## THE HIGGS MECHANISM

Evan Craft

All familiar with Gauge Theory

```
 \begin{split} \mathcal{L}_{SM} &= -\frac{1}{2} \partial_{\nu} g_{0}^{a} \partial_{\nu} g_{\mu}^{a} - g_{s} f^{abc} \partial_{\mu} g_{0}^{a} g_{b}^{b} g_{\nu}^{c} - \frac{1}{4} g_{s}^{2} f^{abc} f^{adc} g_{b}^{b} g_{\nu}^{c} g_{\mu}^{d} g_{\nu}^{c} - \partial_{\nu} W_{\mu}^{+} \partial_{\nu} W_{\mu}^{-} \\ M^{2} W_{\mu}^{+} W_{\mu}^{-} - \frac{1}{2} \partial_{\nu} Z_{0}^{\mu} \partial_{\nu} Z_{0}^{\mu} - \frac{1}{2c_{\nu}^{c}} M^{2} Z_{\mu}^{\mu} Z_{\mu}^{\mu} - \frac{1}{2} \partial_{\mu} A_{\nu} \partial_{\mu} A_{\nu} - i g c_{w} (\partial_{\nu} Z_{\mu}^{\mu} (W_{\mu}^{+} W_{\nu}^{-} - W_{\mu}^{+} W_{\mu}^{-}) + \frac{1}{2c_{\nu}^{c}} M^{2} Z_{\mu}^{\mu} Z_{\mu}^{\mu
                                                    \begin{array}{l} W_{\nu}^{-} \partial_{\nu} W_{\nu}^{+} )) - \frac{1}{2} g^{2} W_{\mu}^{+} W_{\nu}^{+} W_{\nu}^{+} W_{\nu}^{+} + \frac{1}{2} g^{2} W_{\mu}^{+} W_{\nu}^{-} W_{\nu}^{+} W_{\nu}^{-} + \frac{\mu}{2} g^{2} C_{w}^{2} (Z_{\nu}^{0} W_{\nu}^{+} Z_{\nu}^{0} W_{\nu}^{-} - Z_{\mu}^{0} Z_{\mu}^{0} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w}^{2} (A_{\mu} W_{\mu}^{+} A_{\nu} W_{\nu}^{-} - A_{\mu} A_{\mu} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w} c_{w} (A_{\mu} Z_{\nu}^{0} (W_{\mu}^{+} W_{\nu}^{-} - A_{\mu} X_{\nu}^{0} W_{\nu}^{-}) + g^{2} s_{w}^{2} c_{w} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w}^{2} c_{w}^{2} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-} - A_{\mu} X_{\nu}^{0} W_{\nu}^{-}) + g^{2} s_{w}^{2} c_{w}^{2} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w}^{2} c_{w}^{2} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-} - A_{\mu} X_{\nu}^{0} W_{\nu}^{-}) + g^{2} s_{w}^{2} c_{w}^{2} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w}^{2} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w}^{2} (A_{\mu} W_{\nu}^{+} W_{\nu}^{-}) + g^{2} s_{w}^
                                     W_{\nu}^{+}W_{\mu}^{-}) - 2A_{\mu}Z_{\mu}^{0}W_{\nu}^{+}W_{\nu}^{-}) - \frac{1}{2}\partial_{\mu}H\partial_{\mu}H - 2M^{2}\alpha_{h}H^{2} - \partial_{\mu}\phi^{+}\partial_{\mu}\phi^{-} - \frac{1}{2}\partial_{\mu}\phi^{0}\partial_{\mu}\phi^{0} - \frac
                                                                                                                                                                                                                                                 \beta_h \left( \frac{2M^2}{a^2} + \frac{2M}{a}H + \frac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-) \right) + \frac{2M^4}{a^2}\alpha_h - \frac{2M^4}{a^2}
                                                                                                                                                                                                                                                                                                                                                                                                          g\alpha_h M (H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-)
                                                                                                      \tfrac{1}{8}g^2\alpha_h\left(H^4+(\phi^0)^4+4(\phi^+\phi^-)^2+4(\phi^0)^2\phi^+\phi^-+4H^2\phi^+\phi^-+2(\phi^0)^2H^2\right)-
                                                                                                                                                                                                                                                                                                                                                                                                                            gMW_{\mu}^{+}W_{\mu}^{-}H - \frac{1}{2}g\frac{M}{c^{2}}Z_{\mu}^{0}Z_{\mu}^{0}H -
                                                                                                                                                                                                                              \frac{1}{2}ig\left(W_{\mu}^{+}(\phi^{0}\partial_{\mu}\phi^{-}-\phi^{-}\partial_{\mu}\phi^{0})-W_{\mu}^{-}(\phi^{0}\partial_{\mu}\phi^{+}-\phi^{+}\partial_{\mu}\phi^{0})\right)+
         \frac{1}{2}g\left(W_{\mu}^{+}(H\partial_{\mu}\tilde{\phi}^{-}-\tilde{\phi}^{-}\partial_{\mu}H)+W_{\mu}^{-}(H\partial_{\mu}\phi^{+}-\phi^{+}\partial_{\mu}H)\right)+\frac{1}{2}g\frac{1}{c_{m}}(Z_{\mu}^{0}(H\partial_{\mu}\phi^{0}-\phi^{0}\partial_{\mu}H)+
M\left(\frac{1}{c_{vv}}Z_{\mu}^{0}\partial_{\mu}\phi^{0}+W_{\mu}^{+}\partial_{\mu}\phi^{-}+W_{\mu}^{-}\partial_{\mu}\phi^{+}\right)-ig\frac{s_{vv}^{2}}{c_{vv}}MZ_{\mu}^{0}(W_{\mu}^{+}\phi^{-}-W_{\mu}^{-}\phi^{+})+igs_{w}MA_{\mu}(W_{\mu}^{+}\phi^{-}-W_{\mu}^{-}\phi^{+})
                                                                                            W_{\mu}^{-}\phi^{+}) - ig rac{1-2c_{w}^{2}}{2c_{w}} Z_{\mu}^{0}(\phi^{+}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{+}) + igs_{w}A_{\mu}(\phi^{+}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{+}) - igs_{w}A_{\mu}(\phi^{+}\partial_{\mu}\phi^{-} - \phi^{-}\partial_{\mu}\phi^{-}) -
                           \frac{1}{4}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{8}g^2\frac{1}{c^2}Z_{\mu}^0Z_{\mu}^0(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2(2s_w^2-1)^2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{\mu}W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^+\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)-\frac{1}{6}g^2W_{\mu}^{-}(H^2+(\phi^0)^2+2\phi^-\phi^-)
             \tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z_\mu^0\phi^0(W_\mu^+\phi^-+W_\mu^-\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z_\mu^0H(W_\mu^+\phi^--W_\mu^-\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^+\phi^-+W_\mu^-\phi^+) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^+\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^0(W_\mu^-\phi^-+W_\mu^-\phi^-) + \tfrac{1}{2}g^2s_wA_\mu\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-+W_\mu^-\phi^-
                                                                                                                             W_{\mu}^{-}\phi^{+}) + \frac{1}{2}ig^{2}s_{w}A_{\mu}H(W_{\mu}^{+}\phi^{-}W_{\mu}^{-}\phi^{+}) - g^{2}\frac{s_{w}}{c_{w}}(2c_{w}^{2}-1)Z_{\mu}^{0}A_{\mu}\phi^{+}\phi^{-} - W_{\mu}^{-}\phi^{+})
                           g^2 s_w^2 A_\mu A_\mu \phi^+ \phi^- + \tfrac{1}{2} i g_s \, \lambda_{ij}^a (\bar{q}_i^\sigma \gamma^\mu q_j^\sigma) g_\mu^a - \bar{e}^\lambda (\gamma \partial + m_e^\lambda) \bar{e}^\lambda - \bar{\nu}^\lambda (\gamma \partial + m_\nu^\lambda) \nu^\lambda - \bar{u}_j^\lambda (\gamma \partial + m_\nu^\lambda) \bar{e}^\lambda - \bar{e}^\lambda (\gamma \partial +
                                                                              m_u^{\lambda} u_i^{\lambda} - \bar{d}_i^{\lambda} (\gamma \partial + m_d^{\lambda}) d_i^{\lambda} + igs_w A_{\mu} \left( -(\bar{e}^{\lambda} \gamma^{\mu} e^{\lambda}) + \frac{2}{3} (\bar{u}_i^{\lambda} \gamma^{\mu} u_i^{\lambda}) - \frac{1}{3} (\bar{d}_i^{\lambda} \gamma^{\mu} d_i^{\lambda}) \right) +
                                                                          \frac{ig}{4c_w}Z_u^0\{(\bar{\nu}^{\lambda}\gamma^{\mu}(1+\gamma^5)\nu^{\lambda})+(\bar{e}^{\lambda}\gamma^{\mu}(4s_w^2-1-\gamma^5)e^{\lambda})+(\bar{d}_i^{\lambda}\gamma^{\mu}(\frac{4}{3}s_w^2-1-\gamma^5)d_i^{\lambda})+
(\bar{u}_{j}^{\lambda}\gamma^{\mu}(1-\frac{8}{3}s_{w}^{2}+\gamma^{5})u_{j}^{\lambda})\}+\frac{ig}{2\sqrt{2}}W_{\mu}^{+}((\bar{\nu}^{\lambda}\gamma^{\mu}(1+\gamma^{5})U^{lep}_{\lambda\kappa}e^{\kappa})+(\bar{u}_{j}^{\lambda}\gamma^{\mu}(1+\gamma^{5})C_{\lambda\kappa}d_{j}^{\kappa}))+
                                                                                                                                                                                                                \frac{ig}{2\sqrt{2}}W_{\mu}^{-}\left(\left(\bar{e}^{\kappa}U^{lep}_{\kappa\lambda}^{\dagger}\gamma^{\mu}(1+\gamma^{5})\nu^{\lambda}\right)+\left(\bar{d}_{j}^{\kappa}C_{\kappa\lambda}^{\dagger}\gamma^{\mu}(1+\gamma^{5})u_{j}^{\lambda}\right)\right)+
                                                                                                                                                                           rac{ig}{2M\sqrt{2}}\phi^+\left(-m_e^{\kappa}(ar{
u}^{\lambda}U^{lep}_{\lambda\kappa}(1-\gamma^5)e^{\kappa})+m_{
u}^{\lambda}(ar{
u}^{\lambda}U^{lep}_{\lambda\kappa}(1+\gamma^5)e^{\kappa}\right)+
                                                                \frac{ig}{2M\sqrt{2}}\phi^{-}\left(m_{e}^{\lambda}(\bar{e}^{\lambda}U^{lep}_{\lambda\kappa}^{\dagger}(1+\gamma^{5})\nu^{\kappa})-m_{\nu}^{\kappa}(\bar{e}^{\lambda}U^{lep}_{\lambda\kappa}^{\dagger}(1-\gamma^{5})\nu^{\kappa}\right)-\frac{g}{2}\frac{m_{\nu}^{\lambda}}{M}H(\bar{\nu}^{\lambda}\nu^{\lambda})-
                                                                                                 \frac{g}{2}\frac{m_e^{\lambda}}{M}H(\bar{e}^{\lambda}e^{\lambda}) + \frac{ig}{2}\frac{m_{\nu}^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5\nu^{\lambda}) - \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{e}^{\lambda}\gamma^5e^{\lambda}) - \frac{1}{4}\bar{\nu}_{\lambda}M_{\lambda\kappa}^R(1-\gamma_5)\hat{\nu}_{\kappa} - \frac{ig}{2}\frac{m_e^{\lambda}}{M}H(\bar{e}^{\lambda}e^{\lambda}) + \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5\nu^{\lambda}) - \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{e}^{\lambda}\gamma^5e^{\lambda}) - \frac{1}{4}\bar{\nu}_{\lambda}M_{\lambda\kappa}^R(1-\gamma_5)\hat{\nu}_{\kappa} - \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{e}^{\lambda}\gamma^5e^{\lambda}) + \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5\nu^{\lambda}) - \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{e}^{\lambda}\gamma^5e^{\lambda}) - \frac{1}{4}\bar{\nu}_{\lambda}M_{\kappa}^R(1-\gamma_5)\hat{\nu}_{\kappa} - \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5e^{\lambda}) + \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5e^{\lambda}) + \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5e^{\lambda}) - \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5e^{\lambda}) + \frac{ig}{2}\frac{m_e^{\lambda}}{M}\phi^0(\bar{\nu}^{\lambda}\gamma^5e^{\lambda}
                                         \tfrac{1}{4}\, \overline{\nu_{\lambda}\, M_{\lambda\kappa}^{R}\, (1-\gamma_{5}) \hat{\nu}_{\kappa}} + \tfrac{ig}{2M\sqrt{2}} \phi^{+} \left( -m_{d}^{\kappa}(\bar{u}_{j}^{\lambda} C_{\lambda\kappa}(1-\gamma^{5}) d_{j}^{\kappa}) + m_{u}^{\lambda}(\bar{u}_{j}^{\lambda} C_{\lambda\kappa}(1+\gamma^{5}) d_{j}^{\kappa}) + \right.
                                                                                                      \frac{ig}{2M\sqrt{2}}\phi^{-}\left(m_d^{\lambda}(\bar{d}_j^{\lambda}C_{\lambda\kappa}^{\dagger}(1+\gamma^5)u_j^{\kappa})-m_u^{\kappa}(\bar{d}_j^{\lambda}C_{\lambda\kappa}^{\dagger}(1-\gamma^5)u_j^{\kappa}\right)-\frac{g}{2}\frac{m_u^{\lambda}}{M}H(\bar{u}_j^{\lambda}u_j^{\lambda})-
                                \frac{g}{2}\frac{m_{\dot{a}}^{\lambda}}{M}H(\bar{d}_{\dot{i}}^{\lambda}d_{\dot{i}}^{\lambda}) + \frac{ig}{2}\frac{m_{\dot{a}}^{\lambda}}{M}\phi^{0}(\bar{u}_{\dot{i}}^{\lambda}\gamma^{5}u_{\dot{i}}^{\lambda}) - \frac{ig}{2}\frac{m_{\dot{a}}^{\lambda}}{M}\phi^{0}(\bar{d}_{\dot{i}}^{\lambda}\gamma^{5}d_{\dot{i}}^{\lambda}) + \bar{G}^{a}\partial^{2}G^{a} + g_{s}f^{abc}\partial_{\mu}\bar{G}^{a}G^{b}g_{\mu}^{c} +
ar{X}^{+}(\partial^{2}-M^{2})X^{+}+ar{X}^{-}(\partial^{2}-M^{2})X^{-}+ar{X}^{0}(\partial^{2}-rac{M^{2}}{c^{2}})X^{0}+ar{Y}\partial^{2}Y+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}W_{\mu}^{+}(\partial_{\mu}ar{X}^{0}X^{-}-igc_{w})X^{0}+igc_{w}
                                                                                                                                                                           \partial_{\mu}\bar{X}^{+}X^{0})+igs_{w}W_{\mu}^{+}(\partial_{\mu}\bar{Y}X^{-}-\partial_{\mu}\bar{X}^{+}Y)+igc_{w}W_{\mu}^{-}(\partial_{\mu}\bar{X}^{-}X^{0}-
                                                                                                                                                                                         \partial_{\mu} \bar{X}^0 X^+) + igs_w W_u^- (\partial_{\mu} \bar{X}^- Y - \partial_{\mu} \bar{Y} X^+) + igc_w Z_u^0 (\partial_{\mu} \bar{X}^+ X^+ - igc_w Z_u^0)
                                                                                                                                                                                                                                                                                                                                                                                                                            \partial_{\mu}\tilde{X}^{-}X^{-})+igs_{w}A_{\mu}(\partial_{\mu}\tilde{X}^{+}X^{+}-
\partial_{\mu} \bar{X}^{-} X^{-}) - \frac{1}{2} g M \left( \bar{X}^{+} X^{+} H + \bar{X}^{-} X^{-} H + \frac{1}{c^{2}} \bar{X}^{0} X^{0} H \right) + \frac{1-2c_{w}^{2}}{2c_{w}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{+} X^{0} \phi^{+} - \bar{X}^{-} X^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{0} \phi^{+} - \bar{X}^{0} \phi^{+} - \bar{X}^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{0} \phi^{+} - \bar{X}^{0} \phi^{+} - \bar{X}^{0} \phi^{-} \right) + \frac{1}{c^{2}} ig M \left( \bar{X}^{0} \phi^{+} - \bar{X}^{0} \phi^{+} - \bar{X}^{0} \phi^{+} \right)
                                                                                                                                                         \frac{1}{2c_w}igM(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-) + igMs_w(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-) +
                                                                                                                                                                                                                                                                                                                                                                                                                                 \frac{1}{2}igM(\bar{X}^{+}X^{+}\phi^{0}-\bar{X}^{-}X^{-}\phi^{0}).
```

$$\mathscr{L}=\mathrm{i}\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-\mathrm{m}c^{2}\overline{\psi}\psi$$

• Given the action for a particle

$$\mathscr{L}=\mathrm{i}\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-\mathrm{m}c^{2}\overline{\psi}\psi$$

Given the action for a particle



$$\psi \rightarrow e^{i\theta(x)}\psi$$
 (local phase transformation)

 Want this to be invariant under some local transformation

$$\mathscr{L} = i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^2 \overline{\psi} \psi$$

Given the action for a particle



$$\psi \rightarrow e^{i\theta(x)}\psi$$
 (local phase transformation)



 Want this to be invariant under some local transformation

$$\partial_{\mu}(e^{i\theta}\psi) = i(\partial_{\mu}\theta)e^{i\theta}\psi + e^{i\theta}\partial_{\mu}\psi$$

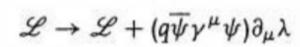
 However the derivative adds an extra term

$$\mathscr{L}=i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-mc^{2}\overline{\psi}\psi$$

Given the action for a particle



$$\psi \to e^{i\theta(x)}\psi$$
 (local phase transformation)



SOI

 Want this to be invariant under some local transformation

Hence we have:



$$\partial_{\mu}(e^{i\theta}\psi)=i(\partial_{\mu}\theta)e^{i\theta}\psi+e^{i\theta}\partial_{\mu}\psi$$

 However the derivative adds an extra term

$$\mathscr{L}=\mathrm{i}\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-mc^{2}\overline{\psi}\psi$$

• **Idea:** Rewrite the original Lagrangian

$$\mathscr{L}=\mathrm{i}\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-mc^{2}\overline{\psi}\psi$$

• Idea: Rewrite the original Lagrangian



$$\mathcal{L} = [i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^{2} \overline{\psi} \psi] - (q \overline{\psi} \gamma^{\mu} \psi) A_{\mu}$$

$$\mathscr{L}=\mathrm{i}\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-mc^{2}\overline{\psi}\psi$$

Idea: Rewrite the original Lagrangian



$$\mathscr{L} = [i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^{2} \overline{\psi} \psi] - (q \overline{\psi} \gamma^{\mu} \psi) A_{\mu}$$

• Where: 
$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$

$$\mathscr{L}=i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-mc^{2}\overline{\psi}\psi$$

Idea: Rewrite the original Lagrangian



This **cancels** the offending term!

$$\mathcal{L} = [i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^{2} \overline{\psi} \psi] - (q \overline{\psi} \gamma^{\mu} \psi) A_{\mu}$$

• Where: 
$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$

$$\mathscr{L}=\mathrm{i}\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi-mc^{2}\overline{\psi}\psi$$

Idea: Rewrite the original Lagrangian



This **cancels** the offending term!

The Lagrangian is **changed** however

$$\mathscr{L} = [i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^{2} \overline{\psi} \psi] - (q \overline{\psi} \gamma^{\mu} \psi) A_{\mu}$$

• Where: 
$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$

$$\lambda(x) \equiv -\frac{\hbar c}{q} \theta(x)$$

• **Define:** Some new variable

$$\lambda(x) \equiv -\frac{\hbar c}{q}\theta(x)$$

• **Define:** Some new variable



**Define:** The covariant derivative

$$\mathcal{D}_{\mu} \equiv \partial_{\mu} + i \frac{q}{\hbar c} A_{\mu}$$

$$\lambda(x) \equiv -\frac{\hbar c}{q} \theta(x)$$

• **Define:** Some new variable



Wherever there is a derivative, replace it with the **covariant derivative** 

• **Define:** The covariant derivative

$$\mathcal{D}_{\mu} \equiv \partial_{\mu} + i \frac{q}{\hbar c} A_{\mu}$$

 The vector field has its own equations of motion which must be included in the Lagrangian

 The vector field has its own equations of motion which must be included in the Lagrangian

The field obeys the Proca action

 The vector field has its own equations of motion which must be included in the Lagrangian

The field obeys the Proca action

$$\mathcal{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_{AC}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

 The vector field has its own equations of motion which must be included in the Lagrangian

The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_{AC}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$



 The vector field has its own equations of motion which must be included in the Lagrangian

The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_{AC}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$

$$A^{\nu}A_{\nu}$$

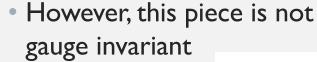
 However, this piece is not gauge invariant

 The vector field has its own equations of motion which must be included in the Lagrangian

 The field obeys the Proca action

$$\mathcal{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_{AC}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$





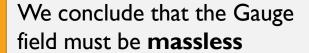
$$\mathcal{L} = \left[i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^{2} \overline{\psi} \psi\right] - \left[\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}\right] - (q \overline{\psi} \gamma^{\mu} \psi) A_{\mu}$$

 The vector field has its own equations of motion which must be included in the Lagrangian

The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_{AC}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu} \lambda$$



 However, this piece is not gauge invariant



$$\mathcal{L} = [i\hbar c \overline{\psi} \gamma^{\mu} \partial_{\mu} \psi - mc^{2} \overline{\psi} \psi] - \left[ \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} \right] - (q \overline{\psi} \gamma^{\mu} \psi) A_{\mu}$$

# QUESTION: HOW CAN WE GIVE **GAUGE FIELDS MASS?**

# **QUESTION**: HOW CAN WE GIVE GAUGE FIELDS MASS?



PART I: WHAT DO MASS TERMS LOOK LIKE?





PART I: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

Question: What is the mass of this field?

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

- Seems to be zero!
- No terms proportional to the wave function squared

Question: What is the mass of this field?

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

• Question: What is the mass of this field?

- Seems to be zero!
- No terms proportional to the wave function squared



$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + 1 - \alpha^2 \phi^2 + \frac{1}{2} \alpha^4 \phi^4 - \frac{1}{6} \alpha^6 \phi^6 + \cdots$$

Expanding the exponential we se otherwise

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

Question: What is the mass of this field?

- Seems to be zero!
- No terms proportional to the wave function squared



$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + 1 - \alpha^2 \phi^2 + \frac{1}{2} \alpha^4 \phi^4 - \frac{1}{6} \alpha^6 \phi^6 + \cdots$$

$$m = \sqrt{2}\alpha\hbar/c$$



• Expanding the exponential we se otherwise

# **QUESTION**: HOW CAN WE GIVE GAUGE FIELDS MASS?



PART I: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN



$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + 1 - \alpha^2 \phi^2 + \frac{1}{2} \alpha^4 \phi^4 - \frac{1}{6} \alpha^6 \phi^6 + \cdots$$

#### GETTING MASS INTO LAGRANGIAN

$$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{mc}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

 These are Gauge Fields and hence obey the Proca action

$$F^{\mu\nu} \equiv \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$

#### GETTING MASS INTO LAGRANGIAN

$$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{\textit{mc}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

 These are Gauge Fields and hence obey the Proca action

$$F^{\mu\nu} \equiv \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$



$$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

From the previous analysis, they must be massless

#### GETTING MASS INTO LAGRANGIAN

$$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{\textit{mc}}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

 These are Gauge Fields and hence obey the Proca action

$$F^{\mu\nu} \equiv \partial^{\mu}A^{\nu} - \partial^{\nu}A^{\mu}$$



$$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

No hidden perturbation....

From the previous analysis, they must be massless

## **QUESTION**: HOW CAN WE GIVE GAUGE FIELDS MASS?



PART I: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN



$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + 1 - \alpha^2 \phi^2 + \frac{1}{2} \alpha^4 \phi^4 - \frac{1}{6} \alpha^6 \phi^6 + \cdots$$



#### GETTING MASS INTO LAGRANGIAN (2<sup>ND</sup> ATTEMPT)

• Question: What's the simplest Lagrangian you can think of?

GETTING MASS INTO LAGRANGIAN (2<sup>ND</sup> ATTEMPT)

• Question: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi)$$

#### GETTING MASS INTO LAGRANGIAN (2ND ATTEMPT)

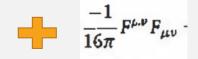
• Question: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathscr{L}=rac{1}{2}(\partial_{\mu}\phi)^{*}(\partial^{\mu}\phi)$$



$$\mathscr{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$

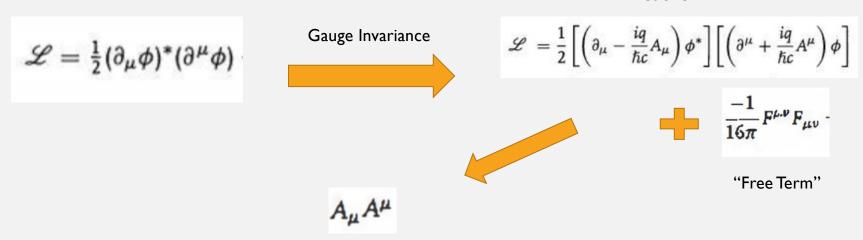


"Free Term"

### GETTING MASS INTO LAGRANGIAN (2<sup>ND</sup> ATTEMPT)

• Question: What's the simplest Lagrangian you can think of?

"Kinetic Term"



Expanding this out gives a term proportional the inner product!

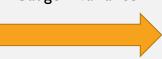
### GETTING MASS INTO LAGRANGIAN (2<sup>ND</sup> ATTEMPT)

• Question: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathscr{L}=rac{1}{2}(\partial_{\mu}\phi)^{*}(\partial^{\mu}\phi)$$

Gauge Invariance



$$\mathscr{L} \ = rac{1}{2} \left[ \left( \partial_{\mu} - rac{iq}{\hbar c} A_{\mu} 
ight) \phi^* 
ight] \left[ \left( \partial^{\mu} + rac{iq}{\hbar c} A^{\mu} 
ight) \phi 
ight]$$



$$\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"



Au

It's also proportional to this, and therefore not a constant

Expanding this out gives a term proportional the inner product!

### GETTING MASS INTO LAGRANGIAN (2ND ATTEMPT)

• Question: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathscr{L}=rac{1}{2}(\partial_{\mu}\phi)^{*}(\partial^{\mu}\phi)$$

Gauge Invariance

$$\mathscr{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$



 $A_{\mu}A^{\mu}$ 

 $\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$ 

"Free Term"

It's also proportional to this, and therefore not a constant

Expanding this out gives a term proportional the inner product!

But it seems as if we are onto something...

GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

• Question: What's the simplest Lagrangian you can think of?

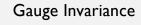
$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi)$$

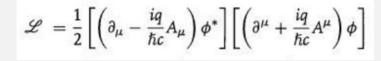
## GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

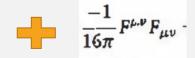
• Question: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathscr{L}=rac{1}{2}(\partial_{\mu}\phi)^{*}(\partial^{\mu}\phi)$$







"Free Term"

# GETTING MASS INTO LAGRANGIAN $(2^{ND}$ ATTEMPT)

Question: What's the simplest Lagrangian you can think of?

 $\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi)$ 

Gauge Invariance



$$\mathscr{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$



$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

Define a couple of new variables

### GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

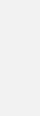
Question: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi)$$

Gauge Invariance



$$\mathscr{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$





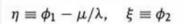
 $\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$ 

"Free Term"

$$\mathcal{L} = \left[ \frac{1}{2} (\partial_{\mu} \eta) (\partial^{\mu} \eta) - \mu^{2} \eta^{2} \right] + \left[ \frac{1}{2} (\partial_{\mu} \xi) (\partial^{\mu} \xi) \right]$$

$$+ \left[ -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left( \frac{q}{\hbar c} \frac{\mu}{\lambda} \right)^{2} A_{\mu} A^{\mu} \right]$$

Expanding this out gives



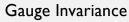
$$\phi \equiv \phi_1 + i\phi_2$$

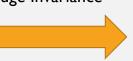
Define a couple of new variables

### GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

Question: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi)$$





$$\mathscr{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$

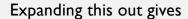




 $\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$  -

"Free Term"

$$\mathscr{L} = \left[\frac{1}{2}(\partial_{\mu}\eta)(\partial^{\mu}\eta) - \mu^{2}\eta^{2}\right] + \left[\frac{1}{2}(\partial_{\mu}\xi)(\partial^{\mu}\xi)\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^{2}A_{\mu}A^{\mu}\right]$$

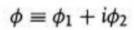




$$m_A = 2\sqrt{\pi} \left( \frac{q\mu}{\lambda c^2} \right)$$

There's a mass term!

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$



Define a couple of new variables

# **QUESTION**: CHANGE OF **VARIABLES**

# **QUESTION**: CHANGE OF VARIABLES



PART I: IS IT LEGAL?

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

# **QUESTION**: CHANGE OF VARIABLES



PART I: IS IT LEGAL?

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

**PART 2**: HOW DO WE DETERMINE THE VALUE?

$$m_A = 2\sqrt{\pi} \left(\frac{q\mu}{\lambda c^2}\right)$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

 Re-Question: What is the mass of this field?

Expand



$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + 1 - \alpha^2 \phi^2 + \frac{1}{2} \alpha^4 \phi^4 - \frac{1}{6} \alpha^6 \phi^6 + \cdots$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi) (\partial^{\mu} \phi) + e^{-(\alpha \phi)^2}$$

 Re-Question: What is the mass of this field?

Expand



$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + 1 - \alpha^2 \phi^2 + \frac{1}{2} \alpha^4 \phi^4 - \frac{1}{6} \alpha^6 \phi^6 + \cdots$$

$$m = \sqrt{2}\alpha\hbar/c$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

**New Question**: What is the mass of this field?



• The sign is wrong hence the mass is **imaginary**!

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)(\partial^{\mu} \phi) + \frac{1}{2} \mu^2 \phi^2 - \frac{1}{4} \lambda^2 \phi^4$$

New Question: What is the mass of this field?



There's always something...

• The sign is wrong hence the mass is **imaginary**!

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!

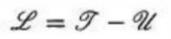
$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

$$\mathcal{L} = \mathcal{T} - \mathcal{U}$$

- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!

Re-Question: What is the mass of this field?

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$



- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!

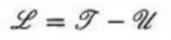


Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$

Re-Question: What is the mass of this field?

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$



- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!



Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$



$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

$$\phi = \pm \mu/\lambda$$

• This occurs at:

Re-Question: What is the mass of this field?

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$

$$\mathcal{L}=\mathcal{T}-\mathcal{U}$$

- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!

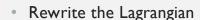


PROPERTY AND THE SAME PARTY NAMED IN

Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\eta)(\partial^{\mu}\eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \frac{1}{4}\lambda^2\eta^4 + \frac{1}{4}(\mu^2/\lambda)^2$$



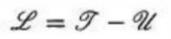


$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

• This occurs at:

Re-Question: What is the mass of this field?

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$



- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!



Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\eta)(\partial^{\mu}\eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \frac{1}{4}\lambda^2\eta^4 + \frac{1}{4}(\mu^2/\lambda)^2$$

Rewrite the Lagrangian

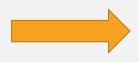




$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

$$\phi = \pm \mu/\lambda$$

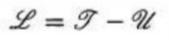




$$m=\sqrt{2}\mu\hbar/c$$

Re-Question: What is the mass of this field?

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + \frac{1}{2}\mu^{2}\phi^{2} - \frac{1}{4}\lambda^{2}\phi^{4}$$



- Feynman calculus is perturbative. Hence we should perturb about the ground state.
- This must not be the ground state as mass cannot be imaginary!



Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\frac{1}{2}\mu^{2}\phi^{2} + \frac{1}{4}\lambda^{2}\phi^{4}$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \eta) (\partial^{\mu} \eta) - \mu^2 \eta^2 \pm \mu \lambda \eta^3 - \frac{1}{4} \lambda^2 \eta^4 + \frac{1}{4} (\mu^2 / \lambda)^2$$

Rewrite the Lagrangian

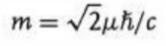






$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

$$\phi = \pm \mu/\lambda$$



This is our change of variables!

• This occurs at:

# **QUESTION**: CHANGE OF **VARIABLES**

# **QUESTION**: CHANGE OF VARIABLES



PART I: IS IT LEGAL?

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

# **QUESTION**: CHANGE OF VARIABLES



PART I: IS IT LEGAL?

 $\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$ 

$$\phi \equiv \phi_1 + i\phi_2$$

**PART 2**: HOW DO WE DETERMINE THE VALUE?

$$\mathscr{U}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$$



# QUESTION: HOW CAN WE GIVE **GAUGE FIELDS MASS?**

# **QUESTION**: HOW CAN WE GIVE GAUGE FIELDS MASS?



PART I: WHAT DO MASS TERMS LOOK LIKE?





PART I: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN

$$\mathcal{L} = \frac{1}{2}(\partial_{\mu}\phi)^*(\partial^{\mu}\phi) + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$

• First: Write out the Lagrangian

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

• First: Write out the Lagrangian

$$\mathcal{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$
$$+ \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2 - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

First: Write out the Lagrangian

• **Second**: Gauge Invariance

$$\mathcal{L} = \frac{1}{2} \left[ \left( \frac{\partial_{\mu}}{\partial \mu} \right)^{2} + \frac{1}{2} \mu^{2} \right]$$

$$\mathcal{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$
$$+ \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2 - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

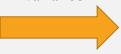
Third: Minimize the "Potential" Energy

$$\tfrac{1}{2}\mu^2(\phi^*\phi)-\tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

• First: Write out the Lagrangian

 Second: Gauge Invariance



$$\mathcal{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$
$$+ \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2 - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

• Third: Minimize the "Potential" Energy

$$\frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$



$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\mathcal{L} = \left[ \frac{1}{2} (\partial_{\mu} \eta) (\partial^{\mu} \eta) - \mu^{2} \eta^{2} \right] + \left[ \frac{1}{2} (\partial_{\mu} \xi) (\partial^{\mu} \xi) \right]$$

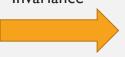
$$+ \left[ -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left( \frac{q}{\hbar c} \frac{\mu}{\lambda} \right)^{2} A_{\mu} A^{\mu} \right]$$

• Fourth: Rewrite in terms of ground state

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

First: Write out the Lagrangian

 Second: Gauge Invariance



$$\mathcal{L} = \frac{1}{2} \left[ \left( \partial_{\mu} - \frac{iq}{\hbar c} A_{\mu} \right) \phi^* \right] \left[ \left( \partial^{\mu} + \frac{iq}{\hbar c} A^{\mu} \right) \phi \right]$$
$$+ \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2 - \frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

$$m_A = 2\sqrt{\pi} \left(\frac{q\mu}{\lambda c^2}\right)$$

• Third: Minimize the "Potential" Energy

$$\frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$



$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\mathcal{L} = \left[ \frac{1}{2} (\partial_{\mu} \eta) (\partial^{\mu} \eta) - \mu^{2} \eta^{2} \right] + \left[ \frac{1}{2} (\partial_{\mu} \xi) (\partial^{\mu} \xi) \right] + \left[ -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left( \frac{q}{\hbar c} \frac{\mu}{\lambda} \right)^{2} A_{\mu} A^{\mu} \right]$$

• Fourth: Rewrite in terms of ground state

## **SOMETHINGS FISHY: NOTHING IN** PHYSICS HAS ZERO DRAWBACKS

### WHITE LIES

 Full Lagrangian: Started with two particles now three??

$$\mathcal{L} = \left[ \frac{1}{2} (\partial_{\mu} \eta) (\partial^{\mu} \eta) - \mu^{2} \eta^{2} \right] + \left[ \frac{1}{2} (\partial_{\mu} \xi) (\partial^{\mu} \xi) \right]$$

$$+ \left[ -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2} \left( \frac{q}{\hbar c} \frac{\mu}{\lambda} \right)^{2} A_{\mu} A^{\mu} \right]$$

$$+ \left[ \frac{q}{\hbar c} [\eta (\partial_{\mu} \xi) - \xi (\partial_{\mu} \eta)] A^{\mu} + \frac{\mu}{\lambda} \left( \frac{q}{\hbar c} \right)^{2} \eta (A_{\mu} A^{\mu})$$

$$+ \frac{1}{2} \left( \frac{q}{\hbar c} \right)^{2} (\xi^{2} + \eta^{2}) (A_{\mu} A^{\mu}) - \lambda_{\mu} \iota (\eta^{3} + \eta \xi^{2}) - \frac{1}{4} \lambda_{\mu}^{2} (\eta^{4} + 2\eta^{2} \xi^{2} + \xi^{4}) \right]$$

$$+ \left( \frac{\mu}{\lambda} \frac{q}{\hbar c} \right) (\partial_{\mu} \xi) A^{\mu} + \left( \frac{\mu^{2}}{2\lambda} \right)^{2}$$

$$(10.131)$$

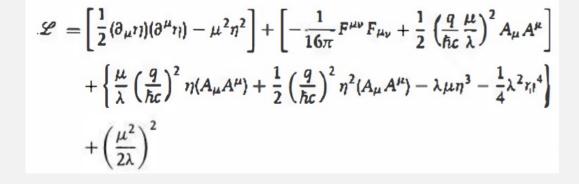
### WHITE LIES

$$\phi \to \phi' = (\cos \theta + i \sin \theta)(\phi_1 + i \phi_2)$$
$$= (\phi_1 \cos \theta - \phi_2 \sin \theta) + i(\phi_1 \sin \theta + \phi_2 \cos \theta)$$

Choose a new gauge

We want the wavefunction to just be real

$$\theta = -\tan^{-1}(\phi_2/\phi_1)$$



The particle is gauged away. What is the meaning?

PHYSICAL REVIEW

VOLUME 127, NUMBER 3

AUGUST 1, 1962

### Broken Symmetries\*

JEFFREY GOLDSTONE
Trinity College, Cambridge University, Cambridge, England

AND

ABDUS SALAM AND STEVEN WEINBERG†
Imperial College of Science and Technology, London, England
(Received March 16, 1962)

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

**GOLDSTONE** 

### **GOLDSTONE**

$$T_{ij}^{\alpha}\langle\phi_j\rangle_0=0.$$

### **GOLDSTONE**

$$T_{ij}^{\alpha}\langle\phi_j\rangle_0=0.$$

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

### GOLDSTONE

$$\phi_i = \chi_i + \eta_i$$

$$T_{ij}^{\alpha}\langle\phi_j\rangle_0=0.\quad \langle\chi_i\rangle_0\equiv0.\quad \eta_i\equiv\langle\phi_i\rangle_0,$$

$$\langle \chi_i \rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle \phi_i \rangle_0,$$

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

### **GOLDSTONE**

$$\phi_i = \chi_i + \eta_i$$

$$T_{ij}^{\alpha}\langle\phi_{j}\rangle_{0}=0.\quad \langle\chi_{i}\rangle_{0}\equiv0.\quad \eta_{i}\equiv\langle\phi_{i}\rangle_{0},$$

$$\langle \chi_i \rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle \phi_i \rangle_0$$

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

### **GOLDSTONE**

$$\phi_i = \chi_i + \eta_i$$

$$T_{ij}^{\alpha}\langle\phi_j\rangle_0\neq0$$

$$T_{ij}^{\alpha}\langle\phi_{j}\rangle_{0}=0. \quad \langle\chi_{i}\rangle_{0}\equiv0. \quad \eta_{i}\equiv\langle\phi_{i}\rangle_{0},$$

$$\langle \chi_i \rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle \phi_i \rangle_0$$
,

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

### **GOLDSTONE**

$$\phi_i = \chi_i + \eta_i$$

$$T_{ij}^{\alpha}\langle\phi_j\rangle_0\neq0$$

$$T_{ij}^{\alpha}\langle\phi_{j}\rangle_{0}=0. \quad \langle\chi_{i}\rangle_{0}\equiv0. \quad \eta_{i}\equiv\langle\phi_{i}\rangle_{0},$$

$$\langle \chi_i \rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle \phi_i \rangle_0,$$

there must exist spinless particles of zero mass.

$$\mathscr{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

### **GOLDSTONE**

$$\phi_i = \chi_i + \eta_i$$

$$T_{ij}^{\alpha}\langle\phi_j\rangle_0\neq0$$

$$T_{ij}^{\alpha}\langle\phi_{j}\rangle_{0}=0. \quad \langle\chi_{i}\rangle_{0}\equiv0. \quad \eta_{i}\equiv\langle\phi_{i}\rangle_{0},$$

$$\langle \chi_i \rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle \phi_i \rangle_0,$$

there must exist spinless particles of zero mass.

$$\mathcal{L} = \frac{1}{2} (\partial_{\mu} \phi)^* (\partial^{\mu} \phi) + \frac{1}{2} \mu^2 (\phi^* \phi) - \frac{1}{4} \lambda^2 (\phi^* \phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\left[\frac{1}{2}(\partial_{\mu}\xi)(\partial^{\mu}\xi)\right]$$

# QUESTION: WHAT TO DO WITH THE EXTRA BOSONS?



as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just

### BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

### Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland (Received 31 August 1964)



### BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs

Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland (Received 31 August 1964)

as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just

## Goldstone bosons become longitudinal 3<sup>RD</sup> DOF for the Gauge Field!

### **QUESTION:** CAN WE USE THIS TO GIVE THE W,Z BOSONS MASS AND DOF?

- I. Write out the Lagrangian
- 2. Minimize the potential
- 3. Apply Gauge Invariance (Covariant Derivative)
- 4. Insert the Ground State into the new Lagrangian
- 5. Simplify
- 6. Read off Masses of the Gauge Bosons

$$L = (D_{\mu}\phi)^{\dagger}D^{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi)^{2}$$

**First**: Consider Higgs Lagrangian in SU2

$$L = (D_{\mu}\phi)^{\dagger}D^{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi)^{2}$$

First: Consider Higgs Lagrangian in

SU2



$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

We have the potential:

$$L = (D_{\mu}\phi)^{\dagger}D^{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi)^{2}$$

First: Consider Higgs Lagrangian in

SU2



$$V(\phi) = -\mu^2 \phi^{\dagger} \phi + \lambda \left( \phi^{\dagger} \phi \right)^2$$



We have the potential:

$$<\phi^{\dagger}\phi> = \frac{v^2}{2}$$

Second: Minimize the potential

$$L = (D_{\mu}\phi)^{\dagger}D^{\mu}\phi + \mu^{2}\phi^{\dagger}\phi - \lambda(\phi^{\dagger}\phi)^{2}$$

First: Consider Higgs Lagrangian in

SU2



$$V(\phi) = -\mu^2 \phi^{\dagger} \phi + \lambda \left( \phi^{\dagger} \phi \right)^2$$

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

We have the potential:

**Third**: Obtain the new ground state



$$<\phi^{\dagger}\phi> = \frac{v^2}{2}$$

Second: Minimize the potential

I. Write out the Lagrangian



2. Minimize the potential



- 3. Apply Gauge Invariance (Covariant Derivative)
- 4. Insert the Ground State into the new Lagrangian
- 5. Simplify
- 6. Read off Masses of the Gauge Bosons

We have the ground state so we can "break" the Lagrangian!

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_{\mu}\phi = \left(\partial_{\mu} + ig \ T^{i}W_{\mu}^{i} + i\frac{1}{2}g'B_{\mu}\right)\phi$$

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_{\mu}\phi = \left(\partial_{\mu} + ig \ T^{i}W_{\mu}^{i} + i\frac{1}{2}g'B_{\mu}\right)\phi$$

**Fourth:** Insert the ground state

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_{\mu}\phi = \left(\partial_{\mu} + ig \ T^{i}W_{\mu}^{i} + i\frac{1}{2}g'B_{\mu}\right)\phi$$

**Fourth:** Insert the ground state

$$(D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) = \left| \left( \partial_{\mu} + \frac{i}{2}g\tau^{k}W_{\mu}^{k} + \frac{i}{2}g'B_{\mu} \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^{2}$$

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_{\mu}\phi = \left(\partial_{\mu} + ig \ T^{i}W_{\mu}^{i} + i\frac{1}{2}g'B_{\mu}\right)\phi$$

**Fourth:** Insert the ground state

$$(D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) = \left| \left( \partial_{\mu} + \frac{i}{2}g\tau^{k}W_{\mu}^{k} + \frac{i}{2}g'B_{\mu} \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^{2}$$

$$\frac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

**Fifth**: Simplify the solution

I. Write out the Lagrangian



2. Minimize the potential



3. Apply Gauge Invariance (Covariant Derivative)



4. Insert the Ground State into the new Lagrangian



5. Simplify



6. Read off Masses of the Gauge Bosons

$$\frac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

$$\left[ rac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

$$W_{\mu}^{\pm}\equivrac{1}{\sqrt{2}}\left(W_{\mu}^{1}\mp iW_{\mu}^{2}
ight)$$

$$\frac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

$$W_{\mu}^{\pm}\equivrac{1}{\sqrt{2}}\left(W_{\mu}^{1}\mp iW_{\mu}^{2}
ight)$$



$$m_W=rac{g\ v}{2}$$
 .

$$\left[ rac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 
ight)^2 + \left( W_{\mu}^2 
ight)^2 
ight) + \left( g W_{\mu}^3 - g' B_{\mu} 
ight)^2 
ight]$$

$$W_{\mu}^{\pm}\equivrac{1}{\sqrt{2}}\left(W_{\mu}^{1}\mp iW_{\mu}^{2}
ight)$$

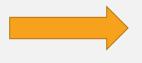


$$m_W=rac{g\ v}{2}$$
 .

$$Z_{\mu} \equiv rac{1}{\sqrt{g^2 + g'^2}} \left( gW_{\mu}^3 - g'B_{\mu} \right)$$

$$\left[ rac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

$$W_{\mu}^{\pm}\equivrac{1}{\sqrt{2}}\left(W_{\mu}^{1}\mp iW_{\mu}^{2}
ight)$$



$$m_W=rac{g\ v}{2}$$
 .

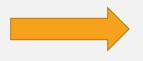
$$Z_{\mu} \equiv rac{1}{\sqrt{g^2 + g'^2}} \left( gW_{\mu}^3 - g'B_{\mu} \right)$$



$$m_Z = rac{v}{2}\sqrt{g^2 + g'^2}$$

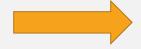
$$\left[ rac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

$$W_{\mu}^{\pm}\equivrac{1}{\sqrt{2}}\left(W_{\mu}^{1}\mp iW_{\mu}^{2}
ight)$$



$$m_W=rac{g\ v}{2}$$
 .

$$Z_{\mu} \equiv rac{1}{\sqrt{g^2 + g'^2}} \left( g W_{\mu}^3 - g' B_{\mu} 
ight)$$

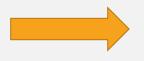


$$m_Z = rac{v}{2}\sqrt{g^2 + g'^2}$$

$$A_{\mu} \equiv \frac{1}{\sqrt{g^2 + g'^2}} \left( g' W_{\mu}^3 + g B_{\mu} \right)$$

$$\frac{v^2}{8} \left[ g^2 \left( \left( W_{\mu}^1 \right)^2 + \left( W_{\mu}^2 \right)^2 \right) + \left( g W_{\mu}^3 - g' B_{\mu} \right)^2 \right]$$

$$W_{\mu}^{\pm}\equivrac{1}{\sqrt{2}}\left(W_{\mu}^{1}\mp iW_{\mu}^{2}
ight)$$

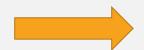


$$m_W=rac{g\ v}{2}$$
 .

$$Z_{\mu} \equiv rac{1}{\sqrt{g^2 + g'^2}} \left( gW_{\mu}^3 - g'B_{\mu} \right)$$

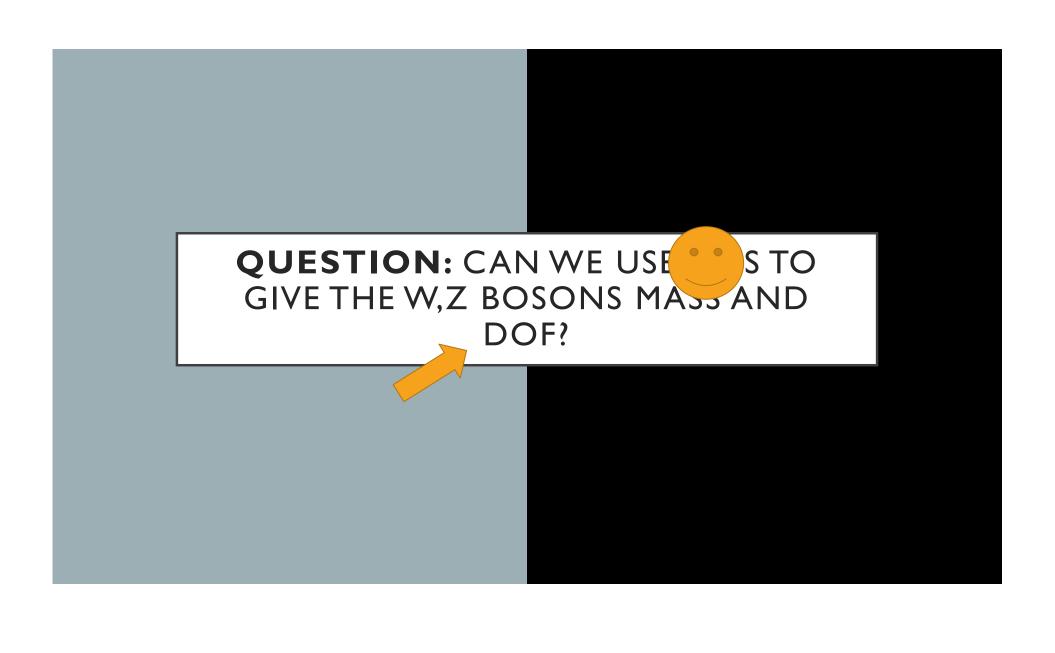
$$m_Z = rac{v}{2}\sqrt{g^2 + g'^2}$$

$$A_{\mu} \equiv rac{1}{\sqrt{g^2 + g'^2}} \left( g' W_{\mu}^3 + g B_{\mu} 
ight)$$



$$m_A = 0$$

### **QUESTION:** CAN WE USE THIS TO GIVE THE W,Z BOSONS MASS AND DOF?



$$\frac{1}{2}\sigma^{1} < \phi > = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^{2} < \phi > = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^{3} < \phi > = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/2\sqrt{2} \end{pmatrix} \neq 0$$

$$\frac{1}{2}I < \phi > = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ v/2\sqrt{2} \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^{1} < \phi > = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^{2} < \phi > = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^{3} < \phi > = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/2\sqrt{2} \end{pmatrix} \neq 0$$

$$\frac{1}{2}I < \phi > = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ v/2\sqrt{2} \end{pmatrix} \neq 0$$

Four Broken Generators



$$T^{3} + Y = \left(\frac{1}{2}\sigma^{2} + \frac{1}{2}I\right) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

$$T^{3} + Y = \left(\frac{1}{2}\sigma^{2} + \frac{1}{2}I\right) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

$$T^{3} + Y = \left(\frac{1}{2}\sigma^{2} + \frac{1}{2}I\right) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

We have that this is U(I) Hypercharge

$$T^{3} + Y = \left(\frac{1}{2}\sigma^{2} + \frac{1}{2}I\right) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

We have that this is U(I) Hypercharge

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$T^{3} + Y = \left(\frac{1}{2}\sigma^{2} + \frac{1}{2}I\right) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

We have that this is U(I) Hypercharge

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

$$T^{3} + Y = \left(\frac{1}{2}\sigma^{2} + \frac{1}{2}I\right) \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

We have that this is U(I) Hypercharge

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

Hence the GWS symmetry degenerates into electromagnetic symmetry