THE HIGGS MECHANISM

Evan Craft

GAUGE THEORY

All familiar with Gauge Theory

$$
\begin{gathered}
\mathcal{L}_{SM} = -\tfrac{1}{2}\partial_\nu g^a_\mu \partial_\nu g^a_\mu - g_s f^{abc}\partial_\mu g^a_\nu g^b_\mu g^c_\nu - \tfrac{1}{4}g_s^2 f^{abc}f^{ade}g^b_\mu g^c_\nu g^d_\mu g^e_\nu - \partial_\nu W^+_\mu \partial_\nu W^-_\mu - \\
M^2 W^+_\mu W^-_\mu - \tfrac{1}{2}\partial_\nu Z^0_\mu \partial_\nu Z^0_\mu - \tfrac{1}{2c_w^2}M^2 Z^0_\mu Z^0_\mu - \tfrac{1}{2}\partial_\mu A_\nu \partial_\mu A_\nu - igc_w(\partial_\nu Z^0_\mu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - Z^0_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + Z^0_\mu(W^+_\nu \partial_\nu W^-_\mu - W^-_\nu \partial_\nu W^+_\mu)) - \\
igs_w(\partial_\nu A_\mu(W^+_\mu W^-_\nu - W^+_\nu W^-_\mu) - A_\nu(W^+_\mu \partial_\nu W^-_\mu - W^-_\mu \partial_\nu W^+_\mu) + A_\mu(W^+_\nu \partial_\nu W^-_\mu - \\
W^-_\nu \partial_\nu W^+_\mu)) - \tfrac{1}{2}g^2 W^+_\mu W^-_\mu W^+_\nu W^-_\nu + \tfrac{1}{2}g^2 W^+_\mu W^-_\nu W^+_\mu W^-_\nu + g^2 c_w^2(Z^0_\mu W^+_\mu Z^0_\nu W^-_\nu - \\
Z^0_\mu Z^0_\mu W^+_\nu W^-_\nu) + g^2 s_w^2(A_\mu W^+_\mu A_\nu W^-_\nu - A_\mu A_\mu W^+_\nu W^-_\nu) + g^2 s_w c_w(A_\mu Z^0_\nu(W^+_\mu W^-_\nu - \\
W^+_\nu W^-_\mu) - 2A_\mu Z^0_\mu W^+_\nu W^-_\nu) - \tfrac{1}{2}\partial_\mu H \partial_\mu H - 2M^2\alpha_h H^2 - \partial_\mu \phi^+ \partial_\mu \phi^- - \tfrac{1}{2}\partial_\mu \phi^0 \partial_\mu \phi^0 - \\
\beta_h\left(\tfrac{2M^2}{g^2} + \tfrac{2M}{g}H + \tfrac{1}{2}(H^2 + \phi^0\phi^0 + 2\phi^+\phi^-)\right) + \tfrac{2M^4}{g^2}\alpha_h - \\
g\alpha_h M\left(H^3 + H\phi^0\phi^0 + 2H\phi^+\phi^-\right) - \\
\tfrac{1}{8}g^2\alpha_h\left(H^4 + (\phi^0)^4 + 4(\phi^+\phi^-)^2 + 4(\phi^0)^2\phi^+\phi^- + 4H^2\phi^+\phi^- + 2(\phi^0)^2H^2\right) - \\
gMW^+_\mu W^-_\mu H - \tfrac{1}{2}g\tfrac{M}{c_w^2}Z^0_\mu Z^0_\mu H - \\
\tfrac{1}{2}ig\left(W^+_\mu(\phi^0\partial_\mu\phi^- - \phi^-\partial_\mu\phi^0) - W^-_\mu(\phi^0\partial_\mu\phi^+ - \phi^+\partial_\mu\phi^0)\right) + \\
\tfrac{1}{2}g\left(W^+_\mu(H\partial_\mu\phi^- - \phi^-\partial_\mu H) + W^-_\mu(H\partial_\mu\phi^+ - \phi^+\partial_\mu H)\right) + \tfrac{1}{2}g\tfrac{1}{c_w}(Z^0_\mu(H\partial_\mu\phi^0 - \phi^0\partial_\mu H) + \\
M\left(\tfrac{1}{c_w}Z^0_\mu\partial_\mu\phi^0 + W^+_\mu\partial_\mu\phi^- + W^-_\mu\partial_\mu\phi^+\right) - ig\tfrac{s_w^2}{c_w}MZ^0_\mu(W^+_\mu\phi^- - W^-_\mu\phi^+) + igs_w MA_\mu(W^+_\mu\phi^- - \\
W^-_\mu\phi^+) - ig\tfrac{1-2c_w^2}{2c_w}Z^0_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) + igs_w A_\mu(\phi^+\partial_\mu\phi^- - \phi^-\partial_\mu\phi^+) - \\
\tfrac{1}{4}g^2W^+_\mu W^-_\mu\left(H^2 + (\phi^0)^2 + 2\phi^+\phi^-\right) - \tfrac{1}{8}g^2\tfrac{1}{c_w^2}Z^0_\mu Z^0_\mu\left(H^2 + (\phi^0)^2 + 2(2s_w^2 - 1)^2\phi^+\phi^-\right) - \\
\tfrac{1}{2}g^2\tfrac{s_w^2}{c_w}Z^0_\mu\phi^0(W^+_\mu\phi^- + W^-_\mu\phi^+) - \tfrac{1}{2}ig^2\tfrac{s_w^2}{c_w}Z^0_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) + \tfrac{1}{2}g^2 s_w A_\mu\phi^0(W^+_\mu\phi^- + \\
W^-_\mu\phi^+) + \tfrac{1}{2}ig^2 s_w A_\mu H(W^+_\mu\phi^- - W^-_\mu\phi^+) - g^2\tfrac{s_w}{c_w}(2c_w^2 - 1)Z^0_\mu A_\mu\phi^+\phi^- - \\
g^2 s_w^2 A_\mu A_\mu\phi^+\phi^- + \tfrac{1}{2}ig_s\lambda^a_{ij}(\bar{q}^\sigma_i\gamma^\mu q^\sigma_j)g^a_\mu - \bar{e}^\lambda(\gamma\partial + m^\lambda_e)e^\lambda - \bar{\nu}^\lambda(\gamma\partial + m^\lambda_\nu)\nu^\lambda - \bar{u}^\lambda_j(\gamma\partial + \\
m^\lambda_u)u^\lambda_j - \bar{d}^\lambda_j(\gamma\partial + m^\lambda_d)d^\lambda_j + igs_w A_\mu\left(-(\bar{e}^\lambda\gamma^\mu e^\lambda) + \tfrac{2}{3}(\bar{u}^\lambda_j\gamma^\mu u^\lambda_j) - \tfrac{1}{3}(\bar{d}^\lambda_j\gamma^\mu d^\lambda_j)\right) + \\
\tfrac{ig}{4c_w}Z^0_\mu\{(\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{e}^\lambda\gamma^\mu(4s_w^2 - 1 - \gamma^5)e^\lambda) + (\bar{d}^\lambda_j\gamma^\mu(\tfrac{4}{3}s_w^2 - 1 - \gamma^5)d^\lambda_j) + \\
(\bar{u}^\lambda_j\gamma^\mu(1 - \tfrac{8}{3}s_w^2 + \gamma^5)u^\lambda_j)\} + \tfrac{ig}{2\sqrt{2}}W^+_\mu\left((\bar{\nu}^\lambda\gamma^\mu(1+\gamma^5){U^{lep}}_{\lambda\kappa}e^\kappa) + (\bar{u}^\lambda_j\gamma^\mu(1+\gamma^5)C_{\lambda\kappa}d^\kappa_j)\right) + \\
\tfrac{ig}{2\sqrt{2}}W^-_\mu\left((\bar{e}^\kappa {U^{lep}}^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)\nu^\lambda) + (\bar{d}^\kappa_j C^\dagger_{\kappa\lambda}\gamma^\mu(1+\gamma^5)u^\lambda_j)\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_e(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1-\gamma^5)e^\kappa) + m^\lambda_\nu(\bar{\nu}^\lambda {U^{lep}}_{\lambda\kappa}(1+\gamma^5)e^\kappa\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_e(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1+\gamma^5)\nu^\kappa) - m^\kappa_\nu(\bar{e}^\lambda {U^{lep}}^\dagger_{\lambda\kappa}(1-\gamma^5)\nu^\kappa)\right) - \tfrac{g}{2}\tfrac{m^\lambda_\nu}{M}H(\bar{\nu}^\lambda\nu^\lambda) - \\
\tfrac{g}{2}\tfrac{m^\lambda_e}{M}H(\bar{e}^\lambda e^\lambda) + \tfrac{ig}{2}\tfrac{m^\lambda_\nu}{M}\phi^0(\bar{\nu}^\lambda\gamma^5\nu^\lambda) - \tfrac{ig}{2}\tfrac{m^\lambda_e}{M}\phi^0(\bar{e}^\lambda\gamma^5 e^\lambda) - \tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa - \\
\tfrac{1}{4}\bar{\nu}_\lambda M^R_{\lambda\kappa}(1-\gamma_5)\hat{\nu}_\kappa + \tfrac{ig}{2M\sqrt{2}}\phi^+\left(-m^\kappa_d(\bar{u}^\lambda_j C_{\lambda\kappa}(1-\gamma^5)d^\kappa_j) + m^\lambda_u(\bar{u}^\lambda_j C_{\lambda\kappa}(1+\gamma^5)d^\kappa_j\right) + \\
\tfrac{ig}{2M\sqrt{2}}\phi^-\left(m^\lambda_d(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1+\gamma^5)u^\kappa_j) - m^\kappa_u(\bar{d}^\lambda_j C^\dagger_{\lambda\kappa}(1-\gamma^5)u^\kappa_j)\right) - \tfrac{g}{2}\tfrac{m^\lambda_u}{M}H(\bar{u}^\lambda_j u^\lambda_j) - \\
\tfrac{g}{2}\tfrac{m^\lambda_d}{M}H(\bar{d}^\lambda_j d^\lambda_j) + \tfrac{ig}{2}\tfrac{m^\lambda_u}{M}\phi^0(\bar{u}^\lambda_j\gamma^5 u^\lambda_j) - \tfrac{ig}{2}\tfrac{m^\lambda_d}{M}\phi^0(\bar{d}^\lambda_j\gamma^5 d^\lambda_j) + \bar{G}^a\partial^2 G^a + g_s f^{abc}\partial_\mu\bar{G}^a G^b g^c_\mu + \\
\bar{X}^+(\partial^2 - M^2)X^+ + \bar{X}^-(\partial^2 - M^2)X^- + \bar{X}^0(\partial^2 - \tfrac{M^2}{c_w^2})X^0 + \bar{Y}\partial^2 Y + igc_w W^+_\mu(\partial_\mu\bar{X}^0 X^- - \\
\partial_\mu\bar{X}^+ X^0) + igs_w W^+_\mu(\partial_\mu\bar{Y}X^- - \partial_\mu\bar{X}^+ Y) + igc_w W^-_\mu(\partial_\mu\bar{X}^- X^0 - \\
\partial_\mu\bar{X}^0 X^+) + igs_w W^-_\mu(\partial_\mu\bar{X}^- Y - \partial_\mu\bar{Y}X^+) + igc_w Z^0_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) + igs_w A_\mu(\partial_\mu\bar{X}^+ X^+ - \\
\partial_\mu\bar{X}^- X^-) - \tfrac{1}{2}gM\left(\bar{X}^+X^+H + \bar{X}^-X^-H + \tfrac{1}{c_w^2}\bar{X}^0X^0H\right) + \tfrac{1-2c_w^2}{2c_w}igM\left(\bar{X}^+X^0\phi^+ - \bar{X}^-X^0\phi^-\right) + \\
\tfrac{1}{2c_w}igM\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + igMs_w\left(\bar{X}^0X^-\phi^+ - \bar{X}^0X^+\phi^-\right) + \\
\tfrac{1}{2}igM\left(\bar{X}^+X^+\phi^0 - \bar{X}^-X^-\phi^0\right).
\end{gathered}
$$

GAUGE THEORY

$$\mathscr{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- Given the action for a particle

GAUGE THEORY

$$\mathscr{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- Given the action for a particle

$$\psi \to e^{i\theta(x)}\psi \qquad \text{(local phase transformation)}$$

- Want this to be invariant under some local transformation

GAUGE THEORY

$$\mathscr{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- Given the action for a particle

$$\psi \to e^{i\theta(x)}\psi \qquad \text{(local phase transformation)}$$

- Want this to be invariant under some local transformation

$$\partial_{\mu}(e^{i\theta}\psi) = i(\partial_{\mu}\theta)e^{i\theta}\psi + e^{i\theta}\partial_{\mu}\psi$$

- However the derivative adds an extra term

GAUGE THEORY

$$\mathcal{L} = i\hbar c\bar{\psi}\gamma^\mu\partial_\mu\psi - mc^2\bar{\psi}\psi$$

- Given the action for a particle

$$\psi \to e^{i\theta(x)}\psi \qquad \text{(local phase transformation)}$$

- Want this to be invariant under some local transformation

$$\partial_\mu(e^{i\theta}\psi) = i(\partial_\mu\theta)e^{i\theta}\psi + e^{i\theta}\partial_\mu\psi$$

- However the derivative adds an extra term

- Hence we have:

$$\mathcal{L} \to \mathcal{L} + (q\bar{\psi}\gamma^\mu\psi)\partial_\mu\lambda$$

GAUGE THEORY

$$\mathscr{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- **Idea:** Rewrite the original Lagrangian

GAUGE THEORY

$$\mathcal{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- **Idea:** Rewrite the original Lagrangian

$$\mathcal{L} = [i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi] - (q\overline{\psi}\gamma^{\mu}\psi)A_{\mu}$$

GAUGE THEORY

$$\mathscr{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- **Idea:** Rewrite the original Lagrangian

$$\mathscr{L} = [i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi] - (q\overline{\psi}\gamma^{\mu}\psi)A_{\mu}$$

- Where: $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$

GAUGE THEORY

$$\mathcal{L} = i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi$$

- **Idea:** Rewrite the original Lagrangian

$$\mathcal{L} = [i\hbar c\overline{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\overline{\psi}\psi] - (q\overline{\psi}\gamma^{\mu}\psi)A_{\mu}$$

- Where: $A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$

This **cancels** the offending term!

GAUGE THEORY

$$\mathcal{L} = i\hbar c\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\bar{\psi}\psi$$

- **Idea:** Rewrite the original Lagrangian

$$\mathcal{L} = [i\hbar c\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - mc^2\bar{\psi}\psi] - (q\bar{\psi}\gamma^{\mu}\psi)A_{\mu}$$

- Where:

$$A_{\mu} \rightarrow A_{\mu} + \partial_{\mu}\lambda$$

This **cancels** the offending term!

The Lagrangian is **changed** however

GAUGE THEORY

$$\lambda(x) \equiv -\frac{\hbar c}{q}\theta(x)$$

- **Define:** Some new variable

GAUGE THEORY

$$\lambda(x) \equiv -\frac{\hbar c}{q}\theta(x)$$

- **Define:** Some new variable

- **Define:** The covariant derivative

$$\mathscr{D}_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}A_\mu$$

GAUGE THEORY

$$\lambda(x) \equiv -\frac{\hbar c}{q}\theta(x)$$

- **Define:** Some new variable

- **Define:** The covariant derivative

$$\mathscr{D}_\mu \equiv \partial_\mu + i\frac{q}{\hbar c}A_\mu$$

Wherever there is a derivative, replace it with the **covariant derivative**

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

- The field obeys the Proca action

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

- The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

- The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

$$A_\mu \to A_\mu + \partial_\mu \lambda$$

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

- The field obeys the Proca action

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

$$A^\nu A_\nu$$

- However, this piece is not gauge invariant

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

- The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

$$A_\mu \rightarrow A_\mu + \partial_\mu \lambda$$

$$A^\nu A_\nu$$

- However, this piece is not gauge invariant

$$\mathscr{L} = [i\hbar c\overline{\psi}\gamma^\mu \partial_\mu \psi - mc^2\overline{\psi}\psi] - \left[\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}\right] - (q\overline{\psi}\gamma^\mu \psi)A_\mu$$

GAUGE THEORY

- The vector field has its own equations of motion which must be included in the Lagrangian

- The field obeys the Proca action

$$\mathscr{L} = \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{m_A c}{\hbar}\right)^2 A^\nu A_\nu$$

$$A_\mu \to A_\mu + \partial_\mu \lambda$$

$$A^\nu A_\nu$$

We conclude that the Gauge field must be **massless**

- However, this piece is not gauge invariant

$$\mathscr{L} = [i\hbar c \bar{\psi} \gamma^\mu \partial_\mu \psi - mc^2 \bar{\psi} \psi] - \left[\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}\right] - (q \bar{\psi} \gamma^\mu \psi) A_\mu$$

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

PART 1: WHAT DO MASS TERMS LOOK LIKE?

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

PART 1: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Question**: What is the mass of this field?

IDENTIFYING MASS TERMS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Question**: What is the mass of this field?

- Seems to be zero!
- No terms proportional to the wave function squared

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Question**: What is the mass of this field?

- Seems to be zero!
- No terms proportional to the wave function squared

$$\mathscr{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^\mu \phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4 - \frac{1}{6}\alpha^6\phi^6 + \cdots$$

- Expanding the exponential we se otherwise

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Question**: What is the mass of this field?

- Seems to be zero!
- No terms proportional to the wave function squared

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4 - \frac{1}{6}\alpha^6\phi^6 + \cdots$$

- Expanding the exponential we se otherwise

$$m = \sqrt{2}\alpha\hbar/c$$

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

PART 1: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN

$$\mathscr{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4 - \frac{1}{6}\alpha^6\phi^6 + \cdots$$

GETTING MASS INTO LAGRANGIAN

$$\mathscr{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi} \left(\frac{mc}{\hbar}\right)^2 A^\nu A_\nu$$

- These are Gauge Fields and hence obey the Proca action

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

GETTING MASS INTO LAGRANGIAN

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A^\nu A_\nu$$

- These are Gauge Fields and hence obey the Proca action

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu$$

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

- From the previous analysis, they must be massless

GETTING MASS INTO LAGRANGIAN

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{8\pi}\left(\frac{mc}{\hbar}\right)^2 A^{\nu} A_{\nu}$$

- These are Gauge Fields and hence obey the Proca action

$$F^{\mu\nu} \equiv \partial^{\mu} A^{\nu} - \partial^{\nu} A^{\mu}$$

$$\mathcal{L} = -\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

- From the previous analysis, they must be massless

- **No hidden perturbation….**

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

PART 1: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN

$$\mathscr{L} = \frac{1}{2}(\partial_{\mu}\phi)(\partial^{\mu}\phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4 - \frac{1}{6}\alpha^6\phi^6 + \cdots$$

GETTING MASS INTO LAGRANGIAN (2^{ND} ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

GETTING MASS INTO LAGRANGIAN (2^{ND} ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

GETTING MASS INTO LAGRANGIAN (2^{ND} ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \ \frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

GETTING MASS INTO LAGRANGIAN (2^{ND} ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

$$A_\mu A^\mu$$

Expanding this out gives a term proportional the inner product!

GETTING MASS INTO LAGRANGIAN (2^{ND} ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \; \frac{-1}{16\pi} F^{\mu\nu} F_{\mu\nu}$$

"Free Term"

$$A_\mu A^\mu$$

Expanding this out gives a term proportional the inner product!

$$\phi^* \phi$$

It's also proportional to this, and therefore not a constant

GETTING MASS INTO LAGRANGIAN (2ND ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance

$$\mathcal{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

$$A_\mu A^\mu$$

Expanding this out gives a term proportional the inner product!

$$\phi^*\phi$$

It's also proportional to this, and therefore not a constant

But it seems as if we are onto something…

GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance $\Longrightarrow$

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \quad \frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

GETTING MASS INTO LAGRANGIAN (2ND ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance →

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

Define a couple of new variables

GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$$+ \quad \frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

Define a couple of new variables

$$\mathscr{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right]$$

Expanding this out gives

GETTING MASS INTO LAGRANGIAN (3RD ATTEMPT)

- **Question**: What's the simplest Lagrangian you can think of?

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi)$$

Gauge Invariance $\longrightarrow$

"Kinetic Term"

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right]$$

$+$

$$\frac{-1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

"Free Term"

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\phi \equiv \phi_1 + i\phi_2$$

Define a couple of new variables

$$\mathscr{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right]$$

Expanding this out gives $\longrightarrow$

$$m_A = 2\sqrt{\pi}\left(\frac{q\mu}{\lambda c^2}\right)$$

There's a mass term!

QUESTION: CHANGE OF VARIABLES

QUESTION: CHANGE OF VARIABLES

PART 1: IS IT LEGAL?

$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$

$\phi \equiv \phi_1 + i\phi_2$

QUESTION: CHANGE OF VARIABLES

PART 1: IS IT LEGAL?

$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$

$\phi \equiv \phi_1 + i\phi_2$

PART 2: HOW DO WE DETERMINE THE VALUE?

$m_A = 2\sqrt{\pi}\left(\frac{q\mu}{\lambda c^2}\right)$

IDENTIFYING MASS TERMS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Re-Question**: What is the mass of this field?

IDENTIFYING MASS TERMS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Re-Question**: What is the mass of this field?

- Expand

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4 - \frac{1}{6}\alpha^6\phi^6 + \cdots$$

IDENTIFYING MASS TERMS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + e^{-(\alpha\phi)^2}$$

- **Re-Question**: What is the mass of this field?

- Expand

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + 1 - \alpha^2\phi^2 + \frac{1}{2}\alpha^4\phi^4 - \frac{1}{6}\alpha^6\phi^6 + \cdots$$

$$m = \sqrt{2}\alpha\hbar/c$$

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- **New Question**: What is the mass of this field?

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- **New Question**: What is the mass of this field?

- The sign is wrong hence the mass is **imaginary**!

IDENTIFYING MASS TERMS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- **New Question**: What is the mass of this field?

- The sign is wrong hence the mass is **imaginary**!

There's always something…

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- **Re-Question**: What is the mass of this field?

IDENTIFYING MASS TERMS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

- **Re-Question**: What is the mass of this field?

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

- **Re-Question**: What is the mass of this field?

$$\mathscr{L} = \mathscr{T} - \mathscr{U}$$

IDENTIFYING MASS TERMS

- **Re-Question**: What is the mass of this field?

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

$$\mathcal{L} = \mathcal{T} - \mathcal{U}$$

- Minimize the "Potential" Energy

$$\mathcal{U}(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

IDENTIFYING MASS TERMS

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

- **Re-Question**: What is the mass of this field?

$$\mathscr{L} = \mathscr{T} - \mathscr{U}$$

- Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

$$\phi = \pm\mu/\lambda$$

- This occurs at:

IDENTIFYING MASS TERMS

- **Re-Question**: What is the mass of this field?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

$$\mathscr{L} = \mathscr{T} - \mathscr{U}$$

- Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

$$\phi = \pm\mu/\lambda$$

- This occurs at:

$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \tfrac{1}{4}\lambda^2\eta^4 + \tfrac{1}{4}(\mu^2/\lambda)^2$$

- Rewrite the Lagrangian

IDENTIFYING MASS TERMS

- **Re-Question**: What is the mass of this field?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

$$\mathscr{L} = \mathscr{T} - \mathscr{U}$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

- Minimize the "Potential" Energy

$$\mathscr{U}(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

$$\phi = \pm\mu/\lambda$$

- This occurs at:

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \tfrac{1}{4}\lambda^2\eta^4 + \tfrac{1}{4}(\mu^2/\lambda)^2$$

- Rewrite the Lagrangian

$$m = \sqrt{2}\mu\hbar/c$$

IDENTIFYING MASS TERMS

- **Re-Question**: What is the mass of this field?

$$\mathscr{L} = \frac{1}{2}(\partial_\mu \phi)(\partial^\mu \phi) + \frac{1}{2}\mu^2\phi^2 - \frac{1}{4}\lambda^2\phi^4$$

$$\mathscr{L} = \mathscr{T} - \mathscr{U}$$

- Minimize the ”Potential” Energy

$$\mathscr{U}(\phi) = -\tfrac{1}{2}\mu^2\phi^2 + \tfrac{1}{4}\lambda^2\phi^4$$

- This occurs at:

$$\phi = \pm\mu/\lambda$$

$$\eta \equiv \phi \pm \frac{\mu}{\lambda}$$

This is our change of variables!

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2 \pm \mu\lambda\eta^3 - \tfrac{1}{4}\lambda^2\eta^4 + \tfrac{1}{4}(\mu^2/\lambda)^2$$

- Feynman calculus is perturbative. Hence we should perturb about the **ground state**.
- This must not be the ground state as mass cannot be imaginary!

- Rewrite the Lagrangian

$$m = \sqrt{2}\mu\hbar/c$$

QUESTION: CHANGE OF VARIABLES

QUESTION: CHANGE OF VARIABLES

PART 1: IS IT LEGAL?

$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$

$\phi \equiv \phi_1 + i\phi_2$

QUESTION: CHANGE OF VARIABLES

PART 1: IS IT LEGAL?

$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$

$\phi \equiv \phi_1 + i\phi_2$

PART 2: HOW DO WE DETERMINE THE VALUE?

$\mathscr{U}(\phi) = -\frac{1}{2}\mu^2\phi^2 + \frac{1}{4}\lambda^2\phi^4$

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

PART 1: WHAT DO MASS TERMS LOOK LIKE?

QUESTION: HOW CAN WE GIVE GAUGE FIELDS MASS?

PART 1: WHAT DO MASS TERMS LOOK LIKE?

PART 2: HOW CAN WE GET THEM INTO THE LAGRANGIAN

PROCESS

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$

- **First**: Write out the Lagrangian

PROCESS

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

- **First**: Write out the Lagrangian

- **Second**: Gauge Invariance

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right] + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

PROCESS

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

- **First**: Write out the Lagrangian

- **Second**: Gauge Invariance

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right] + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

- **Third**: Minimize the ”Potential” Energy

$$\tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

PROCESS

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu\phi)^*(\partial^\mu\phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

- **First**: Write out the Lagrangian

- **Second**: Gauge Invariance

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right] + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

- **Third**: Minimize the "Potential" Energy

$$\tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\mathscr{L} = \left[\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right]$$

- **Fourth**: Rewrite in terms of ground state

PROCESS

- **First**: Write out the Lagrangian

$$\mathscr{L} = \frac{1}{2}(\partial_\mu\phi)^*(\partial^\mu\phi) + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$

- **Second**: Gauge Invariance

$$\mathscr{L} = \frac{1}{2}\left[\left(\partial_\mu - \frac{iq}{\hbar c}A_\mu\right)\phi^*\right]\left[\left(\partial^\mu + \frac{iq}{\hbar c}A^\mu\right)\phi\right] + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2 - \frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu}$$

- **Third**: Minimize the "Potential" Energy

$$\frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

$$\mathscr{L} = \left[\frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \mu^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu\xi)(\partial^\mu\xi)\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right]$$

- **Fourth**: Rewrite in terms of ground state

$$m_A = 2\sqrt{\pi}\left(\frac{q\mu}{\lambda c^2}\right)$$

SOMETHINGS FISHY: NOTHING IN PHYSICS HAS ZERO DRAWBACKS

WHITE LIES

- Full Lagrangian: Started with two particles now three??

$$
\begin{aligned}
\mathcal{L} = & \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right] \\
& + \left[-\frac{1}{16\pi} F^{\mu\nu} F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right] \\
& + \left\{\frac{q}{\hbar c}[\eta(\partial_\mu \xi) - \xi(\partial_\mu \eta)]A^\mu + \frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2 \eta(A_\mu A^\mu)\right. \\
& \left. + \frac{1}{2}\left(\frac{q}{\hbar c}\right)^2 (\xi^2 + \eta^2)(A_\mu A^\mu) - \lambda\mu(\eta^3 + \eta\xi^2) - \frac{1}{4}\lambda^2(\eta^4 + 2\eta^2\xi^2 + \xi^4)\right\} \\
& + \left(\frac{\mu}{\lambda}\frac{q}{\hbar c}\right)(\partial_\mu \xi)A^\mu + \left(\frac{\mu^2}{2\lambda}\right)^2 \qquad (10.131)
\end{aligned}
$$

WHITE LIES

$$\phi \to \phi' = (\cos\theta + i\sin\theta)(\phi_1 + i\phi_2)$$
$$= (\phi_1 \cos\theta - \phi_2 \sin\theta) + i(\phi_1 \sin\theta + \phi_2 \cos\theta)$$

- Choose a new gauge

- We want the wavefunction to just be real

$$\mathcal{L} = \left[\frac{1}{2}(\partial_\mu \eta)(\partial^\mu \eta) - \mu^2\eta^2\right] + \left[-\frac{1}{16\pi}F^{\mu\nu}F_{\mu\nu} + \frac{1}{2}\left(\frac{q}{\hbar c}\frac{\mu}{\lambda}\right)^2 A_\mu A^\mu\right]$$
$$+ \left\{\frac{\mu}{\lambda}\left(\frac{q}{\hbar c}\right)^2 \eta(A_\mu A^\mu) + \frac{1}{2}\left(\frac{q}{\hbar c}\right)^2 \eta^2(A_\mu A^\mu) - \lambda\mu\eta^3 - \frac{1}{4}\lambda^2\eta^4\right\}$$
$$+ \left(\frac{\mu^2}{2\lambda}\right)^2$$

$$\theta = -\tan^{-1}(\phi_2/\phi_1)$$

- The particle is gauged away. What is the meaning?

PHYSICAL REVIEW VOLUME 127, NUMBER 3 AUGUST 1, 1962

Broken Symmetries*

JEFFREY GOLDSTONE
Trinity College, Cambridge University, Cambridge, England

AND

ABDUS SALAM AND STEVEN WEINBERG†
Imperial College of Science and Technology, London, England

(Received March 16, 1962)

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

OUR PROCESS

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$$T_{ij}{}^{\alpha}\langle\phi_j\rangle_0 = 0.$$

OUR PROCESS

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$$T_{ij}{}^{\alpha}\langle\phi_j\rangle_0=0.$$

OUR PROCESS

$$\mathscr{L} = \tfrac{1}{2}(\partial_\mu\phi)^*(\partial^\mu\phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$$\phi_i = \chi_i + \eta_i,$$

$$T_{ij}{}^{\alpha}\langle\phi_j\rangle_0 = 0.$$

$$\langle\chi_i\rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle\phi_i\rangle_0,$$

OUR PROCESS

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^*(\partial^\mu\phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$$\phi_i = \chi_i + \eta_i,$$

$$T_{ij}{}^{\alpha} \langle \phi_j \rangle_0 = 0. \qquad \langle \chi_i \rangle_0 \equiv 0. \qquad \eta_i \equiv \langle \phi_i \rangle_0,$$

OUR PROCESS

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$$\phi_i = \chi_i + \eta_i,$$

$$T_{ij}{}^{\alpha} \langle \phi_j \rangle_0 \neq 0$$

$$T_{ij}{}^{\alpha} \langle \phi_j \rangle_0 = 0.$$

$$\langle \chi_i \rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle \phi_i \rangle_0,$$

OUR PROCESS

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$$\phi_i = \chi_i + \eta_i,$$

$$T_{ij}{}^{\alpha}\langle\phi_j\rangle_0 \neq 0$$

$$T_{ij}{}^{\alpha}\langle\phi_j\rangle_0 = 0.$$

$$\langle\chi_i\rangle_0 \equiv 0.$$

$$\eta_i \equiv \langle\phi_i\rangle_0,$$

there must exist spinless particles of zero mass.

OUR PROCESS

$$\mathcal{L} = \tfrac{1}{2}(\partial_\mu\phi)^*(\partial^\mu\phi) + \tfrac{1}{2}\mu^2(\phi^*\phi) - \tfrac{1}{4}\lambda^2(\phi^*\phi)^2$$

$$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$$

Some proofs are presented of Goldstone's conjecture, that if there is continuous symmetry transformation under which the Lagrangian is invariant, then either the vacuum state is also invariant under the transformation, or there must exist spinless particles of zero mass.

GOLDSTONE

$\phi_i = \chi_i + \eta_i,$

$T_{ij}^{\alpha} \langle \phi_j \rangle_0 \neq 0$

$T_{ij}^{\alpha} \langle \phi_j \rangle_0 = 0.$

$\langle \chi_i \rangle_0 \equiv 0.$

$\eta_i \equiv \langle \phi_i \rangle_0,$

there must exist spinless particles of zero mass.

OUR PROCESS

$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^*(\partial^\mu \phi) + \frac{1}{2}\mu^2(\phi^*\phi) - \frac{1}{4}\lambda^2(\phi^*\phi)^2$

$\eta \equiv \phi_1 - \mu/\lambda, \quad \xi \equiv \phi_2$

$\left[\frac{1}{2}(\partial_\mu \xi)(\partial^\mu \xi)\right]$

QUESTION: WHAT TO DO WITH THE EXTRA BOSONS?

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just

Goldstone bosons become longitudinal 3RD DOF for the Gauge Field!

QUESTION: CAN WE USE THIS TO GIVE THE W,Z BOSONS MASS AND DOF?

ELECTROWEAK BOSONS

1. Write out the Lagrangian
2. Minimize the potential
3. Apply Gauge Invariance (Covariant Derivative)
4. Insert the Ground State into the new Lagrangian
5. Simplify
6. Read off Masses of the Gauge Bosons

ELECTROWEAK BOSONS

$$L = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

First: Consider Higgs Lagrangian in SU2

ELECTROWEAK BOSONS

$$L = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

First: Consider Higgs Lagrangian in SU2

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

We have the potential:

ELECTROWEAK BOSONS

$$L = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

First: Consider Higgs Lagrangian in SU2

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

We have the potential:

$$< \phi^\dagger \phi >= \frac{v^2}{2}$$

Second: Minimize the potential

ELECTROWEAK BOSONS

$$L = (D_\mu \phi)^\dagger D^\mu \phi + \mu^2 \phi^\dagger \phi - \lambda(\phi^\dagger \phi)^2$$

First: Consider Higgs Lagrangian in SU2

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda \left(\phi^\dagger \phi\right)^2$$

We have the potential:

$$\langle \phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Third: Obtain the new ground state

$$< \phi^\dagger \phi >= \frac{v^2}{2}$$

Second: Minimize the potential

ELECTROWEAK BOSONS

1. Write out the Lagrangian
2. Minimize the potential
3. Apply Gauge Invariance (Covariant Derivative)
4. Insert the Ground State into the new Lagrangian
5. Simplify
6. Read off Masses of the Gauge Bosons

ELECTROWEAK BOSONS

We have the ground state so we can "break" the Lagrangian!

ELECTROWEAK BOSONS

Consider the Covariant derivative

We have the ground state so we can "break" the Lagrangian!

ELECTROWEAK BOSONS

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W_\mu^i + i\frac{1}{2} g' B_\mu \right) \phi$$

ELECTROWEAK BOSONS

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W^i_\mu + i\frac{1}{2} g' B_\mu \right) \phi$$

Fourth: Insert the ground state

ELECTROWEAK BOSONS

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W^i_\mu + i\frac{1}{2} g' B_\mu \right) \phi$$

Fourth: Insert the ground state

$$\left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) = \left| \left(\partial_\mu + \frac{i}{2} g \tau^k W^k_\mu + \frac{i}{2} g' B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

ELECTROWEAK BOSONS

We have the ground state so we can "break" the Lagrangian!

Consider the Covariant derivative

$$D_\mu \phi = \left(\partial_\mu + ig\, T^i W_\mu^i + i\frac{1}{2} g' B_\mu \right) \phi$$

Fourth: Insert the ground state

$$\left(D^\mu \phi\right)^\dagger \left(D_\mu \phi\right) = \left| \left(\partial_\mu + \frac{i}{2} g \tau^k W_\mu^k + \frac{i}{2} g' B_\mu \right) \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix} \right|^2$$

$$\frac{v^2}{8} \left[g^2 \left(\left(W_\mu^1\right)^2 + \left(W_\mu^2\right)^2 \right) + \left(g W_\mu^3 - g' B_\mu\right)^2 \right]$$

Fifth: Simplify the solution

ELECTROWEAK BOSONS

1. Write out the Lagrangian
2. Minimize the potential
3. Apply Gauge Invariance (Covariant Derivative)
4. Insert the Ground State into the new Lagrangian
5. Simplify
6. Read off Masses of the Gauge Bosons

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2+\left(W_\mu^2\right)^2\right)+\left(gW_\mu^3-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2+\left(W_\mu^2\right)^2\right)+\left(gW_\mu^3-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2+\left(W_\mu^2\right)^2\right)+\left(gW_\mu^3-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

$$\Rightarrow \quad m_W = \frac{g\,v}{2}\,.$$

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2+\left(W_\mu^2\right)^2\right)+\left(gW_\mu^3-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right)$$

$$m_W = \frac{g\,v}{2}\,.$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right)$$

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2+\left(W_\mu^2\right)^2\right)+\left(gW_\mu^3-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \quad \Rightarrow \quad m_W = \frac{g\,v}{2} .$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right) \quad \Rightarrow \quad m_Z = \frac{v}{2}\sqrt{g^2+g'^2} ,$$

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W_\mu^1\right)^2+\left(W_\mu^2\right)^2\right)+\left(gW_\mu^3-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

$$W_\mu^\pm \equiv \frac{1}{\sqrt{2}}\left(W_\mu^1 \mp iW_\mu^2\right) \quad \Rightarrow \quad m_W = \frac{g\,v}{2}\,.$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(gW_\mu^3 - g'B_\mu\right) \quad \Rightarrow \quad m_Z = \frac{v}{2}\sqrt{g^2+g'^2}\,;$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g'W_\mu^3 + gB_\mu\right)$$

ELECTROWEAK BOSONS

$$\frac{v^2}{8}\left[g^2\left(\left(W^1_\mu\right)^2+\left(W^2_\mu\right)^2\right)+\left(gW^3_\mu-g'B_\mu\right)^2\right]$$

Sixth: Read off Masses

$$W^\pm_\mu \equiv \frac{1}{\sqrt{2}}\left(W^1_\mu \mp iW^2_\mu\right) \quad \Rightarrow \quad m_W = \frac{g\,v}{2}\,.$$

$$Z_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(gW^3_\mu - g'B_\mu\right) \quad \Rightarrow \quad m_Z = \frac{v}{2}\sqrt{g^2+g'^2}\,;$$

$$A_\mu \equiv \frac{1}{\sqrt{g^2+g'^2}}\left(g'W^3_\mu + gB_\mu\right) \quad \Rightarrow \quad m_A = 0$$

QUESTION: CAN WE USE THIS TO GIVE THE W,Z BOSONS MASS AND DOF?

QUESTION: CAN WE USE S TO GIVE THE W,Z BOSONS MASS AND DOF?

ELECTROWEAK BOSONS

$$\frac{1}{2}\sigma^1 < \phi > \; = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^2 < \phi > \; = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^3 < \phi > \; = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/2\sqrt{2} \end{pmatrix} \neq 0$$

$$\frac{1}{2}I < \phi > \; = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ v/2\sqrt{2} \end{pmatrix} \neq 0$$

ELECTROWEAK BOSONS

$$\frac{1}{2}\sigma^1 < \phi > \; = \begin{pmatrix} 0 & 1/2 \\ 1/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^2 < \phi > \; = \begin{pmatrix} 0 & -i/2 \\ i/2 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/2\sqrt{2} \\ 0 \end{pmatrix} \neq 0$$

$$\frac{1}{2}\sigma^3 < \phi > \; = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/2\sqrt{2} \end{pmatrix} \neq 0$$

$$\frac{1}{2}I < \phi > \; = \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ v/2\sqrt{2} \end{pmatrix} \neq 0$$

- **Four Broken Generators** $\Rightarrow$
- **Expect Four Goldstone Bosons!**

ELECTROWEAK BOSONS

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

ELECTROWEAK BOSONS

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

ELECTROWEAK BOSONS

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0.$$

However one linear combination remains unbroken

We have that this is U(1) Hypercharge

ELECTROWEAK BOSONS

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0$$

However one linear combination remains unbroken

We have that this is U(1) Hypercharge

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

ELECTROWEAK BOSONS

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0.$$

However one linear combination remains unbroken

We have that this is U(1) Hypercharge

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

ELECTROWEAK BOSONS

$$T^3 + Y = (\frac{1}{2}\sigma^2 + \frac{1}{2}I)\begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = 0.$$

However one linear combination remains unbroken

We have that this is U(1) Hypercharge

$$Q = T_3 + Y = \begin{pmatrix} 1/2 & 0 \\ 0 & -1/2 \end{pmatrix} + \begin{pmatrix} 1/2 & 0 \\ 0 & 1/2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

$$SU(2)_L \otimes U(1)_Y \rightarrow U(1)_Q$$

Hence the GWS symmetry degenerates into electromagnetic symmetry