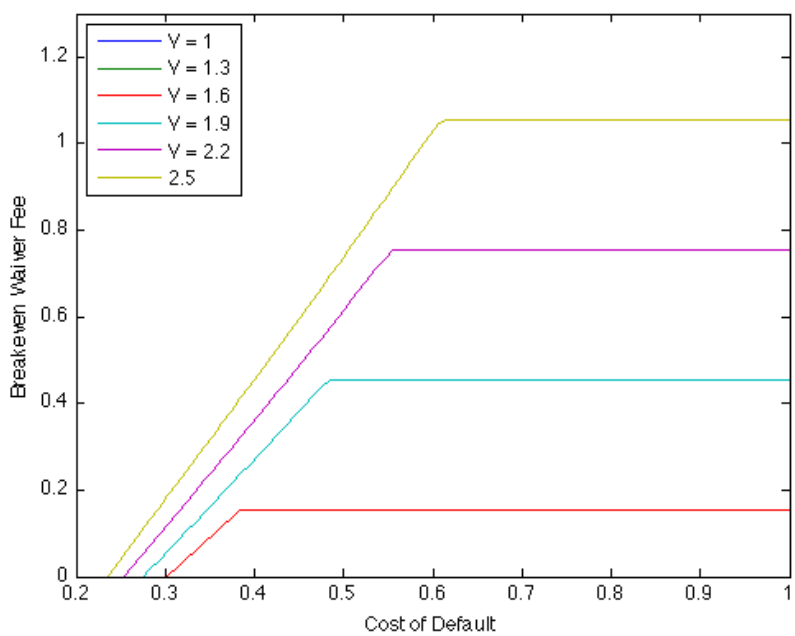
claim has a non-zero recovery upon a future default. This is so because the value of recovery upon a future default is strictly less than the value of recovery upon immediate foreclosure –

Figure 4.9: Effect of Default Costs on the Breakeven Equity Waiver Fee at Varying Default Boundaries



Parameters: $r = .03, c = .04, \mu = .05,\ \rho = .01, d = .02\ \ \sigma = .4,\ \ell = .5,\ A = 1.5, and\ w = 0$

Figure 4.10: Effect of Default Costs on the Breakeven Equity Waiver Fee at Varying Asset Values



Parameters: $r = .03, c = .04, \mu = .05, \rho = .01, d = .02, \sigma = .4, \ell = .5,\ \bar{V} = .25\ and\ w = 0$

thus the reduction of value due to the cost of default is strictly less as well[4]. When the equity holder receives no payout upon an immediate foreclosure, the breakeven equity waiver fee is unaffected by a marginal increase in the cost of default, however, because the debt claimants absorb the marginal value reduction from the increase. These conclusions are graphically illustrated through simulation in Figures 4.9 and 4.10.

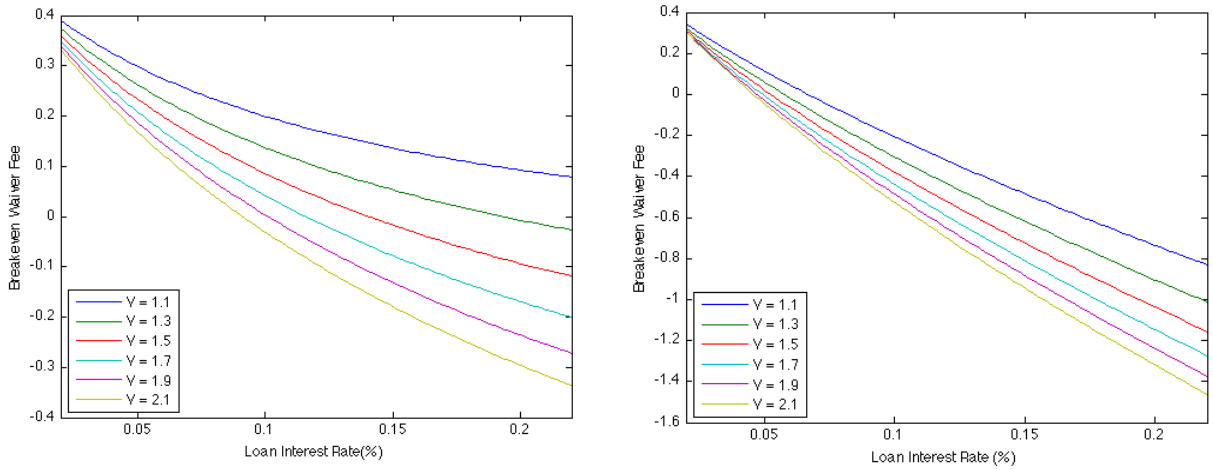
Net, the intersection of the breakeven bank and equity waiver fees suggest that the probability of waiver is relatively constant, while the realized waiver fee is increasing, as the cost of default approaches $1 - \frac{\ell}{V}$ from below. Once the cost of default increases sufficiently (i.e. as it approaches the value of the assets), a waiver is strongly optimal and no waiver fee will be paid, however.

4.4 Comparative Statistics: Capital Structure Characteristics

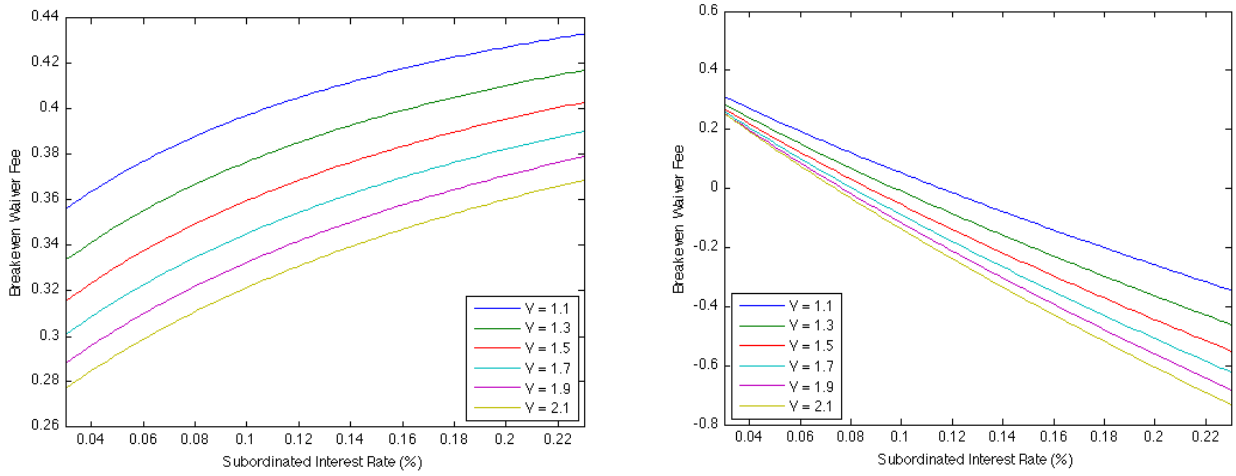
The defining capital structure parameters in the model are the composition of bank debt, subordinated debt and equity in the capital structure and the interest rate (and dividend yield) paid to each of these claims. Interest and dividends have an intuitive effect: the loan interest rate displays a negative correlation with the loan breakeven waiver and the subordinated rate and dividend rate display a positive correlation, while the dividend rate displays a positive correlation with the breakeven equity waiver and loan and subordinated rates display a negative correlation. These relationships are graphically depicted through simulation in Figures 4.11-4.16. The economic interpretation of these relationships are straightforward: the shareholder/bank is willing to pay a larger/accept a smaller waiver fee if the shareholder/bank is paid more per period upon a waiver and demands to pay less/receive more if the other claimants are paid more per period.

[4] This follows because $V > \bar{V}$ and because of the time value of money

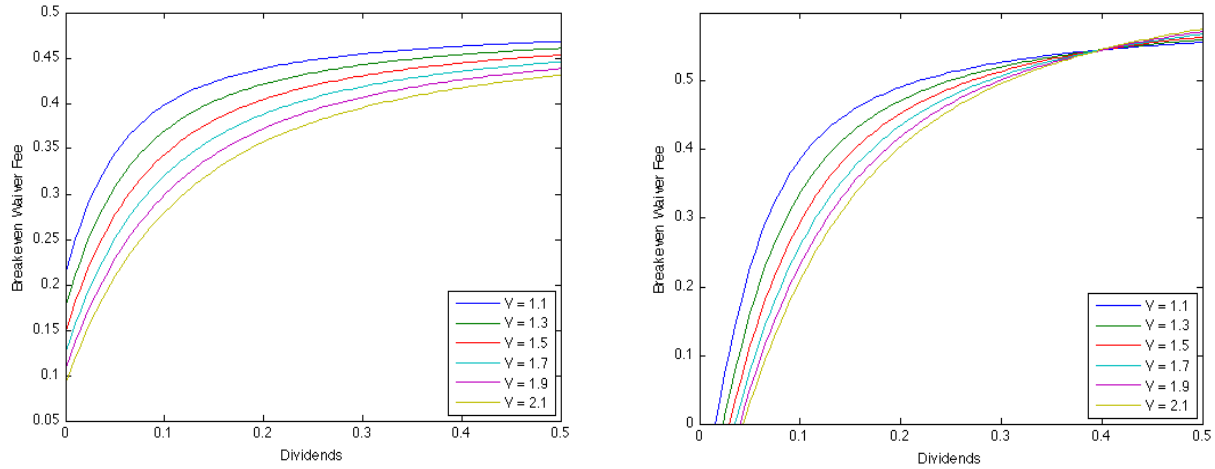
Figures 4.11 and 4.12: The effect of the Loan Interest Rate on the Breakeven Loan Waiver (Left) and Breakeven Equity Waiver (Right) as varying asset values



Figures 4.13 and 4.14: The effect of the Subordinated Debt Interest Rate on the Breakeven Loan Waiver (Left) and Breakeven Equity Waiver (Right) as varying asset values



Figures 4.15 and 4.16: The effect of the Dividend rate on the Breakeven Loan Waiver (Left) and the Breakeven Equity Waiver (Right) as varying asset values



Parameters: $r = .03,\ \rho = .01,\ \sigma = .4,\ \ell = .5,\ \bar{V} = .01,\ \Psi = .9\,, w = 0,\ d = .02(top, middle), c = .04(middle, bottom)\ and\ \mu = c + .01(top, bottom)$

Proposition 7 $\lim_{\mu\to\infty} \mathbb{W}_B(\mu) = \min(\ell, \Psi V) - \min(\ell, \Psi\bar{V})$.

Proposition 8 $\lim_{\mu\to\infty} \mathbb{W}_E(\mu) = 0$ if $V\Psi \leq 1$ and $\lim_{\mu\to\infty} \mathbb{W}_E(\mu) = 1 - V\Psi$ otherwise

Increasing the loan's interest rate causes both the breakeven loan and equity waivers to decrease and increasing the dividend rate causes both the breakeven loan and equity waivers to increase. Hence, while increasing the loan interest will reduce the magnitude of the waiver fee paid and increasing the dividend rate will increase it, the net effect of increasing either on the probability of a waiver is indeterminate. Increasing the subordinate debt rate, however, increases the breakeven bank waiver and decreases the breakeven equity waiver – hence definitively decreasing the probability of a waiver. Proposition 7 and 8 formally state the effect of the subordinated rate on the breakeven equity and bank waivers as it approaches infinity. Because the breakeven bank waiver is conclusively greater than the breakeven equity waiver as the as the subordinated rate approaches infinity, the probability of a waiver approaches zero. Proofs for these propositions are given in the Appendix.

The composition of the capital structure has a more notable effect on the probability of a waiver. In particular, the defining capital structure characteristic that affects the probability of a waiver is the difference between the asset value and the loan principal outstanding. As the distance between the loan principal outstanding and the asset value increases, the breakeven bank waiver fee increases for economically relevant parameter values. This effect is formalized in Proposition 9, a proof of which is given in the Appendix.

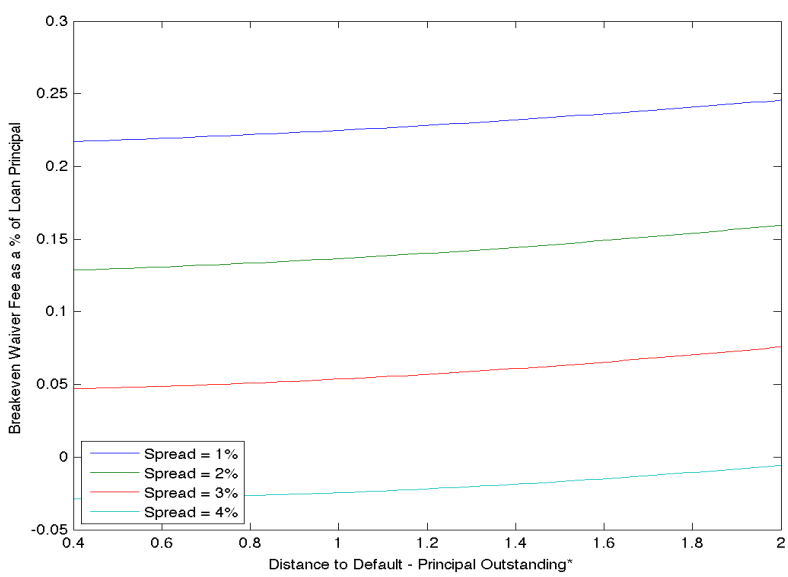
Proposition 9[5] $\frac{d\,\mathbb{W}_B}{d\ell} > 0$ when $V < \bar{V}\left(1 - \frac{r}{c}\right)^{-1/\Omega}$ and $V\Psi > \ell$

[5] As a reference, for the parameter values used in Figure 4.17 and a spread of 1%, the constraint $V < \bar{V}\left(1 - \frac{r}{c}\right)^{-1/\Omega}$ holds for $V < 6.3$, a leverage ratio of .19.

This contradicts with past research, which suggests that the probability of default upon a violation should be proxied by the "distance to default", calculated per equation (4.3) – with D representing either the total principal outstanding or an arbitrary fixed fraction of it calibrated using empirical data. To clarify, the distance to default is not an estimate of the true default boundary in this context, but instead, a supposed indicator of the bank's optimal decision. In the framework of this model, the distance to default at a fixed volatility depends solely on the value of the firm assets, V, given the assumption that the total principal outstanding is 1. Likewise, the distance to the loan principal outstanding depends solely on debt composition (i.e. the loan principal outstanding) at a fixed asset value and volatility.

$$Distance\ to\ Default = \frac{\log(V/D)}{\sigma} \qquad (4.3)$$

Figure 4.17: The effect of the Distance to Default as for a fixed difference between the asset value and loan principal on the Breakeven Loan Waiver



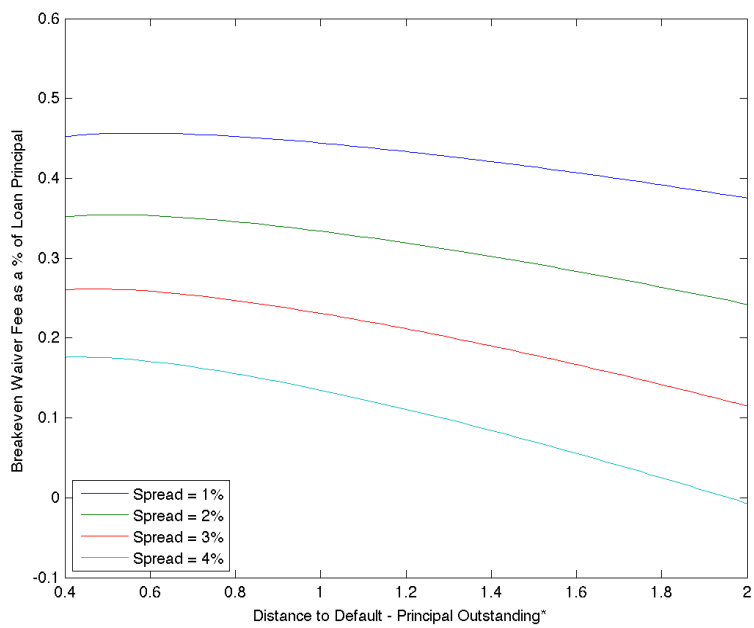
Parameters: $r = .03, \mu = c + .005,\ \rho = .0001, d = .029\ \ \sigma = .3,\ \ell = V - 1,\ \bar{V} = 1 * .6, w = 0, \Psi = .9\ and\ V = [1.01,2]$

Figure 4.17 displays the breakeven waiver fee as a percentage of the loan principal outstanding with respect to the distance to default, as defined in equation (4.3), while varying

the loan principal outstanding and the default threshold. In particular, I set the default threshold equal to 60% of the loan principal outstanding and allow the loan principal to increase in step with the asset value while the total principal outstanding remains fixed (thus, in effect, I reduce the subordinated principal outstanding as I increase the asset value). This is a justifiable approximation of the post-waver default threshold if the bank, and only the bank, has power to foreclose on the company post waiving the current covenant violation. The breakeven bank waiver fee is increasing in the distance to default given these assumptions – implying, all else equal, that the bank demands additional compensation for its waiver as the distance to default increases.

Even if the assumption that the bank has discretion over future corporate bankruptcy is relaxed and the default boundary is set to a fixed threshold, as it has been in the previous

Figure 4.18: The effect of the Distance to Default for a fixed difference between the asset value and loan principal and a fixed default threshold on the Breakeven Loan Waiver

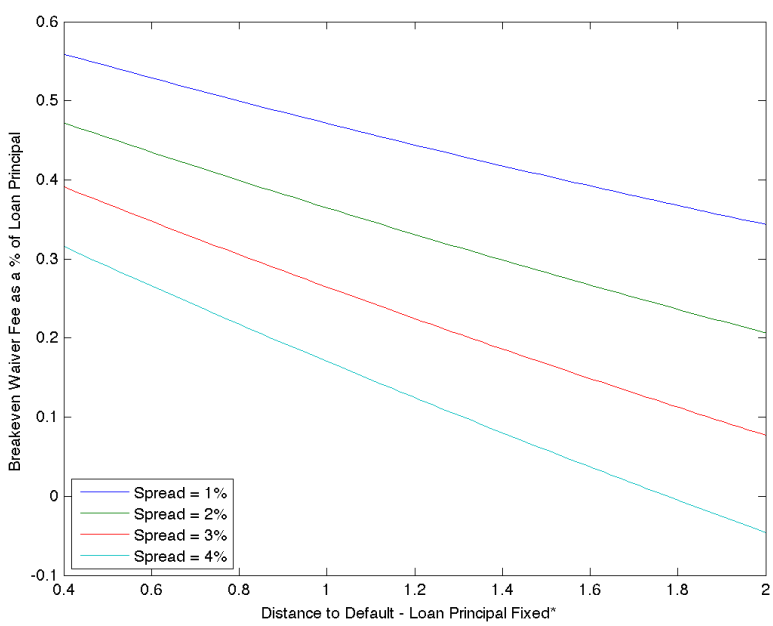


Parameters: $r = .03, \mu = c + .005,\ \rho = .0001, d = .029\ \ \sigma = .3,\ \ell = V - 1,\ \bar{V} = .01, w = 0, \Psi = .9\ and\ V = [1.01,2]$

figures, increasing the distance to default still has a negligible effect on reducing the breakeven waiver fee as a percent of the loan outstanding. I depict this graphically in Figure 4.18. When the distant to default is varied but the loan principal remains fixed (hence the difference between the loan principal outstanding and the total asset value increases as the asset value increases), there is a substantial negative effect on the breakeven bank waiver, however. I Illustrate this in Figure 4.19. Hence, the distance to the loan principal outstanding, not the distance to total principal outstanding as traditionally used in the distance to default calculation, is the capital structure characteristic that determines the breakeven bank waiver fee.

Figures 4.20 – 4.22 illustrate the breakeven equity waiver as the distance to default is varied under the same case assumptions concerning the loan principal and default threshold. The breakeven equity waiver has a negative correlation of similar magnitude to the distance

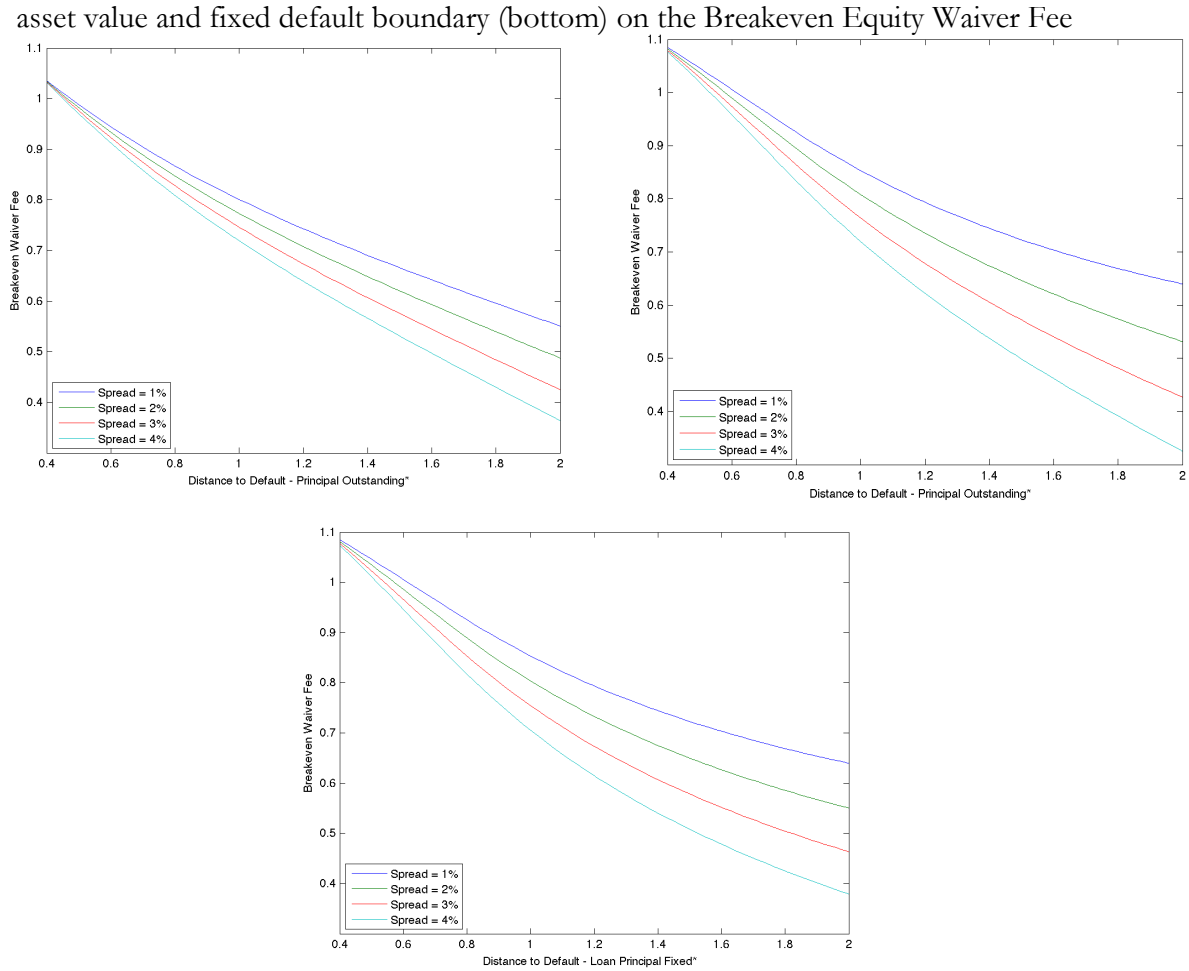
Figure 4.19: The effect of the Distance to Default for a fixed difference between the loan principal outstanding and fixed default boundary on the Breakeven Loan Waiver



Parameters: $r = .03, \mu = c + .005,\ \rho = .0001, d = .029\ \ \sigma = .3,\ \ell = .5\ \bar{V} = .01, w = 0, \Psi = .9\ and\ V = [1.01,2]$

to default in all scenarios. Thus, in total, increasing the distance to default while holding the distance to the loan principal outstanding constant should not have a substantial effect on the probability of a waiver. Increasing the distance to the loan principal outstanding, however, should increase the probability of a waiver. Upon reflection, this is unsurprising – the bank and the equity holder are the only actors in the waiver/foreclosure decision. Hence, the risk recovery tradeoff of the subordinated debt holders should not affect the prospect of a covenant waiver.

Figure 4.20-4.22: The effect of the Distance to Default with a fixed difference between the loan principal and asset value (top right), with a fixed difference between the loan principal and asset value and a fixed default boundary (top left) and with a variable difference between the loan principal and asset value and fixed default boundary (bottom) on the Breakeven Equity Waiver Fee



Parameters: $r = .03, \mu = c + .005,\ \rho = .0001, d = .029\ \ \sigma = .3,\ \ell = .5\ \bar{V} = .01, w = 0, \Psi = .9\ and\ V = [1.01,2]$

4.5 Testable Hypotheses

The key implication of the model is that the volatility-adjusted difference between the asset value and the bank loan outstanding, not the total principal outstanding or some fraction of it, is the important determinant of the probability of a waiver. That being said, Sections 4.3 and 4.4 suggest a number of implications that represent new, and in some cases, non-intuitive, predictions for the effect of the asset and capital structure characteristics on the probability of a waiver and the magnitude of the waiver fee. I outline these predictions below:

(1) the probability of a waiver is decreasing in the ratio of the loan value to total firm assets and the waiver fee paid in instances in which the covenant is waived is increasing

(2) the probability of a waiver is approximately constant in volatility while the waiver fee paid in instances in which the covenant is waived is increasing

(3) the probability of a waiver is approximately constant in the bank loan interest rate while the waiver fee paid in instances in which the covenant is waived is decreasing

(4) the probability of a waiver is decreasing in the subordinated debt interest rate

V. Empirical Strategy and Data

The objective of the empirical work is to test the validity of the hypotheses produced by the model and outlined in the comparative statistics section. Though I provide evidence concerning all the hypothesized relationships, special emphasis is given to the effect of the bank debt share on the probability of a waiver and the magnitude of the waiver fee paid.

Two separate series of regressions are needed to test these relationships. The first uses the coefficient estimates from a probit regression of the form presented in equation (5.1) to test the impact of the variants on the probability of a waiver and the second uses the coefficient estimates from a standard linear regression of the form presented in equation (5.2) to test the impact of the variants on the magnitude of the waiver fee paid in instances in which a waiver is granted.

$$\mathcal{W} = \Phi(a_0 + a_1\lambda + a_n Controls\ + \epsilon) \tag{5.1}$$

$$W = a_0 + a_1\lambda + a_n Controls\ + \epsilon \tag{5.2}$$

where $\mathcal{W}$ is an indicator variable equal to 1 if a waiver is granted, $\Phi(\cdot)$ is the standard normal cumulative distribution function, λ is the variable of interest, W is the incremental waiver payment as a percent of the loan outstanding and controls is a vector of other variables relevant to the regression.

5.1 Data

I created the sample of covenant violations used by narrowing the library of covenant violations compiled by Nini, Sufi and Smith (2011). In their work, Nini, Sufi and Smith use a Perl script to search the 10-Q and 10-K filings for the universe of U.S. listed non-financial firms on Compustat from 1996 to 2009 for evidence of financial covenant violations. In total, their methodology produces a dataset of 14,050 violations.

From the library compiled by Nuni, Sufi and Smith, I randomly drew 125 covenant violation instances. I manually read the filings in which the respective covenant violation was disclosed for each violation instance to record:

(1) the covenant, or covenants, violated

(2) the result of the violation – whether the covenant was waived, a forbearance period was granted or the loan was foreclosed upon

(3) the waiver fee paid (if any)

(4) the step-up in the respective debt tranche, or tranches, interest rate as a result of the waiver (if any)

(5) the type of the debt whose covenant was violated (revolver, term loan, etc. – henceforth referred to as the "relevant debt tranche")

(6) the principal outstanding of the debt tranche, or tranches, whose covenant was violated

(7) the coupon rate of the debt tranche, or tranches, whose covenant was violated

In instances in which the covenant violation disclosed was not resolved by quarter end (9 instances), I read the 10-Q/10-K filing from the succeeding quarter to establish whether a waiver was ultimately granted. In instances in which the interest rate of the debt whose covenant was violated was not listed (11 instances), I read the preceding 10-K filing to locate the interest rate of the debt instrument. There were two instances in which the interest rate of the relevant debt tranche was not disclosed and one instance in which the principal outstanding of the relevant debt tranche was not disclosed. These observations were removed from the dataset.

I next removed instances in which the relevant debt tranche was a revolving loan or line of credit. I did so because the theoretical model of Section 3 models the creditor's

decision as "all or none" with respect to foreclosure. This treatment accurately captures the decision of the creditor for a standard loan or bond because the principal is lent upfront and the repayment schedule is contractually predetermined. Because a debtor draws on the principal available under a line of credit over time, as opposed to receiving the principal upfront, the creditor has a third option upon a covenant violation, however – namely, he can reduce the remaining principal available. Hence, the hypotheses of the model do not necessarily capture the decision of the creditor upon a line of credit covenant violation because of his ability to "partially foreclose". This screen removes 63 observations from the dataset.

Table 5.1 Overview of the Dataset

Instance	Number of Observations
Foreclosure	10
Waiver Fee Paid	16
Interest Rate Changed	6
Subordinated Debt Present	46
Total Observations	59

For each of the remaining 59 violation instances, I downloaded the corresponding firm's fundamental accounting and firm classification data for the relevant quarter from Compustat using the GV key of each firm. Likewise, I downloaded daily share price data from two years before the disclosure of the violation till 10 days after the disclosure from CRSP. For both the Compustat and CRSP data pulls, I downloaded and matched central index keys (CIK) to confirm that the GV keys had correctly identified the companies of interest. Lastly, I downloaded U.S. prime rates, U.S. treasury rates and LIBOR rates from the date of the violation disclosure from the Global Financial Database.

Table 5.2 Summary Statistics for Market and Accounting Data

Statistic	Mean	Median	Minimum	Maximum
Waiver Fee	0.2	0.0	0.0	3.9
Change in Interest Rate	+0.13%	+0.00%	+0.00%	+2.00%
Market Capitalization	377	20	0	6,651
Enterprise Value	658	42	2	15,057
Debt to Equity Ratio	4.4	0.9	0.0	81.7
Book Value of Assets	974	58	2	29,918
Equity Volatility	91.7%	78.3%	18.7%	388.5%
Bank Debt Interest Rate	9.2%	8.5%	3.0%	26.5%
Subordinated Debt Interest Rate	11.9%	9.7%	0.5%	29.8%

All currency figures in USD millions

There are a number of possible concerns with the dataset used. Firstly, by excluding of firms that provide insufficient data concerning their debt principal and interest rate, I could theoretically bias the dataset. Given how few instances I exclude (3), however, this effect should be negligible. Secondly, firms have partial discretion in disclosing violations. SEC Regulation S-X requires that any covenant violation as of the financial statement filing date that has not been subsequently cured must be disclosed in the statement. Furthermore, the SEC extended this requirement in a 2003 interpretation, stating that any firm that is "reasonably likely" to be in violation must declare so[1]. These requirements do give leeway on the disclosure of minor violations that are quickly waived, though; and, thus, the library of violations from which the dataset is drawn may underrepresent minor violations that are quickly waived. Sufi and Roberts (2009) demonstrate that patterns in observable credit quality metrics for the library of violations closely match those of previous studies, suggesting that the reported violations correspond closely to actual violations, and, hence, that any underreporting is minimal, however. Lastly, by excluding line of credit and revolving credit agreement violations, I remove the substantial portion of observations from the

[1] Nini, Sufi and Smith (2011)

sample. Though this exclusion should not bias the dataset, it does limit the applicability of the theoretical framework and the findings of the regressions.

5.2 Dependent Variable Construction

Since the aim is to test the determinants of the waiver/foreclosure decision and the waiver fee magnitude, the dependent variables of interest are an indicator variable, equal to 1 if a waiver is granted, and the incremental waiver payment made by the firm to the bank to receive a waiver.

Indicator Waiver Variable For each violation, the indicator variable is set to 1 if a waiver, either waiving the covenant for a specified time period or modifying the covenant threshold, is granted, and to 0 if the firm files for bankruptcy or the bank either forecloses or stipulates a forbearance agreement that preserves default. There are 49 waivers and 10 foreclosures in the dataset.

Incremental Waiver Payment I construct the incremental waiver payment to capture the value the creditor extracts from the debtor as a percentage of the creditor's outstanding obligation. A bank can extract additional payment from the debtor for its waiver through either raising the interest rate on its loan or charging a direct one-time waiver fee. Though there are distinct pros and cons to each method of extracting payment, the model developed focuses only on quantifying the total economic value extracted for a waiver, not the channel through which this value is extracted. Hence, the comparative statistics section assumes no change in the interest rate and examines only the necessary waiver fee. In reality, however, the bank will decide the means of extracting value from the debtor based on the debtor's ability to pay the waiver fee upfront, its confidence in its assessment of the firm's assets, the effect of each option on its lending relationship with the debtor and a host of other reasons that are not investigated in this work. Hence, to capture the total economic value extracted

for the waiver, I approximate the incremental payment made by the firm by summing the waiver fee and the incremental interest expense, defined as the increase in the interest rate multiplied by the loan principal outstanding, in the first year proceeding the waiver. This means of approximating the incremental interest expense has precedent in the literature, first appearing in the methodology of Beneish and Press (1998). I then construct the incremental waiver payment variable by dividing the incremental payment by the loan principal outstanding to express the payment as a percentage of the loan value.

Table 5.3 Summary Statistics for the Incremental Waiver Payment

	Mean	Median	Minimum	Maximum
Incremental Waiver Payment (All Waivers)	0.75%	0.00%	0.00%	10.98%
Incremental Waiver Payment (Subordinated Debt Present)	0.53%	0.00%	0.00%	6.33%

5.3 Independent Variable Construction

The independent variables stand in for the inputs to the theoretical model developed in Section 3. Hence, they are the determinants of the waiver/foreclosure decision and the waiver fee magnitude if the model is an accurate representation of the interplay between the bank and equity holder following a covenant violation. The independent variables of interest are loan principal to enterprise value, subordinated principal to enterprise value, asset volatility, bank loan spread, subordinated debt spread, dividend yield and industry classification.

Loan Principal to Enterprise Value I construct loan principal to enterprise value to capture the value of the principal owed to the creditor whose covenant was violated in relation to the value of the firm's assets. To do so, I first calculate market capitalization by multiplying the share price at the close of the quarter in which the violation was reported

(henceforth denoted as the "respective quarter") by the diluted common shares outstanding from Compustat, if available, and the common shares outstanding, if not. I then calculate enterprise value by summing the market capitalization and book value of debt less cash and short-term investments. Using this estimate of enterprise value, I construct the Loan Principal to Enterprise Value variable by dividing the recorded loan principal by the enterprise value.

Subordinated Principal to Enterprise Value I construct subordinated principal to enterprise value to capture the value of the principal owed to creditors whose covenants were not violated in relation to the value of the firm's assets. To do so, I subtract the loan principal outstanding from the total debt figure from Compustat. I then divide this difference by the estimate of enterprise value described above.

Asset Volatility I construct asset volatility to capture the volatility of the underlying firm assets. Because asset volatility is not an observable characteristic, I do so by employing the framework of the structural model to write an implicit function that relates the observable equity volatility to the underlying asset volatility. In particular, I use the common assumption employed by Moody's KMV in its CreditMonitor Tool that the default boundary is equal to the short-term liabilities plus one-half of the long-term liabilities[2]. Moreover, for simplicity, I assume the assets evolve in accordance with a geometric Brownian motion. Given these assumptions, the relevant equation is (5.3), which I solve implicitly for the asset volatility using numerical methods.

$$E\sigma_E = V\sigma_V \Phi\left(\frac{1}{t\sigma_V}\left[\ln\left(\frac{V}{D}\right) + (r + .5{\sigma_V}^2)t\right]\right) \tag{5.3}$$

where σ_E is the observed equity volatility, σ_V is the asset volatility, D is the default threshold and $\Phi(\cdot)$ is the standard normal cumulative distribution function. I calculate the observed

[2] Crosbie and Bohn (2013)

equity volatility used in this equation by taking standard deviation of the daily log returns over the year preceding the covenant violation and annualizing this figure.

Though I believe this estimate of asset volatility roughly approximates the true asset volatility, it is mis-specified given the implications of the model I develop in Section 3. The model implies that the creditors select the true default boundary as a response, in part, to the asset volatility. Hence, the default boundary is a function of the asset volatility rather than a fixed value, as assumed in equation (5.3), and the asset volatility cannot be extrapolated in an internally consistent manner from equation (5.3). To address this, I calculate an estimate of asset volatility using the "practitioners" method. This method approximates asset volatility by multiplying the equity volatility by the value of the equity over the value of the enterprise. There are no substantive differences in the regression results when the two estimates of asset volatility are interchanged.

Bank Loan Spread I construct bank loan spread to capture the premium above the risk free rate that the creditor whose covenant was violated is earning of his claim. To do so, I first calculate the effective rate for variable rate debt by adding the appropriate spread to the underlying index rate, in all cases either the U.S. prime rate or LIBOR, at the end of the respective quarter. This enables me to express the effective rate of both fixed and variable rate debt as an absolute percentage. For instances in which there are covenant defaults in more than one tranche of debt, I use a weighted average of the tranches' interest rates weighted by principal outstanding. I then construct the bank loan spread variable by subtracting the U.S. treasury yield to maturity as of the respective quarter end for the treasury with the maturity closest to the loan in question from the effective rate.

Subordinated Debt Spread I construct the subordinated debt spread variable to capture the loan premium above the risk free rate that the creditors whose covenants have not been

violated are earning on their claims. To do so, I calculate the interest on the subordinated debt by subtracting the quarterly bank loan interest from the quarterly total interest expense

Table 5.4 Summary Statistics for the Independent Variables

Statistic	Mean	Median	Min	Max	Number of Observations
Loan Principal to Enterprise Value	33.7%	22.1%	0.0%	111.2%	59
Subordinated Principal to Enterprise Value	25.1%	22.6%	0.3%	90.2%	59
Asset Volatility	43.8%	35.0%	5.6%	214.7%	59
Bank Loan Spread	5.1%	4.1%	(1.4%)	22.5%	59
Subordinated Debt Spread	7.7%	5.4%	(2.0%)	24.2%	46
Energy Indicator	–	–	–	–	9
Materials Indicator	–	–	–	–	6
Industrials Indicator	–	–	–	–	9
Consumer Discretionary Indicator	–	–	–	–	10
Consumer Staples Indicator	–	–	–	–	4
Health Care Indicator	–	–	–	–	4
Financials Indicator	–	–	–	–	0
Information Technology Indicator	–	–	–	–	16
Telecommunications Indicator	–	–	–	–	0
Utilities Indicator	–	–	–	–	1

figure from Compustat. I calculate the effective subordinated rate by multiplying this quarterly expense by four and dividing it by the subordinated principal outstanding. There are 5 instances in which the rate produced by this calculation is significantly outside a believable range. In each instance, I manually read the corresponding filings. The discrepancies arose because of debt issuances/retirements in the quarter that are reflected in the principal balance outstanding at the quarter end but not yet reflected in the interest expense. For these instances, I calculated the subordinated rate manually by taking the principal weighted average of all the subordinated rates available. Finally, I construct the subordinated debt spread variable is by subtracting the U.S treasury yield to maturity at the end of the respective quarter from the effective rate.

Industry Classification Unlike the other independent variables, industry classification does not correspond to a model input. I include it, however, to control for industry specific affects that could otherwise bias the results. I control for industry classification by constructing a series of indicators variables corresponding to each S&P GIC sector using S&P GIC classification codes. In total, I construct 10 indicator variables: Energy, Materials, Industrials, Consumer Discretionary, Consumer Staples, Health Care, Financials Information Technology, Telecommunications and Utilities.

5.4 Hypothesized Coefficient Values

My model's testable hypotheses, presented in Section 3.5, correspond to predictions about the signs of the independent variable coefficients in each regression. I outline these predictions below and summarize the hypothesized signs in Table (5.5). With respect to the probit regressions testing the probability of a waiver, these hypotheses predict:

(1) A negative coefficient on loan principal to enterprise value even while controlling for total debt. This follows from hypothesis 1 and implies that the composition of debt affects the probability of a waiver. This prediction diverges from traditional structural credit models, which support the null hypothesis.

(2) A null (zero valued) coefficient on asset volatility when subordinated debt exists. This follows from hypothesis 2 and implies that volatility does not affect the probability of a waiver. This prediction likewise diverges from traditional structural credit models, which support a negative coefficient.

(3) A positive coefficient on bank loan spread. This follows from hypothesis 4 and implies that a higher bank loan interest rate increases the probability of a waiver.

(4) A negative coefficient on subordinated debt spread. This follows from hypothesis 5 and implies that high subordinated rates decrease the probability of a waiver.

Table 5.5 The Hypothesized Signs of the Independent Variable Coefficients

	Probability of Waiver	Magnitude of Waiver Fee
Loan Principal to Enterprise Value	−	+
Asset Volatility	Ø	+
Subordinated Debt Spread	−	
Bank Debt Spread	+	−

+ positive value; − negative value; Ø zero value

And with respect to the linear regressions measuring the magnitude of the waiver fee, they predict:

(4) A positive coefficient on loan principal to enterprise value even when controlling for total debt. This follows from hypothesis 1 and implies debt composition affects the magnitude of the waiver fee.

(5) A positive coefficient on asset volatility. This follows from hypothesis 2 and implies that higher volatility increases the magnitude of the waiver fee.

(6) A negative coefficient on the bank loan interest rate. This follows from hypothesis 4 and implies that volatility does not affect the probability of a waiver.

VI. Empirical Results

I test the probit and linear regressions specified in Section 5 on two subsets of the data. I first focus on testing the predictions concerning the coefficients on loan principal to enterprise value, asset volatility and bank loan spread in each regression with the full dataset. I then test the predictions concerning the coefficients on the subordinated debt spread by eliminating observations for which there is no subordinated debt outstanding. Lastly, I discuss the estimated coefficients of the auxiliary and control variables

Table 6.1: The Coefficient Signs estimated from the Data v. their Predicted Values for the Primary Variables of Interest

	Empirical Finding/Prediction	
	Probability of Waiver	Magnitude of Waiver Fee
Loan Principal to Enterprise Value	−/−	+/+
Asset Volatility	I/Ø	+/+
Subordinated Debt Spread	−/−	I/
Bank Debt Spread	I/+	I/−

"+" positive value; "−" negative value; "Ø" zero value; "I" *insignificant at a 10% confidence threshold*

6.1 Bank Debt Share

Table 6.1 reports estimates from a probit regression of the indicator waiver variable. The coefficient on the share of bank debt in the capital structure is negatively correlated with the Indicator Waiver variable in both the base case, in which I include a full set of control variables, and in the debt structure case, in which I only control for total debt outstanding. The p-value of the coefficient in the base case is .098, suggesting it is mildly significant. This aligns with the prediction of a positive coefficient outlined in Section 5.4 and supports the hypothesis that debt composition is a significant determinant of the waiver/foreclosure decision. Moreover, the coefficient on total debt is positive (though insignificant) in both the base case and the debt structure case regressions. Thus, the data support the hypothesis that

Table 6.1: Probit Regression of the Indicator Waiver Variable: Full Dataset

	Base Case		**Only Debt Structure: Bank Debt**		**Only Debt Structure: Suborindated**		**Only Asset Volatility**		**Only Loan Spread**	
Independent Variable	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value
Bank Debt Share (Loan/EV)	-3.199	0.098	-2.572	0.046						
Asset Volatility	0.417	0.550					-0.107	0.858		
Loan Spread	-5.982	0.327							-9.867	0.046
Suboridnated Debt Share (Sub/EV)					2.551	0.048				
Total Debt Share (T-Debt/EV)	0.756	0.648	1.090	0.393	-1.486	0.032				
Energy Indicator	-0.919	0.180								
Materials Indicator	Dropped									
Industrial Indicator	-0.032	0.969								
Consumer Staples Indicator	Dropped									
Consumer Discretionary Indicator	1.086	0.223								
Health Care Indicator	-0.473	0.642								
Information Technology Indicator	Dropped									
Utilities Indicator	Omitted									
Intercept	1.904	0.025	1.382	0.001	1.390	0.001	1.004	0.002	1.528	0.000
Pseudo R-Squared	0.317									
Number of Observations:	59									

Note: The regression estimates are the result of probit regression of the independent variables listed in each case on the Indicator Waiver Variable using the full dataset of violations. Subordinated debt share is not included in the base case or debt structure regressions because total debt share is a linear function (the simple sum) of bank debt share and subordinated debt share. The base case omits the utilities indicator to avoid multicollinearity. The materials indicator, consumer staples indicator and information technology indicator are dropped in the base case because they lead to a complete separation of the data.

Table 6.4: Linear Regression of the Incremental Waiver Payment: All Waiver Instances

	Base Case		Only Debt Structure: Bank Debt		Only Asset Volatility		Only Loan Spread	
Independent Variable	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value
Bank Debt Share (Loan/EV)	0.046	0.000	0.041	0.002				
Asset Volatility	0.030	0.000			0.028	0.000		
Loan Spread	-0.092	0.136					0.034	0.655
Total Debt Share (T-Debt/EV)	-0.006	0.579	-0.007	0.576				
Energy Indicator	-0.012	0.435						
Materials Indicator	-0.013	0.402						
Industrial Indicator	-0.005	0.741						
Consumer Staples Indicator	-0.012	0.374						
Consumer Discretionary Indicator	-0.013	0.447						
Health Care Indicator	-0.020	0.262						
Information Technology Indicator	-0.008	0.586						
Utilities Indicator	Omitted							
Intercept	-0.001	0.944	-0.001	0.861	-0.005	0.200	0.006	0.195
R-Squared	0.587							
Number of Obersevations	49							

Note: The regression estimates are the result of an ordinary least squares linear regression of the independent variables listed in each case on the Increment Waiver Payment using all instances in which a waiver was provided. The base case omits the utilities indicator to avoid multicollinearity.

the private creditors select the optimal waiver/foreclosure decision without regard for total principal outstanding. This is a significant deviation from past research and widely held assumptions about the nature of the default boundary as it provides evidence that the loan principal outstanding, not the total principal outstanding, is the key determinant of the waiver/foreclosure decision and, hence, the creditor induced default threshold.

Table 6.2 reports estimates from a simple ordinary least squares linear regression of the incremental waiver payment for instances in which a waiver was provided. The coefficient on the share of bank debt in the capital structure is positively correlated with the waiver fee payment in both the base case and the debt structure case. The coefficients are significant at a five percent confidence level in both cases –especially so in the base case, in which the p-value is less than 0.000. The coefficient on bank debt share in the base case is .046, implying that a one percent increase in bank debt as a proportion of total enterprise value increases the compensation paid to the bank as a percentage of the loan principal outstanding by 4.6 basis points (.046%). These results align with the prediction of a positive coefficient, outlined in Section 5.4, and, hence, support the hypothesis that compensation for a waiver is increasing in bank debt share. Because waiver compensation is a function of the payment needed for the private creditor to optimally waive the covenant, these results also provide additional support for the primary role of private creditors, and, by extension, the significance of debt composition, in the waiver/foreclosure decision.

6.2 Asset Volatility

As displayed in Table 6.1, the coefficient on asset volatility from the probit regression of the indicator waiver variable is positive in the base case and negative in the asset volatility only case. Moreover, it is thoroughly insignificant in both – with p-values of .55 and .858 in the base and asset volatility only cases, respectively. These estimates align with the prediction of

a null coefficient on asset volatility outlined in Section 5.4. Thus, the data support the hypothesis that the waiver/foreclosure decision, and, hence, the creditor induced default threshold, is unaffected by volatility. Similar to the findings concerning bank debt share in Section 6.1, this represents a substantial deviation from past research. It suggests that the probability of a waiver is not proxied by distance to default, not only because total debt principal is the inappropriate threshold to use, but because volatility is irrelevant to optimal waiver/foreclosure decision.

As displayed in Table 6.2, the coefficients on asset volatility from the linear regression of the waiver fee payment are positive and significant in both the base and asset volatility only cases. The p-values in each case are less than 0.000. The coefficient in the base case is 0.03, implying that a one percent increase in asset volatility increases the compensation paid to the bank as a percentage of the loan principal outstanding by 3.0 basis points (.03%). Thus, the coefficient estimates align with the prediction of a positive coefficient on the asset volatility and support the hypothesis that waiver compensation is increasing in asset volatility in instances in which a waiver is provided. These findings also suggest that the equity holder is willing to provide additional waiver compensation when volatility is high due to the value accretive effects of volatility on the equity claim[8]. Hence, the findings help illuminate why asset volatility does not effect the optimal waiver/foreclosure decision despite the negative correlation between asset volatility and the continuing loan claim. Specifically, the equity holder is willing to offset the value reducing effects of high volatility on the continuing loan claim by offering a substantial waiver fee because of the value accretive effects of high volatility on the continuing equity claim.

[8] The accretive effect of high asset volatility on the equity value comes from equity's de-facto "call-option" like payout in which its downside is capped at zero but its upside is infinite. For more details, see Merton (1974).

Though the estimates of asset volatility match with the hypotheses I derive from my model perfectly, they should be interpreted with circumspection because the asset volatility data I use is imprecise. As discussed in Section 5.3, the default threshold I assume in my asset volatility calculation is mis-specified given the implications of the model. Moreover, I use a simplistic structural framework to calculate the asset volatility that may not accurately reflect the true relationship between asset volatility and equity volatility. Though these concerns may bias the data uniformly, and hence may not affect the sign or significance of the asset volatility coefficient, the data set does appear to overstate asset volatility – resulting in a mean asset volatility of 43.8%.

6.3 Bank Loan Spread

The coefficient on bank loan spread from the probit regression of the indicator waiver variable is negative in both the base and loan spread only cases. In the base case, however, the p-value is .327, thus I cannot draw any significant conclusions from its value. These estimates are summarized in Table 6.1. Directionally, the estimated coefficient aligns with the prediction and, hence, would support the hypothesis that a higher bank loan interest rate increases the probability of a waiver if its value was significant. The high p-value may, hence, be a product of the limited number of observations in the dataset and is a point for further research.

The bank loan spread coefficient from the linear regression of the waiver fee payment is negative in the base case, positive in the loan spread only case and insignificant in both. These estimates are summarized in Table 6.2. Thus, the hypothesis that the waiver fee payment is decreasing in the spread paid to the bank loan is not supported. One possible explanation for the result is a confounding correlation between the bank loan spread and a characteristics relating to risk that is not captured by the control variables – for example, the

liquidity of the firm's assets. Such an explanation has economic appeal. If the bank lender observes said characteristic, he will account for it by demanding a higher interest rate spread when first extending credit to the firm, hence establishing a positive correlation between the risky characteristic and the loan spread. He will also account for it when selecting the optimal waiver/foreclosure decision, hence decreasing the probability of a waiver and increasing the waiver compensation all else equal. If the regression does not control for this characteristic, however, it will not separate the effect of said characteristic from the effect of the loan spread.

6.4 Subordinate Debt Spread

In order to estimate the coefficients on the subordinated debt spread variable, I first filter the dataset to create a subset consisting only of observations in which in the corresponding firm had subordinated debt outstanding at the time of the violation. This filtration eliminated 13 observations from the full dataset and 9 observations from the dataset of

Table 6.3: Probit Regression of the Indicator Waiver Variable: Subordinated Debt Observation Subset

	Base Case		**Only Debt Structure: Bank Debt**		**Only Subordinated Spread**	
Independent Variable	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value
Subordinated Spread	-10.747	0.029			-25.619	0.065
Bank Debt Share (Loan/EV)	-0.510	0.807	-2.219	0.130		
Asset Volatility	-1.793	0.183				
Loan Spread	-8.891	0.386				
Total Debt Share (T-Debt/EV)	-8.891	0.303	0.278	0.852		
Intercept	5.475	0.007	1.825	0.003	1.563	0.000
Pseudo R-Squared	0.460					
Number of Observations:	46					

Note: The regression estimates are the result of a probit regression of the independent variables listed in each case on the Indicator Waiver Variable using the a subset of the data in which all observations for which the subordinated principal outstanding is zero are not tabulated. The industry indicator variables are not included because the majority lead to perfect separation of the data with this subset.

waiver instances. The coefficient estimates for bank debt share and asset volatility are directionally similar, albeit less significant, for this subset, demonstrating the robustness of the results obtained from the full dataset.

Table 6.4: Linear Regression of the Incremental Waiver Payment: Subordinated Debt Observation Subset

	Base Case		**Only Debt Structure: Bank Debt**		**Only Subordinated Spread**	
Independent Variable	Coefficient	P-Value	Coefficient	P-Value	Coefficient	P-Value
Subordinated Spread	-0.004	0.878			0.001	0.982
Bank Debt Share (Loan/EV)	0.297	0.119	0.029	0.003		
Asset Volatility	0.004	0.636				
Loan Spread	-0.180	0.122				
Total Debt Share (T-Debt/EV)	0.001	0.900	-0.008	0.356		
Intercept	0.007	0.380	0.002	0.602	0.005	0.048
R-Squared	0.108					
Number of Observations:	40					

Note: The regression estimates are the result of an ordinary least squares linear regression of the independent variables listed in each case on the Increment Waiver Payment using instances in which a waiver was provided and subordinated debt was outstanding at the time of the violation.

As displayed in the Table 6.3, the estimates of the coefficient on the subordinated debt spread from the probit regression of the indicator waiver variable is negative in both the base and subordinated spread only cases. The p-value of the coefficient in the base case is .0029; hence it is significant at a five percent confidence level. The coefficient value, -10.747, has a larger magnitude than any other coefficient in the base case. Thus, it appears that the correlation between the subordinated debt spread and the probability of a waiver is substantial relative to the other inputs. The sign of the estimate aligns with the prediction of a negative coefficient on the subordinated debt spread and, therefore, supports the hypothesis that the probability of a waiver is decreasing in the subordinated debt spread. Meanwhile, the coefficient in the linear regression of the waiver fee payment is

approximately zero and has an insignificant p-value in both the base case and the subordinated only case. No explicit prediction was made concerning the sign of the subordinated debt spread in the linear regression but this result aligns with the framework of the model – if a waiver is optimal for the bank and equity holder given set of subordinated debt characteristics, the waiver payment should be unaffected by these subordinated debt characteristics because they effect the bank and equity holder symmetrically.

One caveat to consider when interpreting the causal effect of the subordinated debt spread is the potential confounding effect of asset characteristics that are unaccounted for in the regression. Similar to the bank loan spread, the subordinated debt spread should display a positive correlation with asset characteristics that increase the risk of lending. Unlike, the bank loan spread, however, I hypothesize that the probability of a waiver is decreasing in the subordinated spread, and, hence, these confounding effects should reinforce the effect of a higher subordinated spread rather than negate it. Thus, it is difficult to determine what proportion of the estimated correlation is due to a causal relationship and what proportion is a product of these confounding effects.

These confounding effects can be observed by comparing the base case coefficient and the asset volatility only case coefficient from the probit regression. The coefficient estimate is reduced by over half (-10.747 v. -25.619) in the base case, implying that the control variables in the base case are correlated with both the probability of a waiver and the subordinated debt spread. Despite these concerns, the linear regression of the incremental waiver payment estimated a null value for the subordinated debt coefficient, however. This provides imperfect evidence that the effect of these confounding characteristics is limited because the incremental waiver payment should be positively correlated to these

confounding effects as well. A more thorough examination of this relationship is left for future research.

6.5 Auxiliary Variables

The industry indicator variables are included as control variables. There are no financial sector firms in the data set and hence I remove the financial indicator variable from the analysis. In the full sample probit regression, the materials, consumer staples and information technology variables are removed because they cause a complete separation of the data (i.e. they perfectly predict the indicator waiver variable). In all regressions, the coefficients on the industry indicator variables are insignificant. Hence, the data suggest that industry classification captures no relevant information about the waiver/foreclosure decision or the magnitude of the waiver payment when accounting for the primary variables of interest in my regression.

VII. Conclusion

This paper sheds light on the optimal creditor and equity holder decisions upon a covenant violation. I offer a model and evidence supportive of the hypotheses that debt composition, in particular the share of private debt, is a key determinant of the waiver/foreclosure decision, where as asset volatility is not.

My findings are novel. They suggest that previously examined ratios – such as total leverage and distance to default – exhibit a positive correlation with foreclosure not because they are predictive, as Chen and Wei (1993) conclude, but, instead, because they exhibit a confounding correlation with the share of private debt outstanding.

I do not claim that total debt nor asset volatility is irrelevant to the default threshold. Instead, my findings suggest they not causally related to creditor induced default when subordinated debt provides a cushion separating the equity from the private creditor. Even a preliminary inspection of the data demonstrates that equity holders strategically initiate default as well, however. That being said, my findings do provide additional evidence that private creditors play a primary role in inducing default. This re-emphasizes the need to reformulate the default boundary used in both the literature and in practical applications of credit risk assessment.

In closing, I provide several suggestions for extending this work. First, my empirical analysis could be further developed. Because each observation had to be hand constructed by manually reading SEC files, I could only compile a limited data set. Creating a larger data set could provide a more statistically significant estimate of the effect of bank debt share, however. Moreover, the aim of my empirical work was to test whether the data directionally support my hypotheses; hence I only test the data with probit and linear regressions. Testing for non-linear relationships – with respect to the incremental waiver payment, in particular –

could better characterize the magnitude of the effect of a marginal change in each parameter.

Second, my theoretical framework could be expanded to account for more nuanced dynamics of the waiver/foreclosure decision. Extending the framework of my model to treat the post-waiver default boundary as an endogenously determined variable would more accurately imitate the true dynamics of the waiver/foreclosure decision. Moreover, loosening the assumption of complete information could highlight additional determinants of foreclosure. Such an examination may also shed light on the determinants of the optimal method through which a creditor extracts waiver compensation from the debtor – namely, an increase in the interest rate or a one-time waiver fee.

Finally, investigating the implications of my findings on the optimal capital structure of a firm may prove fruitful. As I discuss in Section 3, I assume the capital structure of the firm to be exogenous at the time of the violation. The capital structure is endogenously selected at an earlier point in time, however. Faulkender and Petersen (2006) find that firms with high leverage substitute bond debt for private debt. Thus, a firm may optimally select its capital structure in part because of the role private creditors play in inducing default. Formulating a cogent framework that explains the average and cross sectional capital structure characteristics observed in the data is an open investigation in the theoretical literature.

Appendix – Derivations of the Propositions

Proof of Proposition 1

By definition, the breakeven equity waiver fee, $\mathbb{W}_E$, is the fee that sets the value of the equity post waiver payment equal to the value of the equity upon default, as specified in equation (3.1):

$$E([V - \mathbb{W}_E(V;\sigma)],\sigma) = B_E(V,\Psi) \tag{A.1}$$

Moreover, the value of the continuing equity, per equation (3.25), is:

$$E(V - \mathbb{W}_E) = (V - \mathbb{W}_E) - G(V - \mathbb{W}_E) - F(V - \mathbb{W}_E) - \mathbb{C}(V - \mathbb{W}_E) \tag{A.2}$$

Let $V^* = V - \mathbb{W}_E$ for ease of notation. Then, integrating equations (A.1) and (A.2), I can write $\mathbb{W}_E$ as:

$$\mathbb{W}_E = V - G(V^*) - F(V^*) - \mathbb{C}(V^*) - B_E(V,\Psi) \tag{A.3}$$

Thus the limit of the waiver fee as $\sigma \to \infty$ can be expressed as:

$$\lim_{\sigma\to\infty} \mathbb{W}_E = \lim_{\sigma\to\infty} [V - G(V^*) - F(V^*) - \mathbb{C}(V^*) - B_E(V,\Psi)] \tag{A.4}$$

Because B_E does not depend on volatility, its limit is simply its identity:

$$\lim_{\sigma\to\infty} B_E(V,\Psi) = B_E(V,\Psi) = \max\,(\,\mathrm{V}\Psi - 1{,}0) \tag{A.5}$$

The limit of the continuing loan claim is:

$$\lim_{\sigma\to\infty} F(V^*;\,\sigma) = \lim_{\sigma\to\infty} \left(\min\,(\ell,\Psi\bar{V}) - \frac{c\ell}{r}\right) \frac{(\frac{2C}{V^*\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V^*\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} + \frac{c\ell}{r}$$

$$\lim_{\sigma\to\infty} F(V^*;\,\sigma) = \min\,(\ell,\Psi\bar{V}) \tag{A.6}$$

where the second expression follows from the first because the term containing the confluent hypergeometric function converges to 1.

Similarly, the limit of the subordinated claim:

$$\lim_{\sigma\to\infty} G(V^*;\ \sigma) = \lim_{\sigma\to\infty}\left[\left(\min(1-\ell, \max(0, \Psi\bar{V}-\ell)) - \frac{\mu(1-\ell)}{r}\right)\right.$$
$$\left.\times\frac{(\frac{2C}{V\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} + \frac{\mu(1-\ell)}{r}\right]$$

$$\lim_{\sigma\to\infty} G(V^*;\ \sigma) = \min(1-\ell, \max\,(0, \Psi\bar{V}-\ell)) \tag{A.7}$$

and:

$$\lim_{\sigma\to\infty} \mathbb{C}(V^*;\ \sigma) = \lim_{\sigma\to\infty}\left[(1-\Psi)\bar{V}\,\frac{\frac{2C}{V\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{V\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)}\right]$$

$$\lim_{\sigma\to\infty} \mathbb{C}(V^*;\ \sigma) = (1-\Psi)\bar{V} \tag{A.8}$$

Substituting equation (A.5), (A.6), (A.7) and (A.8) back into equation (A.4), I conclude:

$$\lim_{\sigma\to\infty} \mathbb{W}_E(\sigma) = \mathrm{V} - \min(1-\ell, \max\,(0, \Psi\bar{V}-\ell)) - \min(\ell, \Psi\bar{V}) - (1-\Psi)\bar{V} - \max(\mathrm{V}\Psi - 1, 0) \tag{A.9}$$

Proof of Proposition 2

From equation (4.2) (and the definition of the breakeven bank waiver fee):

$$F([V - \mathbb{W}_B(V,\sigma)], \sigma) + \mathbb{W}_B(V,\sigma) = B_\ell(V, \Psi) \tag{A.10}$$

Taking the limit of both sides and integrating the conclusions from equation (A.6):

$$\min\,(\ell, \Psi\bar{V}) + \lim_{\sigma\to\infty} \mathbb{W}_B(V,\sigma) = \min\,(\ell, \Psi V) \tag{A.11}$$

Thus, solving for the limit of the breakeven bank waiver fee:

$$\lim_{\sigma\to\infty} \mathbb{W}_B(\sigma) = \min\,(\ell, \Psi V) - \min\,(\ell, \Psi\bar{V}) \tag{A.12}$$

Proof of Proposition 3

Reiterating equation (A.10):

$$F([V - \mathbb{W}_B(V,\Psi)],\Psi) + \mathbb{W}_B(V,\Psi) = B_\ell(V,\Psi) \tag{A.13}$$

When $\Psi = 0$, the value of the claim upon foreclosure is 0 and, hence equation (A.13) reduces to:

$$\frac{c\ell}{r}\left(1 - \frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)}\right) + \mathbb{W}_B(V,\Psi) = 0$$

$$-\mathbb{W}_B(V,\Psi) = \frac{c\ell}{r}\left(1 - \frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)}\right) \tag{A.14}$$

Consider the right-hand term in equation (A.14). It must be greater than or equal to zero because, firstly, it is equal to zero when $V = \bar{V}$, secondly, it is monotonically increasing in V and, thirdly, the value of $\bar{V}$ is specified to be strictly less than V at the time of a waiver (see section 3.5).

Thus, I can conclude:

$$\mathbb{W}_B(\Psi = 0) < 0 \tag{A.15}$$

Proof of Proposition 4

Reiterating equation (A.1), following from the definition of the breakeven equity waiver:

$$E([V_\omega - \mathbb{W}_E(V,\Psi)],\Psi) = B_E(V,\Psi) \tag{A.16}$$

Because the asset value approaches 1 as $\Psi \to \frac{1}{V}$, the equity holder is guaranteed zero recovery:

$$\lim_{\Psi\to\frac{1}{V}} B_E(V,\Psi) = \lim_{\Psi\to\frac{1}{V}} \max(V\Psi - 1, 0) = \max(1-1, 0) = 0 \tag{A.17}$$

Thus, the breakeven equity waiver as $\Psi \to \frac{1}{V}$ is the value of the waiver fee that renders the continuing equity claim worthless. Algebraically:

$$\lim_{\Psi \to \frac{1}{V}} E([V_\omega - \mathbb{W}_E(V,\Psi)],\Psi) = \lim_{\Psi \to \frac{1}{V}} B_E(V,\Psi) = 0 \tag{A.18}$$

The value of the equity is positive for all values of $V > \bar{V}$. Hence, I test $V = \bar{V}$:

$$E(\bar{V}) = (\bar{V}) - G(\bar{V}) - F(\bar{V}) - \mathbb{C}(\bar{V})$$

$$E(\bar{V}) = \bar{V} - \min(\ell, \Psi\bar{V}) - \min(1-\ell, \max(0, \Psi\bar{V} - \ell)) - (1-\Psi)\bar{V} \tag{A.19}$$

Which simplifies to:

$$E(\bar{V}) = 0 \text{ for } \Psi\bar{V} < 1 \tag{A.20a}$$

$$E(\bar{V}) = \Psi\bar{V} - 1 \text{ for } \Psi\bar{V} > 1 \tag{A.20b}$$

Thus, as $\Psi \to \frac{1}{V}$, the breakeven equity waiver must approach a magnitude that ensures immediate default. Because $\Psi\bar{V} \to \frac{\bar{V}}{V} < 1$ as $\Psi \to \frac{1}{V}$, I can eliminate the case presented in equation (A.20b). I conclude:

$$\lim_{\Psi \to \frac{1}{V}} \mathbb{W}_E(V,\Psi) = V - \bar{V} \tag{A.21}$$

Proof of Proposition 5

Rearranging equation (A.10):

$$\mathbb{W}_B(V,\Psi) = B_\ell(V,\Psi) - F([V - \mathbb{W}_B(V,\Psi)],\Psi) \tag{A.22}$$

Taking the derivative and expanding the second left-hand side term:

$$\frac{d\mathbb{W}_B}{d\Psi} = \frac{d}{d\Psi} B_\ell(V,\Psi) - \frac{d}{d\Psi}\left(\left(\min(\ell, \Psi\bar{V}) - \frac{c\ell}{r}\right)\frac{\left(\frac{2C}{(V-\mathbb{W}_B)\sigma^2}\right)^{\Omega} * {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{\left(\frac{2C}{\bar{V}\sigma^2}\right)^{\Omega} * {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} + \frac{c\ell}{r}\right) \tag{A.23}$$

Expanding the derivative of the second left-hand term in equation (A.23):

$$\left(\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})\right)\frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)}+$$

$$\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)\left[-(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega-1}\left(\frac{2\Omega C(V-\mathbb{W}_B)^{-2}}{\sigma^2}\right)\frac{d\mathbb{W}_B}{d\Psi}\frac{{}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)}\right. \tag{A.24}$$

$$\left.-\ {}_1F_1\left(\Omega+1;2+2\beta;\frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)\left(\frac{2\Omega C(V-\mathbb{W}_B)^{-2}}{(1+2\beta)\sigma^2}\right)\frac{d\mathbb{W}_B}{d\Psi}\frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega}}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)}\right]$$

For ease of notation, I refer to the first of the two expressions in (A.24) as Ξ and remainder of the second expression after $\frac{d\mathbb{W}_B}{d\Psi}$ is factored out as Θ. Substituting (A.24) back into equation (A.23) after factoring out $\frac{d\mathbb{W}_B}{d\Psi}$:

$$\frac{d\mathbb{W}_B}{d\Psi}=\frac{d}{d\Psi}B_\ell(V,\Psi)-(\Xi+\frac{d\mathbb{W}_B}{d\Psi}\ \Theta\ ) \tag{A.25}$$

which, after rearranging, becomes:

$$\frac{d\mathbb{W}_B}{d\Psi}=\frac{\frac{d}{d\Psi}B_\ell(V,\Psi)-\Xi}{1+\Theta} \tag{A.26}$$

I consider Θ. Each compound fraction within the bracket in expression (A.19) – namely $(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega-1}$, $\left(\frac{2\Omega C(V-\mathbb{W}_B)^{-2}}{\sigma^2}\right)$, $(\frac{2C}{\bar{V}\sigma^2})^{\Omega}$, $\left(\frac{2\Omega C(V-\mathbb{W}_B)^{-2}}{(1+2\beta)\sigma^2}\right)$ and $(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega}$ – must be positive because every variable within these fractions is positive by construction and because $V>\mathbb{W}_B$ by construction. Thus, the analog of the bracketed term in Θ (i.e. the bracketed term in (A.19) after the removal of $\frac{d\mathbb{W}_B}{d\Psi}$), is definitively negative. Moreover, $\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)$ is definitively negative, because, by construction, $c>r$ (see Section 3.1). I conclude that $\Theta>0$ and that the denominator of (A.26) is definitively positive.

Next, I consider the numerator. When $V\Psi > \ell$, $B_\ell = \ell$ and, hence, $\frac{d}{d\Psi}B_\ell = 0$. When $V\Psi < \ell$, $B_\ell = V\Psi$ and, hence, $\frac{d}{d\Psi}B_\ell = V$. Similarly, when $\bar{V}\Psi > \ell$, $\Xi = 0$. When, $\bar{V}\Psi < \ell$, $\Xi = \bar{V}\frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega_*}\,{}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega_*}\,{}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)}$. By construction, $V - \mathbb{W}_B > \bar{V}$ (see Section 3.4), and, hence $\frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega_*}\,{}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega_*}\,{}_1F_1\left(\Omega;1+2\beta;\frac{-2C}{\bar{V}\sigma^2}\right)} < 1$ (see proof of proposition 3). Thus, $\frac{d}{d\Psi}B_\ell > \frac{d}{d\Psi}\Xi$ when both values are non-zero.

Finally, using these conclusions, I can outline the sign of the derivative with respect the cost of default:

$$\mathbb{W}_B{}'(\Psi) > 0 \;\; if \;\; \frac{\ell}{V} < \Psi \tag{A.27a}$$

$$\mathbb{W}_B{}'(\Psi) < 0 \;\; if \;\; \frac{\ell}{\bar{V}} > \Psi > \frac{\ell}{V} \tag{A.27b}$$

$$\mathbb{W}_B{}'(\Psi) = 0 \;\; if \;\; \Psi > \frac{\ell}{\bar{V}} \tag{A.27c}$$

Proof of Proposition 6

From the definition of the breakeven equity waiver fee:

$$V - \mathbb{W}_E - G(V - \mathbb{W}_E) - F(V - \mathbb{W}_E) - \mathbb{C}(V - \mathbb{W}_E) = B_E(V, \Psi) \tag{A.28}$$

Rearranging to solve for $\mathbb{W}_E$ as an implicit function:

$$\mathbb{W}_E = V - G(V - \mathbb{W}_E) - F(V - \mathbb{W}_E) - \mathbb{C}(V - \mathbb{W}_E) - B_E(V, \Psi) \tag{A.29}$$

From equation (A.24), I know:

$$\frac{1}{d\Psi}F(V - \mathbb{W}_E) = \Xi + \frac{d\mathbb{W}_E}{d\Psi}\,\Theta \tag{A.30}$$

where Ξ and Θ are used to denote the same expressions they denote in Proof 5. Similarly, it is easily shown:

$$\frac{1}{d\Psi}G(V-\mathbb{W}_E)=$$

$$\frac{\frac{d}{d\Psi}(\min(1-\ell,\max(0,\Psi\bar{V}-\ell)))\Xi}{\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})}+\frac{d\mathbb{W}_E}{d\Psi}\frac{\left(\min(1-\ell,\max(0,\Psi\bar{V}-\ell))-\frac{\mu(1-\ell)}{r}\right)\Theta}{\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)} \tag{A.31}$$

$$\frac{1}{d\Psi}\mathbb{C}(V-\mathbb{W}_E)=\frac{-\bar{V}\Xi}{\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})}+\frac{d\mathbb{W}_E}{d\Psi}\frac{(1-\Psi)\bar{V}\Theta}{\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)} \tag{A.32}$$

$$\frac{1}{d\Psi}B_E(V,\Psi)=\frac{1}{d\Psi}\max\left(V\Psi-1,0\right) \tag{A.33}$$

Thus:

$$\begin{aligned}\frac{d\mathbb{W}_E}{d\Psi}=&-\frac{d\mathbb{W}_E}{d\Psi}\left(\frac{(1-\Psi)\bar{V}\Theta}{\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)}+\frac{\left(\min(1-\ell,\max(0,\Psi\bar{V}-\ell))-\frac{\mu(1-\ell)}{r}\right)\Theta}{\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)}+\Theta\right)\\&-\left(\Xi+\frac{\frac{d}{d\Psi}(\min(1-\ell,\max(0,\Psi\bar{V}-\ell)))\Xi}{\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})}+\frac{-\bar{V}\Xi}{\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})}\right.\\&\left.+\frac{d}{d\Psi}\max(V\Psi-1,0)\right)\end{aligned} \tag{A.34}$$

And solving for $\frac{d\mathbb{W}_E}{d\Psi}$ explicitly:

$$\frac{d\mathbb{W}_E}{d\Psi}=\frac{-\left(\Xi+\frac{\frac{d}{d\Psi}(\min(1-\ell,\max(0,\Psi\bar{V}-\ell)))\Xi}{\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})}+\frac{-\bar{V}\Xi}{\frac{d}{d\Psi}\min(\ell,\Psi\bar{V})}+\frac{d}{d\Psi}\max(V\Psi-1,0)\right)}{\left(1+\frac{(1-\Psi)\bar{V}\Theta}{\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)}+\frac{\left(\min(1-\ell,\max(0,\Psi\bar{V}-\ell))-\frac{\mu(1-\ell)}{r}\right)\Theta}{\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)}+\Theta\right)} \tag{A.35}$$

I consider the denominator. Θ is positive, as I demonstrated in Proof 5. The second rightmost compound fraction is also definitively positive because $\left(\min(1-\ell,\max(0,\Psi\bar{V}-\ell))-\frac{\mu(1-\ell)}{r}\right)<0$. The rightmost compound fraction, however, is negative for $\Psi<1$ and is monotonically increasing in Ψ. It is strictly less than 1, however, for all $(1-\Psi)\bar{V}<1$ (i.e.

instances in which the default costs are less than the total principal outstanding - which holds for economically rationale cases). Thus, the denominator is positive for reasonable values of $\bar{V}$ and Ψ.

I consider the numerator. When $V\Psi > 1$, $\frac{d}{d\Psi}\max(V\Psi - 1, 0) = V$ and zero otherwise.

I constrain, $\bar{V}\Psi < \ell$. Then:

$$\Xi = \bar{V}\frac{(\frac{2C}{(V - \mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{(V - \mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{A.36a}$$

$$\frac{\frac{d}{d\Psi}(\min(1 - \ell, \max(0, \Psi\bar{V} - \ell)))\Xi}{\frac{d}{d\Psi}\min(\ell, \Psi\bar{V})} = 0 \tag{A.36b}$$

$$\frac{-\bar{V}\Xi}{\frac{d}{d\Psi}\min(\ell, \Psi\bar{V})} = -\bar{V}\frac{(\frac{2C}{(V - \mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{(V - \mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{A.36c}$$

hence, given this constraint, the numerator is strictly negative when $V\Psi > 1$ and zero otherwise.

Likewise, I constrain, $1 > \bar{V}\Psi > \ell$:

$$\Xi = 0 \tag{A.37a}$$

$$\frac{\frac{d}{d\Psi}(\min(1 - \ell, \max(0, \Psi\bar{V} - \ell)))\Xi}{\frac{d}{d\Psi}\min(\ell, \Psi\bar{V})} = \bar{V}\frac{(\frac{2C}{(V - \mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{(V - \mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{A.37b}$$

$$\frac{-\bar{V}\Xi}{\frac{d}{d\Psi}\min(\ell, \Psi\bar{V})} = -\bar{V}\frac{(\frac{2C}{(V - \mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{(V - \mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{A.37c}$$

and, hence, the numerator is strictly negative when $V\Psi > 1$ and zero otherwise given this constraint.

Lastly, I constrain, $\bar{V}\Psi > 1$

$$\Xi = 0 \tag{A.38a}$$

$$\frac{\frac{d}{d\Psi}(\min(1-\ell, \max(0, \Psi\bar{V}-\ell)))\Xi}{\frac{d}{d\Psi}\min(\ell, \Psi\bar{V})} = 0 \tag{A.38b}$$

$$\frac{-\bar{V}\Xi}{\frac{d}{d\Psi}\min(\ell, \Psi\bar{V})} = -\bar{V}\frac{(\frac{2C}{(V-\mathbb{W}_B)\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2C}{\bar{V}\sigma^2})^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \tag{A.38c}$$

hence, because this constraint implies that $V\Psi > 1$, the numerator is strictly negative.

I conclude that there are two cases:

$$\mathbb{W}_E{}'(\Psi) < 0 \ if \ \Psi > 1/\mathrm{V} \tag{A.39a}$$

$$\mathbb{W}_E{}'(\Psi) = 0 \ if \ \Psi \leq 1/\mathrm{V} \tag{A.39b}$$

Proof of Proposition 7

Restating equation (A.22)

$$\mathbb{W}_B(V, \mu) = B_\ell(V, \Psi) - F([V - \mathbb{W}_B(V, \mu)], \mu) \tag{A.41}$$

Taking the limits of both sides:

$$\lim_{\mu\to\infty} \mathbb{W}_B = \lim_{\mu\to\infty} [B_\ell(V, \Psi) - F([V - \mathbb{W}_B(V, \mu)], \mu)] \tag{A.42}$$

Because B_ℓ does not depend on volatility, its limit is simply its identity:

$$\lim_{\mu\to\infty} B_\ell(V, \Psi) = B_\ell(V, \Psi) = \min\ (\mathrm{V}\Psi, \ell) \tag{A.43}$$

I consider the limit of the continuing bank claim:

$$\lim_{\mu\to\infty}\left[\left(\min(\ell,\Psi\bar{V})-\frac{c\ell}{r}\right)\frac{(\frac{2(c\ell-\mu(1-\ell)-\delta)}{(V-\mathbb{W}_B)\sigma^2})^{\Omega}\ {}_1F_1\left(\Omega;1+2\beta;\frac{-2(c\ell-\mu(1-\ell)-\delta)}{(V-\mathbb{W}_B)\sigma^2}\right)}{(\frac{2(c\ell-\mu(1-\ell)-\delta 0}{(\bar{V}-\mathbb{W}_B)\sigma^2})^{\Omega}\ {}_1F_1\left(\Omega;1+2\beta;\frac{-2(c\ell-\mu(1-\ell)-\delta)}{(\bar{V}-\mathbb{W}_B)\sigma^2}\right)}+\frac{c\ell}{r}\right] \tag{A.44}$$

where I write out C is a sum of its components to emphasize its dependence on μ. As $\mu\to\infty$, it denominates the difference between V and $\bar{V}$ in the compound fraction including the confluent hypergeoemtric equation. Thus:

$$\lim_{\mu\to\infty}[F([V-\mathbb{W}_B(V,\mu)],\mu)]=\min(\ell,\Psi\bar{V}) \tag{A.45}$$

I conclude:

$$\lim_{\mu\to\infty}\mathbb{W}_B(V,\mu)=\min(\ell,V\Psi)-\min(\ell,\Psi\bar{V}) \tag{A.46}$$

Proof of Proposition 8

Restating equation (A.29):

$$\mathbb{W}_E=V-G(V-\mathbb{W}_E)-F(V-\mathbb{W}_E)-\mathbb{C}(V-\mathbb{W}_E)-B_E(V,\Psi) \tag{A.47}$$

Let $V^*=V-\mathbb{W}_E$ for ease of notation. Taking the limit of both sides:

$$\lim_{\mu\to\infty}\mathbb{W}_E=\lim_{\mu\to\infty}[V-G(V^*)-F(V^*)-\mathbb{C}(V^*)-B_E(V,\Psi)] \tag{A.48}$$

Because the value in bankruptcy does not depend on the loan interest rate, the limit is its definition:

$$\lim_{\mu\to\infty}B_E(V,\Psi)=B_E(V,\Psi)=\max(V\Psi-1,0) \tag{A.49}$$

Per the discussion in Proof 7, I know:

$$\lim_{\mu\to\infty}F(V^*)=\min(\ell,\Psi\bar{V}) \tag{A.50}$$

Similarly:

$$\lim_{\mu\to\infty}\mathbb{C}(V^*)=(1-\Psi)\bar{V} \tag{A.51}$$

And:

$$\lim_{\mu\to\infty} G(V^*) = V - \bar{V} + \min(1-\ell, \max(0, \Psi\bar{V} - \ell)) \tag{A.52}$$

Plugging these expressions back into (A.48):

$$\lim_{\mu\to\infty} \mathbb{W}_E(\mu) = -\max(V\Psi - 1, 0) \tag{A.53}$$

Thus I conclude:

$$\lim_{\mu\to\infty} \mathbb{W}_E(\mu) = 1 - V\Psi \text{ if } V\Psi > 1 \tag{A.54a}$$

$$\lim_{\mu\to\infty} \mathbb{W}_E(\mu) = 0 \text{ if } V\Psi \leq 1 \tag{A.54b}$$

Proof of Proposition 9

Taking the derivative with respect to ℓ of both sides of equation (A.22)

$$\frac{d}{d\ell}\, \mathbb{W}_B(V,\ell) = \frac{d}{d\ell}(B_\ell - F(V - \mathbb{W}_B)) \tag{A.55}$$

I examine the terms on the right hand side seperately. Looking first at the derivative of the claim value in bankruptcy:

$$\frac{d}{d\ell} B_\ell = \frac{d}{d\ell} \min\,(V\Psi, \ell) \tag{A.56}$$

hence $\frac{dB_\ell}{d\ell} = 1$ if $V\Psi > \ell$ and $\frac{dB_\ell}{d\ell} = 0$ otherwise.

The derivative of continuing claim value is:

$$\begin{aligned}
\frac{d}{d\ell}F(V-\mathbb{W}_B) = {} & \left(\frac{d}{d\ell}\min(\ell,\Psi\bar{V}) - \frac{c}{r}\right)\frac{\left(\frac{2C}{(V-\mathbb{W}_B)\sigma^2}\right)^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{\left(\frac{2C}{\bar{V}\sigma^2}\right)^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} + \\
& \left(\min(\ell,\Psi\bar{V}) - \frac{c\ell}{r}\right)\Bigg[-\left(\frac{2C}{(V-\mathbb{W}_B)\sigma^2}\right)^{\Omega-1}\left(\frac{2\Omega C(V-\mathbb{W}_B)^{-2}}{\sigma^2}\right)\frac{d\mathbb{W}_B}{d\ell}\frac{{}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{\left(\frac{2C}{\bar{V}\sigma^2}\right)^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} \\
& - \ {}_1F_1\left(\Omega+1; 2+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)\left(\frac{2\Omega C(V-\mathbb{W}_B)^{-2}}{(1+2\beta)\sigma^2}\right)\frac{d\mathbb{W}_B}{d\ell}\frac{\left(\frac{2C}{(V-\mathbb{W}_B)\sigma^2}\right)^{\Omega}}{\left(\frac{2C}{\bar{V}\sigma^2}\right)^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)}\Bigg] + \frac{c}{r}
\end{aligned} \tag{A.57}$$

Denoting Θ in the same manner as used in in Proof 5 and denoting the rightmost term in equation (A.57) as γ, I can write:

$$\frac{d\,\mathbb{W}_B}{d\ell} = \frac{\frac{d}{d\ell}\min(V\Psi, \ell) - \gamma - \frac{c}{r}}{1 + \Theta} \tag{A.58}$$

As demonstrated in Proof 5, $\Theta > 0$, hence the denominator of (A.58) is positive.

Moreover, $\gamma < 0$ because $\frac{c}{r} > \frac{d}{d\ell}\min(\ell, \Psi\bar{V})$. The term γ is strictly greater than $-\frac{c}{r}$ but the sum $\frac{c}{r} + \gamma$ is less than 1 if:

$$\frac{c}{r} - \frac{c}{r}\frac{\left(\frac{\bar{V}}{(V - \mathbb{W}_B)}\right)^{\Omega} * \ {}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{(V - \mathbb{W}_B)\sigma^2}\right)}{{}_1F_1\left(\Omega; 1 + 2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)} < 1 \tag{A.59}$$

simplifying and relaxing this constraint, I can conclude that:

$$\frac{d}{d\ell}F(V - \mathbb{W}_B) < 1 \text{ when } V < \bar{V}\left(1 - \frac{r}{c}\right)^{-1/\Omega} \tag{A.60}$$

Which holds because $\frac{c}{r} - \frac{c}{r}\left(\frac{\bar{V}}{V}\right)^{\Omega} < \frac{c}{r} - \frac{c}{r}\frac{\left(\frac{\bar{V}}{(V-\mathbb{W}_B)}\right)^{\Omega} * \ {}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{(V-\mathbb{W}_B)\sigma^2}\right)}{{}_1F_1\left(\Omega; 1+2\beta; \frac{-2C}{\bar{V}\sigma^2}\right)}$

Incorporating these conclusions into (A.56), I conclude:

$$\frac{d\,\mathbb{W}_B}{d\ell} > 0 \text{ when } V < \bar{V}\left(1 - \frac{r}{c}\right)^{-1/\Omega} \text{ and } V\Psi > \ell \tag{A.61}$$

References

Allen, David, and Robert Powell. "Structural Credit Modelling and Its Relationship to Market Value at Risk: An Australian Sectoral Perspective." MPRA Paper. University Library of Munich, Germany, March 2008.

Anderson, R. W., and S. Sundaresan. "Design and Valuation of Debt Contracts." *Review of Financial Studies* 9, no. 1 (January 1, 1996): 37–68.

Baird, Douglas G., and Robert K. Rasmussen. "Private Debt and the Missing Lever of Corporate Governance." SSRN Scholarly Paper. Rochester, NY: Social Science Research Network, March 24, 2005.

Beneish, Messod D., and Eric Press. "Costs of Technical Violation of Accounting-Based Debt Covenants." *The Accounting Review* 68, no. 2 (April 1, 1993): 233–57.

———. "Interrelation Among Events of Default*." *Contemporary Accounting Research* 12, no. 1 (September 1, 1995): 57–84.

———. "The Resolution of Technical Default." *The Accounting Review* 70, no. 2 (April 1, 1995): 337–53.

Black, Fischer, and John C. Cox. "Valuing Corporate Securities: Some Effects of Bond Indenture Provisions." *The Journal of Finance* 31, no. 2 (May 1, 1976): 351–67.

Bourgeon, Jean-Marc, and Georges Dionne. "On Debt Service and Renegotiation When Debt-Holders Are More Strategic." *Journal of Financial Intermediation* 22, no. 3 (July 2013): 353–72.

Brennan, Michael J., and Eduardo S. Schwartz. "Finite Difference Methods and Jump Processes Arising in the Pricing of Contingent Claims: A Synthesis." *The Journal of Financial and Quantitative Analysis* 13, no. 3 (September 1, 1978): 461–74.

———. "Savings Bonds, Retractable Bonds and Callable Bonds." *Journal of Financial Economics* 5, no. 1 (1977): 67–88.

Chava, Sudheer, and Michael R. Roberts. "How Does Financing Impact Investment? The Role of Debt Covenants." *The Journal of Finance* 63, no. 5 (October 1, 2008): 2085–2121.

Chen, Kevin C. W., and K. C. John Wei. "Creditors' Decisions to Waive Violations of Accounting-Based Debt Covenants." *The Accounting Review* 68, no. 2 (April 1, 1993): 218–32.

Crosbie, Peter and John Bohn. "Modeling Default Risk - Modeling Methodology." Moody's KMV Company: 10-30. 2003.

DeFond, Mark L., and James Jiambalvo. "Debt Covenant Violation and Manipulation of Accruals." *Journal of Accounting and Economics* 17, no. 1–2 (January 1994): 145–76.

Denis, David J., and Jing Wang. "Debt Covenant Renegotiations and Creditor Control Rights." *Journal of Financial Economics* 113, no. 3 (September 2014): 348–67.

Dichev, Ilia D., and Douglas J. Skinner. "Large–Sample Evidence on the Debt Covenant Hypothesis." *Journal of Accounting Research* 40, no. 4 (September 1, 2002): 1091–1123.

Faulkender, Michael, and Mitchell A. Petersen. "Does the Source of Capital Affect Capital Structure?" *Review of Financial Studies* 19, no. 1 (March 20, 2006): 45–79.

Gârleanu, Nicolae, and Jeffrey Zwiebel. "Design and Renegotiation of Debt Covenants." *Review of Financial Studies* 22, no. 2 (February 1, 2009): 749–81.

Gilson, S. C., and M. R. Vetsuypens. "Creditor Control in Financially Distressed Firms: The Empirical Evidence," January 1, 1995.

Kahan, Marcel, and Bruce Tuckman. "Private vs. Public Lending: Evidence from Covenants.", July 1, 1993.

Leland, Hayne E. "Corporate Debt Value, Bond Covenants, and Optimal Capital Structure." *The Journal of Finance* 49, no. 4 (September 1, 1994): 1213–52.

Mark S. Carey, Michael B. Gordy. "The Bank as Grim Reaper: Debt Composition and Recoveries on Defaulted Debt." Fed Working Paper, May (2007): 336–54.

Merton, Robert C. "On the Pricing of Corporate Debt: The Risk Structure of Interest Rates*." *The Journal of Finance* 29, no. 2 (May 1, 1974): 449–70.

Nini, Greg, David C. Smith, and Amir Sufi. "Creditor Control Rights and Firm Investment Policy." *Journal of Financial Economics* 92, no. 3 (June 2009): 400–420.

———. "Creditor Control Rights, Corporate Governance, and Firm Value." *Review of Financial Studies* 25, no. 6 (June 1, 2012): 1713–61.

Roberts, Michael R. "The Role of Dynamic Renegotiation and Asymmetric Information in Financial Contracting." NBER Working Paper. National Bureau of Economic Research, Inc, 2014.

Roberts, Michael R., and Amir Sufi. "Control Rights and Capital Structure: An Empirical Investigation." *The Journal of Finance* 64, no. 4 (August 1, 2009): 1657–95

Sansone, Allison and Alicia Taylor, 2007. *The Handbook of Loan Syndications and Trading*. New York: McGraw-Hill, 2006.

Saunders, Anthony, Sascha Steffen, Felix Freudenberg, and Bjorn Imbierowicz. "Covenant Violations, Loan Contracting, and Default Risk of Bank Borrowers." NYU Working Paper, Social Science Research Network, January 1, 2012.

Schaefer, Stephen M. "Conditional Predictions of Bond Prices and Returns: Discussion." *The Journal of Finance* 35, no. 2 (May 1, 1980): 417–19.

Sweeney, Amy Patricia. "Debt-Covenant Violations and Managers' Accounting Responses." *Journal of Accounting and Economics* 17, no. 3 (1994): 281–308.

Zhang, Zhipeng. "Who Pulls the Plug? Theory and Evidence on Corporate Bankruptcy Decisions." MPRA Paper, June 1, 2009.