

Quiz 4

Started: Feb 7 at 7:42pm

Quiz Instructions

Question 1

1 pts

Choose the strongest statement (from among the following options) for the following hash family.

	a	b	c
h_1	0	0	0
h_2	1	0	1
h_3	0	1	1
h_4	1	1	0

- ☐ Universal
☐ 1-Universal
☐ Universal and 1-Universal but not 2-universal
☒ 2-universal
☐ 3-universal

Question 2

1 pts

Choose the strongest statement (from among the following options) for the following hash family.

	a	b	c
h_1	0	0	0
h_2	1	0	1
h_3	0	1	1
h_4	0	0	1

- ☐ universal
☐ 1-universal
☒ universal and 1-universal but not 2-universal
☐ 2-universal
☐

3-universal

Question 3

1 pts

Suppose you have a hash family H that is 3-universal, and it contains hash functions that map $\{0, 1, 2, \dots, n-1\}$ to $\{1, 2, \dots, n\}$.

True or False: the number of distinct functions in H can be $O(n^2)$.

- ☐ True
- ☒ False

Question 4

1 pts

We saw for a **fixed** dictionary S of size N , we can construct a hash function h so that all query operations take constant time, assuming universal hash functions can be evaluated in $O(1)$ time.

True or False: this function h can be found in $O(N)$ expected time.

- ☒ True
- ☐ False

Question 5

1 pts

In the streaming lecture, we saw a deterministic algorithm to return the majority element in a stream of length n (if one exists), based on incrementing and decrementing counters. Each element belongs to a universe Σ .

Which of the following statements is true about this algorithm?

- ☒ Can return false positives
- ☐ Can return false negatives
- ☐ Can return both false positives and false negatives
- ☐ Uses storage of size $\Omega(n)$
- ☐ Uses storage of size $\Omega(|\Sigma|)$

Question 6**1 pts**

We also saw a randomized algorithm to maintain heavy hitters in a stream given both inserts and deletes. Given an element e , this algorithm returns an estimate $est(e)$.

Which of the following statements are always true about this estimate?

- ☐ The estimate is an under-estimate
- ☒ The estimate is an over-estimate
- ☐ The estimate can sometimes be an under-estimate, and sometimes an over-estimate

Question 7**0.5 pts**

In the Karp-Rabin algorithm, suppose we have a text of length m , and want to find a string of length n .

After we pick the random prime, what is the tightest bound of the runtime of this algorithm?

- ☐ $O(\lg(m) + \lg(n))$
- ☒ $O(m + n)$
- ☐ $O(n \log(m))$
- ☐ $O(m \log(n))$
- ☐ $O(mn)$

Question 8**0.5 pts**

The Karp-Rabin algorithm may yield which of the following:

- ☒ false positives
- ☐ false negatives

Question 9**1 pts**

Given an n -bit number x , the number of **distinct** prime divisors of x is at most what? (Pick the smallest correct upper bound from among the following options)

- ☐ $\lg \lg n$
- ☐ $\lg n$
- ☒ n
- ☐ n^2
- ☐ 2^n

Question 10**1 pts**

Given any number x in the range $\{2, 3, \dots, n\}$, the number of **distinct** prime divisors of x is at most what? (Pick the smallest correct upper bound from among the following options)

- ☐ $\lg \lg n$
- ☒ $\lg n$
- ☐ n
- ☐ n^2
- ☐ 2^n

Question 11**1 pts**

Given an n -bit number x , we want to check whether x is a prime or not. One algorithm is to first check that x is 2 or not, and then for each odd number y in the range $[3.. \lfloor \sqrt{x} \rfloor]$, check that y is not a divisor of x .

True or false: This algorithm runs in time polynomial in the number of bits n .

- ☐ True
- ☒ False

