HW 1

Evan Dragich

25 January 2022

1.5

a

$$
\begin{aligned}
A(4) &= 1000 \times (1 - 0.06 \times 4) \\
&= 1000 \times (1.24) \\
&= \$1240.00
\end{aligned}
$$

b

$$
\begin{aligned}
A(4) &= 1000 \times (1 - 0.06 \times 4) \\
&= 1000 \times (0.76) \\
&= \$760.00
\end{aligned}
$$

c

$$
\begin{aligned}
A(4) &= 1000 \times (1 + 0.06)^4 \\
&= 1000 \times (1.262) \\
&= \$1262.48
\end{aligned}
$$

d

$$
\begin{aligned}
r^{(4)} &= 0.06 i(t) && = 1 + \frac{r^{(m)}}{m}^{m} - 1 \\
&= 1 + \frac{0.06}{4}^{4} - 1 \\
&= 1 + 0.015^4 - 1 \\
&= 0.0614 A(4) && = 1000 \times (1 + 0.0614)^4 \\
&= 1000 \times (1.269) \\
&= \$1268.99
\end{aligned}
$$

e

$$
\begin{aligned}
d^{(12)} &= 0.06 i(t) && = 1 - \frac{d^{(m)}}{m}^{-m} - 1 \\
&= 1 - \frac{0.06}{12}^{-12} - 1 \\
&= 1 - 0.005^{-12} - 1 \\
&= 0.0620 A(4) && = 1000 \times (1 + 0.0620)^4 \\
&= 1000 \times (1.272) \\
&= \$1272.01
\end{aligned}
$$

f

$$\begin{aligned}A(4) &= 1000 \times e^{\delta t}\\ &= 1000 \times e^{0.06\times 4}\\ &= 1000 \times e^{0.24}\\ &= 1000 \times (1.271)\\ &= \$1271.25\end{aligned}$$

1.9

Present Value

$$\begin{aligned}A(0) &= \sum_{i=1}^{5} C_t \times (1+i)^{-t}\\ &= \sum_{i=1}^{5} C_t \times (1.02)^{-t}\\ &= 950 \times (1.02)^0 + 800 \times (1.02)^{-1} + 150 \times (1.02)^{-2} + 400 \times (1.02)^{-3} + 120 \times (1.02)^{-4}\\ &= 950 \times 1 + 800 \times 0.9804 + 150 \times 0.9612 + 400 \times 0.9423 + 120 \times 0.9239\\ &= 950 + 784.314 + 144.175 + 376.9289 + 110.8615\\ &= \$2366.28\end{aligned}$$

A(5)

To calculate the value in 5 years, we could begin as above, and accumulate each cash flow individually to $t = 5$ rather thatn $t = 0$. However, it is equivalent to simply accumulate the sum as a whole.

$$\begin{aligned}A(5) &= A(0) \times 1.02^5\\ &= \$2366.28 \times 1.10408\\ &= \$2612.56\end{aligned}$$

1.36

To solve for the interest earned in the fifth year, we can subtract $A(5) - A(4)$

$$
\begin{aligned}
A(5) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt \\
A(4) &= A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
A(5) - A(4) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt - A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
&= A(0) \times (\int_0^5 \frac{1}{10(1+t)^3} dt - \int_0^4 \frac{1}{10(1+t)^3} dt) \\
&= A(0) \times (\int_4^5 \frac{1}{10(1+t)^3} dt) \\
&= 100 \times \frac{1}{10} \times \int_4^5 (1+t)^{-3} dt \\
&= 10 \times \frac{-(1+t)^{-2}}{2} |_4^5 \\
&= 10 \times (\frac{-(1+5)^{-2}}{2} - \frac{-(1+4)^{-2}}{2}) \\
&= 5 \times ((1+4)^{-2} - (1+5)^{-2}) \\
&= 5 \times (\frac{1}{25} - \frac{1}{36}) \\
&= 5 \times (0.0122) \\
&= \$0.0611
\end{aligned}
$$

1.42

$$a(t) = e^{\int_0^t \delta(s) ds}$$

To solve for $a_x(t) = a_y(t)$, we must equate:

$$
\begin{aligned}
e^{\int_0^t \delta_x(s) ds} &= e^{\int_0^t \delta_y(s) ds} \\
\int_0^t \delta_x(s) ds &= \int_0^t \delta_y(s) ds
\end{aligned}
$$

From looking at the graph, $delta_x(s) = 0.1t$ and $delta_y(s) = 0.025t$ from $t \in (0, 2)$ and 0.05 when $t > 2$

Thus, integrating both functions to solve for the t in which they are equal:

$$
\begin{aligned}
\Delta_x(t) &= \int_0^t \delta_x(s) ds \\
&= \int_0^t 0.01(s) ds \\
&= 0.005t^2
\end{aligned}
$$

$$
\begin{aligned}
\Delta_y(t) &= \int_0^t \delta_y(s)ds \\
&= \int_0^t 0.025(s)ds \text{ , } 0 \le \text{t} \le 2 \\
&\quad \int_0^2 0.025(s)ds + \int_2^t 0.05ds \text{ , } 2 \le t \le 10 \\
&= 0.0125t^2 \text{ , } 0 \le \text{t} \le 2 \\
&\quad 0.05 + 0.05t - 0.1 \text{ , } 2 \le t \le 10 \\
&= 0.0125t^2 \text{ , } 0 \le \text{t} \le 2 \\
&\quad -0.05 + 0.05t \text{ , } 2 \le t \le 10 \\
&= 0.0125t^2 \text{ , } 0 \le \text{t} \le 2 \\
&\quad 0.05(t-1) \text{ , } 2 \le t \le 10
\end{aligned}
$$

Thus, to solve for points where $0 \le t \le 2$, we must equate

$$
\begin{aligned}
0.005t^2 &= 0.0125t^2 \\
t &= 0
\end{aligned}
$$

To solve for points where $2 \le t \le 10$, we must equate

$$
\begin{aligned}
0.005t^2 &= -0.05 + 0.05t \\
0.005t^2 - 0.05t + 0.05 = 0 \\
0.005(t^2 - 10 + 10) = 0 \\
t &= 5 \pm \sqrt{15}
\end{aligned}
$$

Given the constraint of $2 \le t \le 10$ for this equality, we find $t = 5 + \sqrt{15} = 8.873$. Thus, the two solutions are $t = \{0, 8.873\}$.

2.5

a

$$
\begin{aligned}
AV(750a_{\overline{8}|4\%}) &= 750 \sum_{k=0}^{7} (1.04)^k \\
&= 750(9.21423) \\
&= \$6910.67
\end{aligned}
$$

b

$$
\begin{aligned}
i_{effective} &= 1.04^2 - 1 \\
&= 0.0816 \\
AV(750a_{\overline{8}|8.16\%}) &= 750 \sum_{k=0}^{3} (1.0816)^k \\
&= 750(3.25146) \\
&= \$2438.59
\end{aligned}
$$

c

$$\begin{aligned}
i_{effective} &= 1.04^{\frac{1}{2}} - 1 \\
&= 0.0198 \\
AV(750a_{\overline{8}|1.98\%}) &= 750\sum_{k=0}^{3}(1.08198)^k \\
&= 750(8.57691) \\
&= \$6432.68
\end{aligned}$$

2.47

a

$Rate(18, -1, 11, 0, 0) = 5.791\%$

b

$Rate(18, -1, 11, 0, 1) = 6.656\%$

c

$Rate(28, -1, 18, 0, 0) = 3.343\%$

d

$Rate(28, -1, 18, 0, 1) = 3.645\%$

2.53

$$\begin{aligned}
s_{\overline{2n}|} &= s_{\overline{n}|}(1+i)^{-1} + s_{\overline{n}|} \\
y &= x(1+i)^{-1} + x \\
y &= x((1+i)^{-1} + 1)
\end{aligned}$$

$$\begin{aligned}
(1+i)^{-1} + 1 &= \frac{y}{x} \\
(1+i)^{-1} &= \frac{y}{x} - 1
\end{aligned}$$

$$\begin{aligned}
s_{\overline{kn}|} &= s_{\overline{n}|}(1+i)^{-(k-1)} + s_{\overline{n}|} \\
&= x(\frac{y}{x} - 1)^{k-1} + x \\
&= x((\frac{y}{x} - 1)^{k-1} + 1)
\end{aligned}$$

Thus, for $k = 1$, $s_{\overline{kn}|} = 1$, and for $k \geq 2$, $s_{\overline{kn}|} = x((\frac{y}{x} - 1)^{k-1} + 1)$