Quiz 3

Started: Feb 1 at 5:03pm

Quiz Instructions

Question 1 — 1 pts

Suppose we implement quick-select in a deterministic way by always picking the first element of the array as the pivot, instead of choosing a uniformly random element as the pivot. What is the worst-case runtime of this algorithm?

- ○ $\Theta(1)$
- ○ $\Theta(\lg n)$
- ○ $\Theta(n)$
- ○ $\Theta(n \lg n)$
- ○ $\Theta(n \lg \lg n)$
- ● $\Theta(n^2)$

The following five questions are about the tree based union-find data structure (with path compression and union-by-rank), where we perform n Makeset operations, followed by an arbitrary number of Find and Union operations.

Question 2 — 0.2 pts

The rank of a root node is always equal to the height of the tree rooted at it.

- ○ True
- ● False

Question 3 — 0.2 pts

The rank of a root node is at least as large as the height of the tree rooted at it.

- [x] True
- [ ] False

Question 4 0.2 pts

Choose the tightest correct upper bound on the rank of any node from the options below.

- [ ] $O(1)$
- [x] $O(\log^* n)$
- [ ] $O(\log n)$
- [ ] $O(\log \log n)$
- [ ] $O(n)$
- [ ] None of the above

Question 5 0.2 pts

Select the tightest upper bound on the worst case cost of a find operation from among the following options:

- [ ] $O\left(1\right)$
- [x] $O\left(\log^* \ n\right)$
- [ ] $O\left(\log \log n\right)$
- [ ] $O\left(\log n\right)$
- [ ] $O\left(n\right)$

Question 6 0.2 pts

Select the tightest upper bound on the amortized cost of a find operation from among the following options:

- (•) $O(1)$
- ( ) $O(\log^* n)$
- ( ) $O(\log \log n)$
- ( ) $O(\log n)$
- ( ) $O(n)$

Question 7 — 0.5 pts

Both Kruskal's and Prim's algorithms find a minimum spanning tree in graphs even with negative edge weights.

- (•) True
- ( ) False

Question 8 — 0.5 pts

Given a graph where all the edge weights are distinct, both Kruskal's algorithm and Prim's algorithm must always return the same answer.

- (•) True
- ( ) False

Question 9 — 1 pts

Given a graph G with distinct edge weights, let T be its minimum spanning tree. Now suppose we change the weight of a single edge in G.

Let T' be the new minimum spanning tree. What is the maximum number of edges that belong to T' but did not belong to T.

- (•) 1
- ( ) 2

- $O(\log^* n)$
- $O(\log n)$
- $O(n)$

Question 10 — 1 pts

Given a graph G where the edge weights are positive real numbers, and let T be its minimum spanning tree. Now suppose we replace each edge weight by its square (i.e. replace 4 by 16, 3 by 9, etc). Then T is still an MST even with the new edge weights.

- (•) True
- ( ) False

Question 11 — 1 pts

Suppose we hash a set of size n into a table of size m using a hash function chosen from a universal hash family. The expected total number of collisions is:

- $\Theta(1)$
- $\Theta(\log n)$
- $\Theta(\frac{n^2}{m})$
- (•) $\Theta(\frac{n}{m})$
- $\Theta((\frac{n}{m})^2)$
- $\Theta(n)$

Question 12 — 1 pts

In the hashing lecture notes, we constructed a universal hash family using the matrix method.

Say we are mapping from a universe of u-bit strings, and the table is of size $M = 2^m$, so that the output is m-bit strings. Which of the following statements are **false** for this particular construction?

- ○ $Pr[h(x) = h(y)] \leq \frac{1}{M}$ for all $x \neq y$
- ◉ $Pr[h(x) = h(y)] = \frac{1}{M}$ for all $x \neq y$
- ○ $Pr[h(x) = h(y)] = 1$ for all $x = y$
- ○ For each x, $h(x)$ is uniformly distributed over all M possible outcomes.
- ○ There are $2^{m \times u}$ hash functions in this family.

In each of the following examples, we are given a hash family of hash functions $\{h_1, h_2, \ldots, \}$ mapping either some set of elements (named $\{p, q, r, \ldots\}$ to $\{0, 1\}$. As usual, you pick one of the hash functions from the family and map the elements using it.

Recall that a hash family H of functions mapping into $U = \{0, 1\}$ is universal if

$$Pr_{h \leftarrow H}[h(x) = h(y)] \leq \frac{1}{|U|} = 1/2$$

Question 13

1 pts

Which of the following three hash families are universal? Mark all that apply. **(You must get all of them correct to get any points, so be careful.)**

Hash Family (a)	p	q
h_1	0	1
h_2	1	0

Hash Family (b)	p	q	r	s
h_1	0	1	0	1
h_2	1	0	1	0

Hash Family (c)	p	q	r
h_1	0	0	1
h_2	0	1	1

- ☐ None
- ☑ (a)
- ☑ (b)

- [ ] (c)

Question 14 — 1 pts

Which is the strongest property that is true for the following hash family?

	a	b
h_1	0	1
h_2	1	0
h_3	0	0
h_4	1	1

The notions of universal and k-universal are explained in lecture.

- ( ) It has no good properties
- ( ) It is universal
- (•) It is 1-universal
- ( ) It is 1-universal and universal, but not 2-universal
- ( ) It is 2-universal
- ( ) It is 3-universal

Question 15 — 1 pts

Which is the strongest property that is true for the following hash family?

Hash Family (a)	p	q
h_1	0	1
h_2	1	0

- ( ) It has no good properties
- (•) It is universal
- ( ) It is 1-universal
- ( ) It is 1-universal and universal, but not 2-universal
- ( ) It is 2-universal
- ( ) It is 3-universal

Quiz saved at 9:49pm Submit Quiz