JOURNAL OF URBAN ECONOMICS 9, 190–211 (1981)

Capital–Land Substitution in Urban Housing: A Survey of Empirical Estimates

JOHN F. MCDONALD[1]

Department of Economics, University of Illinois at Chicago Circle, Chicago, Illinois 60680

Received May 10, 1979; revised February 25, 1980

Recent empirical studies of capital–land substitution in urban housing are examined to determine the best estimate of the elasticity of substitution parameter σ. Studies based upon a cross section of metropolitan areas produce a rather narrow range for σ. Studies of individual metropolitan areas produce a wide range of estimates for σ, suggesting that σ may vary across metropolitan areas. However, all estimates of σ are probably biased toward zero by errors in the measurement of land values.

1. INTRODUCTION

The elasticity of capital–land substitution in urban housing is a key parameter in the explanation of the spatial structure of the urban housing market in the long run. The elasticity of substitution, designated σ throughout this paper, is a determinant of the land rent gradient, the population density gradient, the factor shares of land and housing capital, and the elasticity of supply of housing both in the aggregate and on a particular site. Muth [19, 21] and Kau and Lee [14] have explored these implications rather fully. The purpose of this paper is to review the empirical studies of capital–land substitution in order to determine whether a consensus estimate exists and to make suggestions for further research. The first preliminary estimate of σ was computed by Muth [19], and the first detailed empirical study was also conducted by Muth [22]. Koenker [17] also conducted an empirical study at approximately the same time. No research followed immediately after the appearance of these two studies but interest in the topic has been rekindled by the appearance of the study by Rydell [28] which employed data from the housing supply experiment in Green Bay. I have found nine more studies which appeared in 1976 or later years, excluding the study by Smith [31] that is based upon a different model of housing and the study of capital–land substitution in office buildings by Clapp [7]. It is appropriate now to look at the results of this

[1] I thank Dennis Carlton and George Tolley for pointing out an error in an earlier version of the paper. Also, the referee made a number of helpful suggestions which improved the paper. I extend my thanks.

0094-1190/81/020190-22$02.00/0

burst of research activity. What is the economic model upon which these studies are based? What are the econometric problems which researchers in this area must face? Who has solved these problems?

The plan of the paper is as follows. In the next section the basic model of the urban housing market is reviewed to place the empirical studies in the proper context. The examination of the basic model uncovers some additional hypotheses that have not been tested. The next section contains a detailed examination of the testable implications of the basic model. The fourth section discusses the specification of econometric models and presents the results of the empirical studies. The next section looks into the problem of measurement error bias which stems from the fact that land value must be estimated before an empirical estimate of σ can be obtained. A brief summary concludes the paper.

2. THE BASIC MODEL

The basic model of the urban housing market, as developed and presented by Muth [21, 22] assumes perfectly competitive input and output markets. All actual or potential housing producers have identical production functions in which the capacity to produce housing services, a stock variable, is produced by the stocks of land and capital. Note that this use of stock variables is in contrast to studies of manufacturing which use flow variables. All of the studies examined in this paper use data on stocks.[2] Each producer operates in a region where its production function is homogeneous of degree one. The production function for the firm can be written

$$H = H(K, L), \tag{1}$$

where H is the capacity to provide a flow of housing services, K is the stock of capital, and L is the stock of land. Now consider housing production on a fixed area of land with exogenously fixed accessibility and other amenities. Assume that the land market contains many sites with identical characteristics, so that competition in the land market fixes the land value of the site in question. The price of capital is also exogenous to the site and the producer (or producers) who occupy the site. Given that input prices and the quantity of land are fixed, only one level of H is consistent with long-run equilibrium. This output can be produced and sold because competition in the output market establishes the price of the capacity to produce housing services (given the accessibility and other amenities of the site).

[2] Koenker [17] examined the relationship between property value and gross rent for a sample of 189 properties. Since the R^2 between the two variables was found to be 0.96, he abandoned the attempt to convert the data to flows.

In Muth's standard model, households are assumed to have a utility function which can be written

$$U = U(X, H), \tag{2}$$

where X is the composite commodity, defined as dollars spent on all other goods (including leisure). From (1), the utility function may be written

$$U = U(X, H(K, L)). \tag{2'}$$

Equation (2') makes it clear that Muth's (21) model can be interpreted as a weakly separable utility function rather than as a model of the production of housing. We can imagine that households purchase goods, X, K, and L, but that the choices of K and L are separable from the choice of X in the sense that the marginal rate of substitution of K for L is independent of the quantity of X consumed. An estimate of the elasticity of substitution of K for L can thus either be thought of as an estimate of a parameter of a production function or as an estimate of a parameter of the weakly separable utility function. As shown by Berndt and Christensen [3], the assumption of weak separability implies that the Allen partial elasticities of substitution of X for K and X for L are equal. A test of this hypothesis would be a test of Muth's model, but no one has conducted such a test. Weak separability, combined with the assumption that $H(K, L)$ is homogeneous of degree one, implies that the income elasticities of demand for K and L are equal. (See Muth [20]). Only Muth [22] has tested this hypothesis, and found that the hypothesis cannot be rejected.

Further reflection upon (2) suggests, however, that Muth's formulation may be too restrictive, as Muth [21, p. 18] himself recognized. Arnott [1, p. 296] has stated that

> It should be noted that although H is called a production function, it is really an aggregator reflecting tastes.

If this is accepted, then the form of the function $H = H(K, L)$ possibly varies across households. No one has tested for such a possibility. More generally, it may be reasonable to assume that $H = H(K, L)$ includes both technological and taste factors. For example, suppose that land is a factor in the production of the capacity to produce housing services *and* a good in its own right. The utility function now may be written

$$U = U(X, H(K, L), L). \tag{3}$$

Assume that the marginal rate of substitution of H for L is independent of X, or that U is weakly separable. Now the elasticity of substitution of K for

L, written

$$\sigma = d\ln(K/L)/d\ln(R/r), \tag{4}$$

where R = the value of land and r = the price of capital, is a combination of the elasticity of substitution of K for L in the production of housing and the elasticity of substitution of H for L in the utility function. Clearly in this model σ can vary across households even if the function $H(K, L)$ is the same for all households. Also, while it is reasonable to assume that the function $H(K, L)$ is the same within a metropolitan area, the form of the production function may vary across metropolitan areas because of climatic differences, for example. In a warmer climate interior space and exterior space are probably closer substitutes in production than in colder climates. Also, more K is required per unit of interior space in colder climates. Furthermore, the demand for land as a separate good may be lower in colder climates. While David [8], Kain and Quigley [13], King [15], and Straszheim [32] have estimated demand functions for various attributes of residential real estate, no one has estimated the parameters of the specific models suggested here. The hypotheses that σ may vary across metropolitan areas or across households within a metropolitan area are topics for possible future research.

3. IMPLICATIONS OF THE BASIC MODEL

The empirical work described below is based upon (2′) in that the capacity to produce housing services is assumed to be a commodity that is produced by capital and land and that capital and land do not contribute to utility in a manner that is separate from their contributions to housing output. We explore the implications of this basic model in this section.

The basic implication of the model is that the intensity of land use (measured in various ways) is a function of the prices of the services of capital and land. The simplest versions of the model assume that $H(K, L)$ is a constant elasticity of substitution (CES) production function with constant returns to scale.[3] A variety of implications can be derived simply from the definition of the elasticity of substitution σ in (4). If we write

$$\ln(K/L) = c + \sigma\ln(R/r), \tag{5}$$

we can immediately derive

$$\ln(rK/L) = c + \sigma\ln R + (1-\sigma)\ln r \tag{6}$$

[3]The CES production function is written $H = \gamma[\alpha L^{\rho} + (1-\alpha)K^{\rho}]^{1/\rho}$ where $\sigma = 1/(1-\rho)$. As Nerlove [23] has shown, the first-order conditions for profit maximization give $H/L = \gamma^{-\rho/\sigma}\alpha^{-\sigma}(R/p)^{\sigma}$ and $H/K = \gamma^{-\rho/\sigma}(1-\alpha)^{-\sigma}(R/p)^{\sigma}$.

and

$$\ln(rK/RL) = c + (1-\sigma)\ln r - (1-\sigma)\ln R. \tag{7}$$

These are the two equations that are most widely used in empirical work. Muth [21, pp. 54–55] has also shown that

$$d\ln(pH/L) = (\rho_L + \rho_K\sigma)\, d\ln R, \tag{8}$$

where ρ_L and ρ_K are the shares of land and capital, respectively ($\rho_L + \rho_K = 1$).[4] Also, Muth [19] has derived the equation[5]

$$d\ln(rK/pH) = (1-\sigma)\rho_L\, d\ln(r/R). \tag{9}$$

While most studies have been based upon the CES production function, two recent studies by Sirmans and Redman [3] and Sirmans et al. [29] have assumed a variable elasticity of substitution (VES) production function and Rosen [27] has used a translog cost function for housing services. These two more flexible functional forms are described in detail.

There are many functional forms for a VES production function, but Sirmans and Redman [30] and Sirmans et al. [29] have chosen the function introduced by Revankar [26] because of the relative simplicity of the statistical test used to distinguish the VES from the CES form. Revankar's VES function is written

$$H = \gamma L^{\alpha(1-\delta\rho)}[K + (\rho - 1)L]^{\alpha\delta\rho}, \tag{10}$$

where α, δ, ρ and γ are parameters.[6] This function includes the Cobb–Douglas function ($\rho = 1$) as a special case. The returns-to-scale parameter is α. The marginal product of capital is

$$\frac{\partial H}{\partial K} = \alpha\delta\rho\frac{H}{K + (\rho - 1)L}. \tag{11}$$

[4]Muth's derivation starts by assuming $pH = RL + rK$. Dividing both sides by L, taking natural logs, and differentiating with respect to R produces $d\ln(pH/L) = \rho_L d\ln R + \rho_K d\ln(K/L)$. From the definition of the elasticity of substitution, $d\ln(pH/L) = (\rho_L + \rho_K\sigma) d\ln R$.

[5]This result can be shown easily from (7). If $\ln(rK/RL) = c + (1-\sigma)\ln(r/R)$, then $\ln(rK/pH) = c + (1-\sigma)\ln(r/R) + \ln RL - \ln pH$, or (assuming that $\ln\rho_L$ is approximately equal to ρ_L) $\ln(rK/pH) = c + \rho_L(1-\sigma)\ln(r/R)$.

[6]The parameter values satisfy the following constraints: $\gamma > 0$, $\alpha > 0$, $0 < \delta < 1$, $0 \le \delta\rho \le 1$, and $K/L > (1 - \rho/1 - \delta\rho)$.

The marginal product of land is

$$\frac{\partial H}{\partial L} = \alpha(1 - \delta\rho)\frac{H}{L} + \alpha\delta\rho(\rho - 1)\frac{H}{K + (\rho - 1)L}. \tag{12}$$

Assuming constant returns to scale ($\alpha = 1$) and perfectly competitive input markets, we can write

$$r = p\delta\rho\frac{H}{Z} \tag{13}$$

and

$$R = p(1 - \delta\rho)\frac{H}{L} + p\delta\rho(\rho - 1)\frac{H}{Z}, \tag{14}$$

where $Z = K + (\rho - 1)L$. Calculation of R/r and rearranging terms produces

$$\frac{K}{L} = \frac{1 - \rho}{1 - \delta\rho} + \frac{\delta\rho}{1 - \delta\rho}(R/r). \tag{15}$$

A number of empirically testable implications can be drawn from (15). For example, multiplying by r produces

$$\frac{rK}{L} = \frac{(1 - \rho)}{1 - \delta\rho}r + \frac{\delta\rho}{1 - \delta\rho}R. \tag{16}$$

Also, dividing (16) by R yields

$$\frac{rK}{RL} = \frac{\delta\rho}{1 - \delta\rho} + \frac{(1 - \rho)}{1 - \delta\rho}\frac{r}{R}, \tag{17}$$

and adding RL/RL to both sides of (16) yields

$$\frac{pH}{RL} = 1 + \frac{(1 - \rho)}{1 - \delta\rho}r + \frac{\delta\rho}{1 - \delta\rho}R. \tag{18}$$

The elasticity of substitution can easily be derived from (15) by noting that

$$\sigma = \frac{d(K/L)}{d(R/r)}\frac{(R/r)}{(K/L)} = \frac{\delta\rho}{1 - \delta\rho}\frac{(R/r)}{(K/L)} \tag{19}$$

and substituting for R/r from (13) and (14). Simplification of (19) produces

$$\sigma = 1 + \left(\frac{\rho - 1}{1 - \delta\rho} \right) \frac{L}{K}. \tag{20}$$

Here σ varies with L/K. This result means that the elasticity of substitution is embedded in (15)–(18). For example, if the coefficient of r in (18) is A, then

$$\sigma = 1 - A\frac{L}{K}. \tag{21}$$

One limitation of Revankar's VES function is that $\sigma - 1$ is either nonnegative for all values of L/K or nonpositive for all values of L/K depending upon whether $A \leq 0$ or $A \geq 0$.

An alternative to the VES production function approach is to use the results of modern duality theory. (See Diewert [10] for a survey of this literature.) This approach makes use of the duality between production functions and unit cost functions. For example, it can be shown that:

1. If a production function is homogeneous of degree one and concave, then there exists a unit cost function with the same properties.
2. If a unit cost function is homogeneous of degree one (in input prices) and concave, then there exists a production function with the same properties.
3. Finally, the production function derived from the unit cost function assumed above implies the original unit cost function.

These results mean that the parameters of the production function, such as the elasticity of substitution, can be derived from the parameters of the unit cost function.

The study by Rosen [27] is based upon the translog cost function for the capacity to produce housing, written

$$\begin{aligned} \ln Z = {} & \alpha_0 + \alpha_K \ln r + \alpha_L \ln R + \frac{1}{2}\delta_{KK}(\ln r)^2 \\ & + \delta_{KL} \ln r \ln R + \frac{1}{2}\delta_{LL}(\ln R)^2, \end{aligned} \tag{22}$$

where Z is the unit cost of housing capacity. The usual restrictions placed upon the translog cost function are

$$\begin{aligned} \alpha_K + \alpha_L &= 1, \\ \delta_{KL} &= \delta_{LK}, \\ \delta_{KK} + \delta_{KL} + \delta_{LK} + \delta_{LL} &= 0, \end{aligned} \tag{23}$$

and

$$\delta_{KL} = -\delta_{KK} = -\delta_{LL}.$$

Equation (22) is homogeneous of degree one and concave if these restrictions (23) hold.[7] Differentiating (22) with respect to $\ln R$ produces

$$\frac{\partial Z}{\partial R}\frac{R}{Z} = \alpha_L + \delta_{KL}\ln r + \delta_{LL}\ln R. \tag{24}$$

Since Shepard's lemma implies $\partial Z/\partial R = L/H$, (24) becomes

$$\rho_L = \alpha_L + \delta_{KL}\ln r + \delta_L\delta \ln R = \alpha_L + \delta_{KL}(\ln r/\ln R). \tag{25}$$

Likewise

$$\rho_K = \alpha_K + \delta_{KL}\ln \delta + \delta_{KK}\ln r = \alpha_K + \delta_{KL}(\ln R/\ln r). \tag{26}$$

Thus by (23), it is necessary only to estimate either (25) or (26) to obtain an estimate of the unit cost function (22), except for the constant term α_0. Uzawa [34] has shown that the elasticity of substitution can be derived from the unit cost function as

$$\sigma - 1 = \frac{\partial^2 \ln Z}{\partial R \partial r} \bigg/ \frac{\partial \ln Z}{\partial R}\frac{\partial \ln Z}{\partial r}, \tag{27}$$

which implies that

$$\sigma = 1 + \frac{\delta_{KL}}{\rho_K \rho_L}. \tag{28}$$

Thus, σ can be calculated from an estimate of either (25) or (26). However, it should be noted that the real advantage of the translog function is in its use in the case of three or more factors of production. As discussed by Diewert [9, 10], the CES function cannot assume a form which has an arbitrary set of constant elasticities of substitution if the number of factors is greater than two. The general form of the translog function, written

$$\ln Z = \alpha_0 + \sum_i \alpha_i \ln P_i + \frac{1}{2}\sum_i \sum_j \delta_{ij}\ln P_i \ln P_j, \tag{29}$$

does provide the needed flexibility.

[7]As Burgess [5] has pointed out, the translog cost and production functions are not self-dual, so the choice of the cost function instead of the production function is arbitrary. The translog unit cost function can be regarded as an exact representation of the true cost function in the relevant range or as a second-order local approximation of the true cost function.

There are functional forms in addition to the translog unit cost function which can be used in conjunction with the results of modern duality theory. A notable example is the generalized Leontief unit cost function developed by Diewert [9], written

$$Z = b_{LL}R + 2b_{LK}r^{1/2}R^{1/2} + b_{KK}r. \tag{30}$$

However, this function has not been applied to the study of urban housing.

4. ECONOMETRIC SPECIFICATIONS AND EMPIRICAL RESULTS

The next step in the analysis is to specify a stochastic model that is suitable for empirical estimation. Drawing from the discussion by Nerlove [23], Clapp [7, p. 124] has briefly discussed this aspect of the model. No one else has examined this problem in the present context. Most empirical researchers have followed the strategy of deriving one of the exact equations in Section 3 above (Eqs. (6)–(9), (16)–(18), or (25)–(26), and then assuming that a normal error term with mean zero can be added to the particular exact equation used. This procedure is justified if we assume that the actors in the housing market make decisions without error, but that the researcher has measured the dependent variable with normal error. However, if we assume that there are errors in optimizing behavior, then we must decide how to introduce these errors into the model. Also, we may assume that there is a random error in the production function (or cost function) which makes the level of housing capacity output for a given level of inputs a random variable.

Nerlove [23] has provided a discussion of this last problem in the estimation of CES function. First, consider the possibility that there is a multiplicative random normal error in the production function and that economic agents optimize without error. As Nerlove [23, pp. 104–105] has shown, whether the random error is known ex ante or only ex post, the ratio of inputs is not influenced by the random error and K/L is an exact function of relative factor prices. Thus the random error in the production function cannot be used to rationalize a stochastic equation if the dependent variable is the ratio of inputs K/L, rK/L or rK/RL. An assumption of measurement error of the dependent variable or random imperfections in optimization with respect to either K or L (or both) must be introduced to permit econometric estimation. However, if the dependent variable is the output level H, then the random error in the econometric specification can be assumed to arise from the error in the production function. For example, from Footnote 3 we have

$$H/L = \gamma^{-\rho/\sigma}\alpha^{-\sigma}(R/p)^{\sigma}e^{-u\rho/\sigma}, \tag{31}$$

where e^u is the error term that can be assumed to be part of the constant term of the production function. Equation (31) can be rewritten as

$$pH/L = \gamma^{-\rho/\sigma}\alpha^{-\sigma}R^{\sigma}p^{1-\sigma}e^{-u\rho/\alpha}, \tag{32}$$

from which (8) can be derived using Shepard's lemma as

$$\begin{aligned}\frac{\partial \ln(pH/L)}{\partial \ln R} &= \sigma + (1-\sigma)\frac{d\ln p}{d\ln R}\\ &= \rho_L + \sigma\rho_K \end{aligned}\tag{33}$$

Thus

$$\ln(pH/L) = B + (\rho_L + \sigma\rho_K)\ln R - u\rho/\sigma, \tag{34}$$

where $B = (-\rho/\sigma)\ln\gamma - \sigma\ln\alpha$. This is the equation which Fountain [11] has estimated. However, Fountain [11] assumed that p is constant and believed that he estimated (32). The coefficient of $\ln R$ was found to be 0.57. Since p is variable because R is variable, the elasticity of substitution implied by Fountain's result is 0.38 ($\rho_L = 0.31$ and $\rho_K = 0.69$). The same error has been made by Clapp [6]. The stochastic version of (9) can similarly be derived from the results cited in Footnote 3.

Estimation of the VES production function has thus far been confined to estimating the stochastic versions of (16) and (17) in which the dependent variables are rK/L and rK/RL, respectively. These stochastic specifications must also be based either upon the assumption of measurement error in the dependent variable or errors in optimization with respect to K or L. As Sirmans and Redman [3] and Sirmans, *et al.* [29] have shown, Revankar's VES production function leads to an interesting use of the Box–Cox [4] Transformation to provide a statistical test to distinguish the VES from the CES function. Consider the general form of (6) and (16) assuming r is constant,

$$\frac{(rK/L)^{\lambda} - 1}{\lambda} = b_0 + b_1(R^{\lambda} - 1)/\lambda + u, \tag{35}$$

where u is the normal error term and λ is the crucial functional form parameter. If $\lambda = 1$ the function is VES (linear), and as λ approaches 0 the function approaches the CES (logarithmic) form.[8] The value λ_0 is chosen

[8]At first the term $(R^{\lambda} - 1)/\lambda$ with $\lambda = 0$ may appear to be indeterminate. Let $X = R^{\lambda}$. For X finite and positive, we can write $X = e^{\ln X}$. Now we find the Taylor series expansion for $e^{\ln X}$ around $X_0 = 1\,(\ln X_0 = 0)$, or $e^{\ln X} = e^0/0! + (e^{\ln X}0)'/1!\,(\ln X - 0) + (e^{\ln X}0)''/2!\,(\ln X - 0)^2 + \ldots$ or $e^{\ln X} = 1 + \ln X + 1/2!\,(\ln X)^2 + \ldots$. It follows that $(R^{\lambda} - 1)/\lambda = 1/\lambda[1 + \lambda\ln R + 1/2!\,(\lambda\ln R)^2 + \ldots - 1]$. For $\lambda = 0$ $(R^{\lambda} - 1)/\lambda = \ln R$.

to maximize the value of the log of the likelihood function, and an approximate 95% confidence interval for λ can be established by use of the likelihood ratio test

$$L^*(\lambda_0) - L^*(\lambda) < \frac{1}{2}\chi^2(0.05) = 1.92,$$

where $L^*(\lambda)$ is the log of the likelihood function and χ^2 is the chi-square distribution. All of these points are described in greater detail by Box and Cox [4], Kmenta [16] and Sirmans *et al.* [29].

The stochastic specification of the translog cost function used by Rosen [27] is (25) with a normal error term added to capture errors in optimizing behavior. As was previously noted, the error term could also stem from measurement error in the dependent variable. Rosen's assumption is conventional, as Burgess [5] has noted. However, Burgess [5] has pointed out that the translog cost function can be interpreted either as the exact cost function or as a second-order local approximation to an arbitrary functional form. In the latter case, Burgess [5] suggests that a normal error term be added to the translog cost function itself and that the cost function be estimated jointly with a cost share equation such as (25). However, this option is not feasible for the problem at hand because the unit cost of housing services is not a measurable variable.

The empirical studies of capital–land substitution in urban housing are summarized in Table 1. In Table 1 are listed for each study the data sources, the equations estimated, and the estimate of σ that is preferred by the author of the study (except in those cases in which the author has incorrectly reported the result).

TABLE 1

Empirical Estimates of the Elasticity of Substitution in Urban Housing

Study	Preferred estimate	Type of data	Equations estimated
Muth (19)	0.08 (reported as 0.75)	Aggregate time series (2 dates) for U.S., new FHA insured dwellings, 1946 and 1960. *RL* estimated by FHA.	Eq. (9)
Muth (22)	0.50[a]	Cross section of 47 metropolitan areas for 1966. Observations are mean values for new FHA-insured single-family houses. *RL* estimated by FHA and *r* estimated as a Boeckh construction cost index.	Eq. (7)
Koenker (17)	0.71[a]	Cross section of new multifamily structures in Ann Arbor, 1964–1966. *RL* and *pH* estimated by tax assessor.	Eq. (6) (*r* assumed constant)

TABLE 1—*Continued*

Study	Preferred estimate	Type of data	Equations estimated
Rydell (28)	0.50[a]	Cross section of rental property of all ages in Brown County, Wisconsin, 1974. Devises own measure of r and uses own assessment of R. Owners supplied estimate of pH.	Eq. (5)
Fountain (11)	0.38[a] (reported as 0.57)	Cross section of construction projects in Los Angeles SMSA, 1972–1974. All projects consist of multifamily buildings. Used tax assessor's estimate of rK (which was based on construction costs) and RL. Tax assessor thus provides pH.	Eq. (34)
Clapp (6)	0.97 (reported as 0.98)	Cross section of single-family houses of all ages sold in Chicago during 1970–1972. Allows σ to vary with age of house. Estimate of R obtained from hedonic regression of pH on characteristics of structure and site.	Eq. (34)
Rosen (27)	0.43[a]	Cross section of 10,000 single-family houses from 31 metropolitan areas, 1969. Uses FHA estimates of RL and Boeckh construction cost index for r.	Eq. (25)
Arnott and Lewis (2)	0.36[a]	Cross section of 23 Canadian metropolitan areas, 1975–1976. Observations are mean values for new single-family houses. Measure K/L as ratio of floor area to lot size, RL is mean of prices paid for lots, and rK is $pH - RL$.	$\ln RL/rK = a + \rho \ln K/L$, where $\rho = 1 - \sigma/\sigma$
McDonald (18)	1.13	Cross section of new single-family and multifamily structures sold in Chicago, 1969–1971. Uses Olcott's estimate of R.	Eq. (6) (r assumed constant)
McDonald (18)	0.86	Cross section of single-family houses of all ages sold in Chicago, 1970–1972. Allows σ to vary with age of house. Uses Olcott's estimate of R.	Eq. (6) (r assumed constant)
Polinsky and Ellwood(25)	0.45[a]	Cross section of 10,000 single-family houses from 31 metropolitan areas, 1969. Uses FHA estimates of RL and Boeckh construction cost index for r.	Eq. (7)
Sirmans *et al*. (29)	0.93[a] to 0.66[a] (0.83 at point of means)	Cross section of census tracts for Santa Clara County, Calif. Tract data are mean values for new single-family houses built in 1960. Uses FHA estimate for RL.	Eq. (6), Eq. (16) (r assumed constant)
Sirmans and Redman (30)	0.55[a], 0.52[a], 0.45[a] (at points of means)	Cross section of U.S. metroplitan areas for 1967, 1971, 1975. Observations are mean values for new FHA-insured single family houses. Uses FHA estimate for RL and Boeckh construction cost estimate for r.	Eq. (7) Eq. (17)

[a]The estimate of σ is reported to be significantly different from 1.0.

The initial study by Muth [19] was based upon the equation

$$\Delta \ln\left(\frac{rK}{pH}\right) \Big/ \Delta \ln(r/R) = (1 - \sigma)\rho_L, \tag{36}$$

which is the same as (9) above. Muth [19, p. 229] noted that the average share of land for new FHA-insured single-family houses had increased from 11.5% to 16.6% over the period of 1946 to 1960. Furthermore, the price of equivalent residential sites had increased by 180%, and the price of housing construction had increased by 77% over the same time period. Muth [19, p. 229] stated that these figures imply an elasticity of substitution of 0.75, but the figures actually imply $\sigma = 0.08$. This latter result is derived from Muth's data because rK/pH changed from 88.5% to 84.4% or $\Delta \ln(rK/pH) = -0.059$. Scaling the units so that r/R for 1946 = 1, r/R changed from 1 to 0.632 giving $\Delta \ln(r/R) = -0.459$. Also, $\rho_L = 0.14$ (the mean of 0.115 and 0.166). Substitution of these values into (36) above yields $\sigma = 0.08$. While Muth clearly has regarded his estimate only as preliminary, the discovery of an apparent computational error seriously depreciates the value of these data because all subsequent studies have found that the elasticity of substitution is much larger than 0.08. Muth's figures would imply a value of σ that is biased toward zero if there had been technical change during the period of 1946–1960 that was "capital saving," or, at a given value of K/L, increased the marginal product of land relative to the marginal product of capital. If there had been no technical change, then K/L would have increased more as r/R decreased, and the estimate of σ would have been greater. Indeed, because Muth's estimate of σ is so low compared to the various cross section estimates, there is reason to believe that capital saving technical change did occur during the period of 1946–1960.

The range of the other estimates for σ in Table 1 is 0.36 to 1.13. In 9 out of 12 studies cited the estimate of σ is reported to be significantly less than 1.0. No estimate of σ is reported to be significantly greater than 1.0. Five of the 12 studies report a value for σ in the range 0.4 to 0.6. The studies listed in Table 1 are of two basic types; those based upon data from one metropolitan area and those based upon data from a cross section of metropolitan areas. The range for the estimates of σ in the former group of seven studies is 0.38 to 1.13, while the range in the latter group of five studies is only 0.36 to 0.55. Furthermore, five out of seven studies in the former group report estimates of σ which exceed 0.55, the largest estimate of σ in the latter group. Thus, estimates of σ based upon data from a single metropolitan area tend to be greater than the estimates obtained from a cross section of metropolitan areas. Can this result be explained by "aggregation bias?" A little reflection and consultation with Theil [33, pp.

556–562] will convince one that this cannot be the case. Since all of the studies of a cross section of metropolitan areas are simple OLS regressions (only one independent variable), the estimates of σ produced are the simple means of σ for each of the metropolitan areas in the sample.[9]

On the other hand, this pattern of empirical results would have occurred if σ varies across metropolitan areas and it so happened that the studies of individual metropolitan areas were of metropolitan areas with relatively large values of σ. The seven studies of individual metropolitan areas listed in Table 1 include three studies of Chicago (McDonald [18] and Clapp [6]) and one study each for Ann Arbor, Brown County, Wisconsin (Green Bay), Los Angeles, and Santa Clara County, California. If the studies of Chicago are excluded, the range for σ is reduced to 0.38 to 0.83. Thus, it may simply be the case that Chicago has a relatively large value for σ. However, the acceptance of this method for resolving the apparent discrepancy between the two types of studies implies that further studies based upon a cross section of metropolitan areas should test for variations in σ across those metropolitan areas.[10]

5. MEASUREMENT ERROR BIAS

A potentially serious problem with all of the studies listed in Table 1 is measurement error bias. All of these studies utilize an estimate of land value as a crucial independent variable. It is likely that appraisals of land value by tax assessers or real estate appraisers are made with appreciable error. The purpose of this section is to make a preliminary estimate of the extent of this bias.

The studies by Koenker [17], McDonald [18], Rydell [28], and Sirmans *et al.* [29], are similar in that they utilize data from one metropolitan area and the same basic regression equation has been estimated. First introduced by Koenker [17], the equation states

$$\ln\left(\frac{pH - RL}{L}\right) = a + \sigma \ln R + u, \tag{37}$$

where u is a normal error term with mean zero and constant variance.[11]

[9]The exceptions are the studies by Rosen [27] and Polinsky and Ellwood [25]. In these studies the estimates of σ are weighted means, where the weight for a metropolitan area equals its proportion of the total sample of houses.

[10]The study by Sirmans and Redman [30] permits σ to vary across metropolitan areas with variations in K/L according to the VES function, but makes no further effort in this direction.

[11]Koenker [17] derived the equation to be estimated as follows. By definition of the elasticity of substitution, $\ln(K/L) = c + \sigma\ln(R/r)$. Adding $\ln r$ to both sides and assuming $rK + RL = pH$, we obtain $\ln[(pH - RL)/L] = c + \sigma \ln R + (1 - \sigma)\ln r$. Assuming r is constant at all locations, the equation in the text is obtained.

The price of K is assumed to be constant at all locations. In all five studies an estimated land value is used in place of R and the selling price of the house is used in place of pH.[12] It is thus quite likely that R has been measured with error and that pH has been measured with relatively little error. It is instructive to examine the impact of this possible measurement error. Recalling that $rK = pH - RL$, suppose that $V = Re^{\delta}$ and $i = re^{\gamma}$, where V and i are the measured prices of land and capital, respectively, and δ amd γ are normal error terms. The price of capital (r) is measured with error because R is measured with error. Substitution for r and R in (37) yields

$$\ln\left(\frac{iK}{L}\right) = a + \sigma \ln V + (u - \sigma\delta + \gamma). \tag{38}$$

Here δ and γ can be considered to be omitted variables with which $\ln V$ is correlated. The bias in the estimate of σ can be written

$$p\lim \hat{\sigma} - \sigma = -\sigma\beta_{\delta} + \beta_{\gamma}, \tag{39}$$

where β_{δ} and β_{γ} are the regression coefficients obtained in regressions of δ and γ on $\ln V$.[13] We know that $\beta_{\delta} > 0$, so if r is measured without error, the estimate of σ will be biased toward zero. However, since σ and γ are negatively correlated because rK is measured as a residual, we know that $\beta_{\gamma} < 0$. This adds to the downward bias of $\hat{\sigma}$. Thus, the method of estimation in most common use potentially contains a measurement error bias that is more serious than the usual measurement error bias.[14] The more conventional method for writing the bias introduced by measurement error is (Johnston [12, p. 282])

$$p\lim \hat{\sigma} = \sigma / \left(1 + \sigma_e^2/\sigma_u^2\right), \tag{40}$$

where σ_u^2 is the variance of u and σ_e^2 is the variance of the measurement error. In this case, the latter variance is

$$\sigma_e^2 = \sigma^2\sigma_{\delta}^2 + \sigma_{\gamma}^2 - 2\sigma\sigma_{\delta\gamma} \tag{41}$$

where σ_{δ}^2 and σ_{γ}^2 are the variances of δ and γ, respectively, and $\sigma_{\delta\gamma}$ is the covariance of δ and γ.

[12] The study by Rydell [28] is slightly different from the others in that the dependent variable is $\ln(K/L)$, where K is measured as the difference between total property value and lot value divided by an index of the price of capital improvements. However, the discussion of bias introduced by measurement error in R also applies to Rydell's study.

[13] See Johnston [12, pp. 168–169] for a derivation of this result.

[14] The studies by Clapp [6] and Fountain [11] are based upon the equation $pH/L = B + (\rho_L + \sigma\rho_K) \ln R$, so their studies are subject only to the "usual" measurement error bias.

Econometricians such as Theil [33], Johnston [12], and Kmenta [16] have made a number of suggestions for overcoming measurement error bias, none of which is totally satisfactory (except the suggestion to obtain better data, of course). These suggestions are:

1. Make an "enlightened guess" of the magnitude of σ_e^2/σ_u^2.
2. Use Wald's [35] method for fitting straight lines if both variables are subject to error (the method of group averages).
3. Assume that $\sigma_e^2/\sigma_u^2 = \lambda$, a known constant, and use the method of maximum likelihood estimation.
4. Use an instrumental variable for $\ln V$. Each of these suggestions is discussed in turn.

The data base developed by Wendt and Goldner [36] and used by Sirmans *et al.* [29] is used here to perform an empirical study of measurement error bias. The data are a sample of single-family houses for Santa Clara County, California for 1960. The data, taken from FHA new applications, were grouped into 98 census tracts. The variables provided by Wendt and Goldner [36] include average lot value, average lot value per square foot, average value of lot plus improvements (selling price), and average lot size. The lot value is the FHA appraisal that is based upon sales prices of lots in each locality. The 1960 census provided, for 97 out of the 98 tracts in the sample, information on homeownership, vacancies, deteriorated and dilapidated units, units with no bath, units built before 1939, crowded units, black and Latino population, median income level, and median education level. These variables were added to the data file. The sample size is 97.

Koenker's equation is estimated with these data, yielding

$$\ln\left(\frac{pH - VL}{L}\right) = \underset{(24.10)}{1.015} + \underset{(11.15)}{0.772} \ln V, \tag{42}$$

with $R^2 = 0.562$ and t values in parentheses. This result is not identical to the result obtained by Sirmans *et al.* [29, p. 411] because they used the full sample of 98 observations, but their results are extremely close to those reported above. Note that the estimate of σ of 0.77 is significantly less than 1.0 because the estimate of the standard error of σ is 0.069. Now let us turn to the suggestions for correcting the possible measurement error bias in this estimate of σ.

The first suggestion listed above is to calculate

$$\sigma = \hat{\sigma}\left(1 + \sigma_e^2/\sigma_u^2\right)$$

based upon an "enlightened guess" of σ_e^2/σ_u^2. There is no information with which to make an enlightened guess in this situation, but with $\hat{\sigma} = 0.77$, σ

would equal 1.0 if $\sigma_e^2/\sigma_u^2 = 0.30$. An unwillingness to make the assumption that $\lambda = \sigma_e^2/\sigma_u^2$ is known also eliminates as a serious possibility the third suggestion listed above.

Wald's method of group averages, the second suggestion listed above, as presented by Johnston [12, pp. 283–286], is to rank the observations by the level of the independent variable and to calculate

$$\hat{\sigma} = \frac{\bar{Y}_3 - \bar{Y}_1}{\bar{R}_3 - \bar{R}_1}, \tag{43}$$

where $\bar{Y}_1$ is the mean of the dependent variable for the lowest third of the observations on R, $\bar{Y}_3$ is the mean of the dependent variable for the greatest third of the observations on R, and where $\bar{R}_1$ and $\bar{R}_3$ are defined similarly. Calculation of $\hat{\sigma}$ by this method yields 0.847. However, as Theil [33, p. 611] points out, Neyman and Scott [24] have shown that this method produces a consistent estimate of $\hat{\sigma}$ only if the measurement errors are so small that the grouping according to observed R is identical to the grouping that would correspond to the true values. This requires that the range of the measurement error by finite and sufficiently small so as not to disturb the grouping into thirds. The importance of this problem cannot be assessed. In any case, the estimate of σ of 0.85 suggests that the previous estimate of 0.77 is biased downward as suspected.

The final suggestion listed above is to use an instrumental variable. The idea is to find a variable that is uncorrelated with the measurement error but is highly correlated with $\ln R$. As a first attempt, an instrument was created by regressing observed land value on the variables available in the 1960 census for each of the census tracts. These variables are percentage of homeowners, percent of units vacant, percent dilapidated, percentage of units with no bath, percentage of units built before 1939, percent crowded (1.01 or more persons per room), percent black population, percent Latino population, median income level, and median education level. However, the correlation of these variables with observed land value is very low ($R^2 = 0.08$ for the regression), so this attempt to create an instrumental variable failed. Next, the size of the lot was tried as the instrumental variable. Wendt and Goldner [36, p. 204] pointed out that this variable is highly correlated with observed land value, and it probably is not measured with error. However, is lot size correlated with the measurement error in lot value? This would be the case if, for example, lot values for large lots (small lots) are systematically underestimated (overestimated). For purposes of this study, we shall assume that this correlation does not exist. The regression of observed square-foot land value on lot size yields

$$\ln V = \underset{(8.74)}{6.018} + \underset{(9.54)}{0.742} \ln L, \tag{44}$$

with $R^2 = 0.484$ and t values in parentheses. Designate the instrumental variable calculated from this regression as $(\ln V)^*$. The regression result obtained using $(\ln V)^*$ as the independent variable is

$$\ln\left(\frac{pH - VL}{L}\right) = \underset{(19.23)}{1.170} + \underset{(10.05)}{1.053}\,(\ln V)^*, \tag{45}$$

with $R^2 = 0.510$ and t values in parentheses. While neither of these tests is conclusive, their results both are consistent with the notion that $\hat{\sigma}$ as calculated by Sirmans *et al.* [29, p. 411] is biased downward by measurement error. The instrumental variable technique indicated that σ is very close to 1.0.

The study by Clapp [6] includes an attempt to determine the magnitude of the measurement error bias introduced by the Koenker [17] procedure. Clapp [6] estimated (34) using two alternative measures of land value; the tax assessor's estimate and an estimate derived from a hedonic regression of total property value on the characteristics of the structure and site. The latter method involves deducting from the total property value the values of the structural features of the house. The estimate of σ obtained using the tax assessor's estimate of R is 0.38 while the hedonic regression approach yields an estimate of 0.97. (These figures were reported as 0.57 and 0.98, respectively.) Tax assessor's data for R were used by Fountain [11], Koenker [17], and Rydell [28].

The studies by Muth [22], Polinsky and Ellwood [25], Rosen [27], and Sirmans and Redman [30] utilize data from a cross section of metropolitan areas.[15] In each of these studies the equation to be estimated is of the form

$$G\left(\frac{RL}{rK}\right) = a + bF(R/r) + u, \tag{46}$$

where $G(RL/rK)$ and $F(R/r)$ are transformations of the ratio of factor shares and factor prices, respectively, and u is a normal error term. For example, Muth [22], Polinsky and Ellwood [25], and Sirmans and Redman [30] estimate

$$\ln\left(\frac{RL}{rK}\right) = a + (1 - \sigma)\ln(R/r) + u. \tag{47}$$

Here the expenditure on land is the FHA estimated market price of the site including street improvements, utilities and landscaping. The expenditure

[15] The study by Arnott and Lewis [2] also uses data from a cross section of metropolitan areas, but the equation estimated is different from the others. This study is discussed in detail below.

on structures, rK, is derived as a residual. The price of capital, r on the right-hand side of (47), is an independent estimate (for example, the Boeckh index of brick residential structures).

This procedure presents a slightly more difficult analysis of the possible biases. Assume measurement error in R and r, or $V = Re^{\delta}$ and $i = re^{\gamma}$. In addition, assume that K is measured with error, or $C = Ke^{\eta}$. We are thus able to consider the possibility that r is measured without error, but that rK is measured with error. Assume that γ is uncorrelated with δ and η and that δ is negatively correlated with η because the value of capital is defined as total property value minus estimated land value. Substitution of the expressions for R, r and K yields

$$\ln \frac{VL}{rC} = \alpha + (1 - \sigma)\ln \frac{V}{i} + (u + \delta\sigma + \gamma(1 - \sigma) - \eta). \tag{48}$$

The bias in the estimate of $(1 - \sigma)$ becomes

$$p\lim(1 - \hat{\sigma}) - (1 - \sigma) = \sigma\beta_{\delta} + (1 - \sigma)\beta_{\gamma} - \beta_{\eta}, \tag{49}$$

where β_{δ}, β_{γ} and β_{η} are the regression coefficients obtained in regressions of δ, γ and η on $\ln(V/i)$. The assumptions made above imply that $\beta_{\delta} > 0$ because V is positively correlated with δ and i is uncorrelated with δ. Similarly, $\beta_{\eta} < 0$. Finally, $\beta_{\gamma} < 0$. Thus, if r is measured without error, $\hat{\sigma}$ is biased downward. If r is measured with error, then the bias in $\hat{\sigma}$ will still be negative unless $-\beta_{\gamma}(1 - \sigma) > \sigma\beta_{\delta} - \beta_{\eta}$, a condition which can hold only if $\sigma < 1.0$. Thus this procedure possibly introduces biases which tend to be offsetting.

The study by Rosen [27] estimates

$$\rho_L = \frac{RL}{pH} = a + b\ln(R/r) + u, \tag{50}$$

which is derived from the translog cost function as described above. The elasticity of substitution is calculated as

$$\sigma = (b + \rho_K/\rho_L)/\rho_K\rho_L. \tag{51}$$

Rosen [27] and Polinsky and Ellwood [25] used the same data base and found estimates of σ of 0.43 and 0.45, respectively (where Rosen's estimate of σ is at the mean values of ρ_L and ρ_K). Rosen's estimate of b is subject to bias because of errors in measurement of R and r. These biases may tend to be offsetting, as discussed above. In addition, the calculation of σ from estimates of b, ρ_L and ρ_K may be subject to a bias stemming from errors in the measurement in ρ_L and ρ_K. However, Rosen's estimate of σ is calculated using mean values of ρ_L and ρ_K, so this source of bias will be absent if the measurement errors have mean zero.

The study by Arnott and Lewis [2] is an examination of a cross section of 23 metropolitan areas in Canada in 1975 and 1976. The equation estimated is

$$\ln(R/rK) = \alpha - \rho \ln(K/L) + u, \tag{52}$$

where $\sigma = 1/(1 - \rho)$ as shown above. Here K/L was measured as the ratio of floor area to lot size, rK was measured as sale price less land costs, and R was measured directly. Arnott and Lewis [2] have estimated σ to be equal to 0.37 for 1975 and 0.34 for 1976. However, clearly the measurement of K/L contains measurement error because K has been assumed to equal floor area. This measure does not allow for variations in the quality of a house of a given size. The coefficient of $\ln(K/L)$ is thus biased toward zero. Since $\sigma = 1/(1 - \rho)$, the estimate of σ is also biased toward zero.

To summarize this section, it can be concluded that studies based upon a single metropolitan area produce estimates of σ that are biased toward zero because of errors in the measurement of R. Studies based upon a cross section of metropolitan areas contain another measurement error bias that tends to be offsetting if r is measured with error. However, I believe that R is more likely to be measured with error than is r because R varies greatly within a metropolitan area (and is thus inherently difficult to measure) and r does not. Thus, I believe that the estimates of σ based upon a cross section of metropolitan areas are also biased toward zero. A preliminary estimate of the bias in the case of Santa Clara County data is 0.28, or 27%.

6. CONCLUSION

All of the studies reviewed in this paper have shown that, in the case of newly constructed housing, the intensity of land use is determined by the value of land in accordance with basic microeconomic theory. This result is confirmation of the value of the basic model for understanding urban housing in the long run. However, the analysis of the short run can be considerably more complex. For example, McDonald [18] has shown that the relationship between land value and the intensity of land use tends to break down as the stock of housing ages.

At this time there is no consensus estimate of the elasticity of substitution (σ) between capital and land in the production of the capacity to produce housing services. Indeed, there is good reason to suppose that σ varies across metropolitan areas and within a metropolitan area. The estimates of σ for Chicago tend to be greater than the estimates for other metropolitan areas, for example. Furthermore, it is likely that all of the estimates of σ are biased toward zero because of measurement error in the estimates of land values. A preliminary estimate of the extent of the bias indicates a bias of -0.28 in a study based upon a single metropolitan area. Studies of a cross section of metropolitan areas contain an offsetting bias because of measurement error in the price of capital. In my judgment, the measurement error in land values is probably greater and the more serious econometric problem. Thus, the estimates of σ in Table 1 of 0.36 to 1.13 should be considered to be biased downward. Most of the studies examined indicate that the estimate of σ obtained is significantly less than 1.0. The presence of downward bias calls this conclusion into question. The analysis of the housing market in the long run for some metropolitan areas (such as Santa Clara County and Chicago) need not be based upon the assumption that σ is less than 1.0.

Several questions have been raised in this paper that call for additional research. The possibility of systematic variations in σ should be investigated further. Variations in σ are built into the VES and the translog functions, but these functions may not incorporate all of the reasons for variations in σ. Section 2 above contains some additional hypotheses concerning variations in σ. Also, the early results of Muth [19] suggest that there has been capital-saving technical change in urban housing. A fuller understanding of urban housing markets in the long run surely requires more research into technical progress. Finally, future studies should treat the issue of measurement error with more care. The technique of instrumental variables can be used to test for the presence of the problem.

REFERENCES

1. R. Arnott, The reduced form price elasticity of housing, *J. Urban Econ.*, **5**, 293–304 (1978).
2. R. Arnott and F. Lewis, The transition of land to urban use, *J. Pol. Econ.*, **87**, 161–169 (1979).
3. E. Berndt and L. Christensen, The internal structure of functional relationship: Separability, substitution, and aggregation, *Rev. Econ. Studies*, **40**, 403–410 (1973).
4. G. Box and D. Cox, An analysis of transformations, *J. Roy. Statist. Soc.*, **26**, 211–252 (1964).
5. D. Burgess, Duality theory and pitfalls in the specification of technologies, *J. Econometrics*, **3**, 105–121 (1975).
6. J. Clapp, The elasticity of substitution for land: The effects of measurement errors, *J. Urban Econ.*, **8**, 255–263 (1980).
7. J. Clapp, The substitution of urban land for other inputs, *J. Urban Econ.*, **6**, 122–134 (1979).
8. M. David, "Family Composition and Consumption," North-Holland, Amsterdam (1962).
9. W. Diewert, An application of the Shepard duality theorem: A generalized Leontief production function, *J. Pol. Econ.*, **79**, 481–507 (1971).
10. W. Diewert, Applications of duality theory, *in* "Frontiers of Quantitative Economics, Vol.

II," (M. Intriligator and D. Kendrick, Eds.), pp. 106–171, North-Holland, Amsterdam (1974).

11. J. Fountain, An empirical estimation of the housing production function, Unpublished paper presented at the North American Meetings of the Regional Science Association (1977).
12. J. Johnston, "Econometric Methods," McGraw-Hill, New York (1972).
13. J. Kain and J. Quigley, "Housing Markets and Racial Discrimination: A Microeconomic Analysis," Nat. Bur. Econ. Res., New York (1975).
14. J. Kau and C. Lee, Capital-land substitution and urban land use, *J. Reg. Sci.*, **16**, 83–92 (1976).
15. A.T. King, The demand for housing: A Lancastrian approach, *Southern Econ. J.*, **43**, 1077–1087 (1977).
16. J. Kmenta, "Elements of Econometrics," Macmillan, New York (1971).
17. R. Koenker, An empirical note on the elasticity of substitution between land and capital in a monocentric housing market, *J. Reg. Sci.*, **12**, 299–305 (1972).
18. J. McDonald, "Economic Analysis of an Urban Housing Market," Academic Press, New York (1979).
19. R. Muth, The derived demand curve for a productive factor and the industry supply curve, *Oxford Econ. Papers (New Series)*, **16**, 221–234 (1964).
20. R. Muth, Household production and consumer demand functions, *Econometrica*, **34**, 699–798 (1966).
21. R. Muth, "Cities and Housing," Univ. of Chicago Press, Chicago (1969).
22. R. Muth, The derived demand for urban residential land, *Urban Studies*, **8**, 243–254 (1971).
23. M. Nerlove, Recent empirical studies of the CES and related production functions, *in* "The Theory and Empirical Analysis of Production" (M. Brown, Ed.,), Nat. Bur. Econ. Res., New York (1967).
24. J. Neyman and E. Scott, On certain methods of estimating the linear structural relation, *Ann. Math. Statist.*, **22**, 352–361 (1951).
25. A. M. Polinsky and D. Ellwood, An empirical reconciliation of micro and grouped estimates of the demand for housing, *Rev. Econ. Statist.*, **61**, 199–205 (1979).
26. N. Revanker, A class of variable elasticity of substitution production functions, *Econometrica*, **39**, 61–71 (1971).
27. H. Rosen, Estimating inter-city differences in the price of housing services, *Urban Studies*, **15**, 351–355 (1978).
28. C. Rydell, Measuring the supply response to housing allowances, *Papers Reg. Sci. Assoc.*, **37**, 31–53 (1976).
29. C. Sirmans, J. Kau and C. Lee, The elasticity of substitution in urban housing production: A VES approach, *J. Urban Econ.*, **6**, 407–415 (1979).
30. C. Sirmans and A. Redman, Capital-land substitution and the price elasticity of demand for urban residential land, *Land Econ.*, **55**, 167–176 (1979).
31. B. Smith, The supply of urban housing, *Quar. J. Econ.*, **90**, 389–405 (1976).
32. M. Straszheim, "An Econometric Analysis of the Urban Housing Market," Nat. Bur. Econ. Res., New York (1975).
33. H. Theil, "Principles of Econometrics," Wiley, New York (1971).
34. H. Uzawa, Production functions with constant elasticities of substitution, *Rev. Econ. Studies*, **29**, 291–299 (1962).
35. A. Wald, The fitting of straight lines if both variables are subject to error, *Ann. Math. Statist.*, **11**, 284–300 (1940).
36. P. Wendt and W. Goldner, Land values and the dynamics of residential location, *in* "Essays in Urban Land Economics," University of California, Los Angeles, 188–213 (1966).