HW 1

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25 January 2022

1.5

a

$$\begin{aligned} A(4) &= 1000 \times (1 - 0.06 \times 4) \\ &= 1000 \times (1.24) \\ &= \$1240.00 \end{aligned}$$

b

$$\begin{aligned} A(4) &= 1000 \times (1 - 0.06 \times 4) \\ &= 1000 \times (0.76) \\ &= \$760.00 \end{aligned}$$

c

$$\begin{aligned} A(4) &= 1000 \times (1 + 0.06)^4 \\ &= 1000 \times (1.262) \\ &= \$1262.48 \end{aligned}$$

d

$$\begin{aligned} r^{(4)} &= 0.06i(t) & &= 1 + \frac{r^{(m)}}{m}^{m} - 1 \\ &= 1 + \frac{0.06}{4}^{4} - 1 \\ &= 1 + 0.015^4 - 1 \\ &= 0.0614A(4) & &= 1000 \times (1 + 0.0614)^4 \\ &= 1000 \times (1.269) \\ &= \$1268.99 \end{aligned}$$

e

$$\begin{aligned} d^{(12)} &= 0.06i(t) & &= 1 - \frac{d^{(m)}}{m}^{-m} - 1 \\ &= 1 - \frac{0.06}{12}^{-12} - 1 \\ &= 1 - 0.005^{-12} - 1 \\ &= 0.0620A(4) & &= 1000 \times (1 + 0.0620)^4 \\ &= 1000 \times (1.272) \\ &= \$1272.01 \end{aligned}$$

f

$$
\begin{aligned}
A(4) &= 1000 \times e^{\delta t} \\
&= 1000 \times e^{0.06 \times 4} \\
&= 1000 \times e^{0.24} \\
&= 1000 \times (1.271) \\
&= \$1271.25
\end{aligned}
$$

1.9

Present Value

$$
\begin{aligned}
A(0) &= \sum_{i=1}^{5} C_t \times (1+i)^{-t} \\
&= \sum_{i=1}^{5} C_t \times (1.02)^{-t} \\
&= 950 \times (1.02)^{0} + 800 \times (1.02)^{-1} + 150 \times (1.02)^{-2} + 400 \times (1.02)^{-3} + 120 \times (1.02)^{-4} \\
&= 950 \times 1 + 800 \times 0.9804 + 150 \times 0.9612 + 400 \times 0.9423 + 120 \times 0.9239 \\
&= 950 + 784.314 + 144.175 + 376.9289 + 110.8615 \\
&= \$2366.28
\end{aligned}
$$

A(5)

To calculate the value in 5 years, we could begin as above, and accumulate each cash flow individually to $t = 5$ rather thatn $t = 0$. However, it is equivalent to simply accumulate the sum as a whole.

$$
\begin{aligned}
A(5) &= A(0) \times 1.02^{5} \\
&= \$2366.28 \times 1.10408 \\
&= \$2612.56
\end{aligned}
$$

1.36

To solve for the interest earned in the fifth year, we can subtract $A(5) - A(4)$

$$
\begin{aligned}
A(5) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt \\
A(4) &= A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
A(5) - A(4) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt - A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
&= A(0) \times (\int_0^5 \frac{1}{10(1+t)^3} dt - \int_0^4 \frac{1}{10(1+t)^3} dt) \\
&= A(0) \times (\int_4^5 \frac{1}{10(1+t)^3} dt) \\
&= 100 \times \frac{1}{10} \times \int_4^5 (1+t)^{-3} dt \\
&= 10 \times \frac{-(1+t)^{-2}}{2} |_4^5 \\
&= 10 \times (\frac{-(1+5)^{-2}}{2} - \frac{-(1+4)^{-2}}{2}) \\
&= 5 \times ((1+4)^{-2} - (1+5)^{-2}) \\
&= 5 \times (\frac{1}{25} - \frac{1}{36}) \\
&= 5 \times (0.0122) \\
&= \$0.0611
\end{aligned}
$$

1.42

$$a(t) = e^{\int_0^t \delta(s) ds}$$

To solve for $a_x(t) = a_y(t)$, we must equate:

$$
\begin{aligned}
e^{\int_0^t \delta_x(s) ds} &= e^{\int_0^t \delta_y(s) ds} \\
\int_0^t \delta_x(s) ds &= \int_0^t \delta_y(s) ds
\end{aligned}
$$

From looking at the graph, $delta_x(s) = 0.1t$ and $delta_y(s) = 0.025t$ from $t \in (0, 2)$ and 0.05 when $t > 2$

Thus, integrating both functions to solve for the t in which they are equal:

$$
\begin{aligned}
\Delta_x(t) &= \int_0^t \delta_x(s) ds \\
&= \int_0^t 0.01(s) ds \\
&= 0.005t^2
\end{aligned}
$$

$$
\begin{aligned}
\Delta_y(t) &= \int_0^t \delta_y(s)ds \\
&= \int_0^t 0.025(s)ds \text{ , } 0 \leq \text{t} \leq 2 \\
&\quad \int_0^2 0.025(s)ds + \int_2^t 0.05ds \text{ , } 2 \leq t \leq 10 \\
&= 0.0125t^2 \text{ , } 0 \leq \text{t} \leq 2 \\
&\quad 0.05 + 0.05t - 0.1 \text{ , } 2 \leq t \leq 10 \\
&= 0.0125t^2 \text{ , } 0 \leq \text{t} \leq 2 \\
&\quad - 0.05 + 0.05t \text{ , } 2 \leq t \leq 10 \\
&= 0.0125t^2 \text{ , } 0 \leq \text{t} \leq 2 \\
&\quad 0.05(t-1) \text{ , } 2 \leq t \leq 10
\end{aligned}
$$

Thus, to solve for points where $0 \leq t \leq 2$, we must equate

$$
\begin{aligned}
0.005t^2 &= 0.0125t^2 \\
t &= 0
\end{aligned}
$$

To solve for points where $2 \leq t \leq 10$, we must equate

$$
\begin{aligned}
0.005t^2 &= -0.05 + 0.05t \\
0.005t^2 - 0.05t + 0.05 = 0 & \\
0.005(t^2 - 10 + 10) = 0 & \\
t &= 5 \pm \sqrt{15}
\end{aligned}
$$

Given the constraint of $2 \leq t \leq 10$, we find $t = 5 + \sqrt{15} = 8.873$