# Homework 2

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**Problem 1**

Homework 2

Problem 1

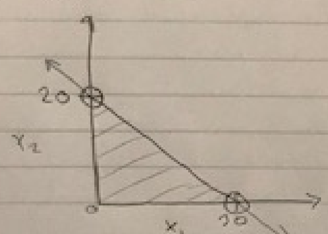
Let $x_1$ be the number of tortes, and $x_2$ be the number of pies.

Maximize $4x_1 + 5x_2$

s.t. $2x_1 + 3x_2 \le 60$

$x_1, x_2 \ge 0$

Graphically, this becomes $x_2 = 20 - \frac{2}{3}x_1$

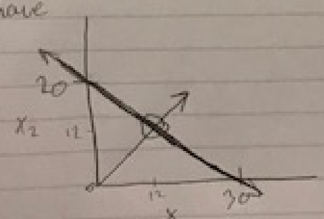



The shaded region represents the feasible region, so we have the corner at $x_1 = 0$, $x_2 = 20$ as a solution, however the corner of $x_1 = 30$, $x_2 = 0$ will be the best solution as Max will get 120 points vs 100 points.

Now add the constraint

$x_1 \ge x_2$

We have




Then the best solution is at the corner $x_1 = 12$ and $x_2 = 12$. This will give Max 108 points meaning a decrease of 12 points with this constraint

## Problem 2

### Part a

Problem 2

Let $x_1$ represent the number of acres of wheat, and $x_2$ represent the number of acres of corn.

Maximize $2000x_1 + 3000x_2$

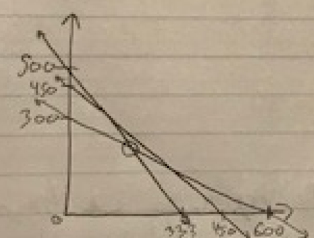
S.t. $3x_1 + 2x_2 \leq 1000$ (water)

$2x_1 + 4x_2 \leq 1200$ (fertilizer)

$x_1 + x_2 \leq 450$

$x_1, x_2 \geq 0$

a) Graphically,




Then solve for the intersection of these lines.

$3x_1 + 2x_2 \leq 1000$

$2x_1 + 4x_2 \leq 1200$

$x_1 = 200, \quad x_2 = 200$

The objective becomes $2000(200) + 3000(200)$

$= \$1,000,000$

\$1 Million in profit

### Part b

Here is the solution in R:

```
library(lpSolve)
A = matrix(c(3,2,1,1,0,2,4,0,1,1),nrow = 5,ncol = 2)
b = c(1000,1200,450,0,0)
```

```r
c = c(2000,3000)
dir = c(rep("<=",3),rep(">=",2))
s = lp("max",c,A,dir,b)
s
```

```
## Success: the objective function is 1e+06
```

```r
s$solution
```

```
## [1] 200 200
```

So we see that the solution matches the graphical solution.

### Part c

We can implement a loop to go through the increments of fertilizer to solve this.

```r
x = 200
for (x in seq(200,2200,by=100)) {
  b = c(1000,x,450,0,0)
  s = lp("max",c,A,dir,b)
  print(s$solution)
  print(s)
}
```

```
## [1] 100   0
## Success: the objective function is 2e+05
## [1] 150   0
## Success: the objective function is 3e+05
## [1] 200   0
## Success: the objective function is 4e+05
## [1] 250   0
## Success: the objective function is 5e+05
## [1] 300   0
## Success: the objective function is 6e+05
## [1] 325.0  12.5
## Success: the objective function is 687500
## [1] 300  50
## Success: the objective function is 750000
## [1] 275.0  87.5
## Success: the objective function is 812500
## [1] 250 125
## Success: the objective function is 875000
## [1] 225.0 162.5
## Success: the objective function is 937500
## [1] 200 200
## Success: the objective function is 1e+06
## [1] 175.0 237.5
## Success: the objective function is 1062500
## [1] 150 275
## Success: the objective function is 1125000
## [1] 125.0 312.5
## Success: the objective function is 1187500
## [1] 100 350
## Success: the objective function is 1250000
```

```
## [1]  75.0 387.5
## Success: the objective function is 1312500
## [1]  50 425
## Success: the objective function is 1375000
## [1]  25.0 462.5
## Success: the objective function is 1437500
## [1]   0 500
## Success: the objective function is 1500000
## [1]   0 500
## Success: the objective function is 1500000
## [1]   0 500
## Success: the objective function is 1500000
```

Here we can see how the decision variables and total profit vary. The farmer discontinues producing wheat at 2000 tons of fertilizer or above, and he stops producing corn at 1000 tons of fertilizer or below.

## Problem 3

Setting up the Linear Programming problem:

### Decision Variables

We will define $x_i$ as the fraction of investment $i$ purchased by Star Oil ($i = 1, 2, 3, 4, 5$).

### Maximize:

$13x_1 + 16x_2 + 16x_3 + 14x_4 + 39x_5$

### Subject to the constraints:

$$11x_1 + 53x_2 + 5x_3 + 5x_4 + 29x_5 \leqslant 40$$
$$3x_1 + 6x_2 + 5x_3 + x_4 + 34x_5 \leqslant 20$$
$$x_1, x_2, x_3, x_4, x_5 \leqslant 1$$
$$x_1, x_2, x_3, x_4, x_5 \geqslant 0$$

Now we solve in R:

```
A = matrix(0,nrow = 12,ncol = 5)
A[1,1:5] = c(11,53,5,5,29)
A[2,1:5] = c(3,6,5,1,34)
A[3:7,] = diag(5)
A[8:12,] = diag(5)

b = c(40,20,rep(1,5),rep(0,5))
c = c(13,16,16,14,39)
dir = c(rep("<=",7),rep(">=",5))
s = lp("max",c,A,dir,b)
s$solution
```

```
## [1] 1.0000000 0.2008600 1.0000000 1.0000000 0.2880835
```

```
s
```

```
## Success: the objective function is 57.44902
```

## Problem 4

### Decision Variables

We will define $x_1$ as the servings of corn, $x_2$ as the servings of milk, and $x_3$ as the servings of bread

### Minimize:

$0.18x_1 + 0.23x_2 + 0.05x_3$

### Subject to the constraints:

$$107x_1 + 500x_2 \geqslant 5000$$
$$107x_1 + 500x_2 \leqslant 50000$$
$$72x_1 + 121x_2 + 65x_3 \geqslant 2000$$
$$72x_1 + 121x_2 + 65x_3 \leqslant 2250$$
$$x_1, x_2, x_3 \geqslant 0$$
$$x_1, x_2, x_3 \leqslant 10$$

Now we solve in R:

```r
A = matrix(0,nrow = 10,ncol = 3)
A[1:2,1] = 107
A[1:2,2] = 500
A[3:4,1] = 72
A[3:4,2] = 121
A[3:4,3] = 65
A[5:7,] = diag(3)
A[8:10,] = diag(3)
b = c(5000,50000,2000,2250,0,0,0,10,10,10)
c = c(.18,.23,.05)
dir = c(">=","<=",">=","<=",rep(">=",3),rep("<=",3))
s = lp("min",c,A,dir,b)
s$solution
```

```
## [1]  1.944444 10.000000 10.000000
```

```
s
```

```
## Success: the objective function is 3.15
```

## Problem 5

### Decision Variables

We will define $x_i$ as the total weight of wood harvested in unit 1, year i, and $y_i$ as the total weight of wood harvested in unit 2, year i ($i = 1, 2, 3$).

**Maximize:**

$x_1 + x_2 + x_3 + y_1 + y_2 + y_3$

**Subject to the constraints:**

$$x_1 + \frac{1}{1.3}x_2 + \frac{1}{1.4}x_3 \leqslant 2$$
$$y_1 + \frac{1}{1.2}y_2 + \frac{1}{1.6}y_3 \leqslant 3$$
$$x_1 + y_1 \geqslant 1.2$$
$$x_2 + y_2 \geqslant 1.5$$
$$x_3 + y_3 \geqslant 2$$
$$x_1 + y_1 \leqslant 2$$
$$x_2 + y_2 \leqslant 2$$
$$x_3 + y_3 \leqslant 3$$

Now we solve in R:

```
A = matrix(0,nrow = 8,ncol = 6)
A[1,1:3] = c(1,1/1.3,1/1.4)
A[2,4:6] = c(1,1/1.2,1/1.6)
A[3:5,1:3] = diag(3)
A[3:5,4:6] = diag(3)
A[6:8,1:3] = diag(3)
A[6:8,4:6] = diag(3)
b = c(2,3,1.2,1.5,2,2,2,3)
c = c(rep(1,6))
dir = c(rep("<=",2),rep(">=",3),rep("<=",3))
s = lp("max",c,A,dir,b)
s
```

```
## Success: the objective function is 6.586538
```

```
s$solution
```

```
## [1] 0.4615385 2.0000000 0.0000000 1.1250000 0.0000000 3.0000000
```