Evan Dragich
Math 590
HW 5 - Due 4/1/2022

6.5 a. Letting the unknown premium paid be P:

$$L_0^N = \begin{cases} 100000\, v^{(K_{[50]}+1)} - P\ddot{a}_{\overline{\min(K_{[50]}+1,10)}|}, & K_{[50]} < 10 \\ -P\ddot{a}_{\overline{\min(K_{[50]}+1,10)}|} & , K_{[50]} \geq 10 \end{cases}$$

b. To solve for premium, we know:

$$P = 100000 \cdot \left(\frac{A^{\ \ 1}_{[50]:\overline{10}|}}{\ddot{a}_{[50]:\overline{10}|}} \right)$$

$$= 100000 \left(\frac{.01439}{8.05665} \right)$$

$$= \boxed{\$178.57}$$

6.11 Note that gross premium means we must take into account expenses, and set:

$$EPV(\text{benefits}) = EPV(\text{gross premium income}) - EPV(\text{expenses})$$

$EPV(\text{gross premium income}) - EPV(\text{expenses})$:

correction for first payment

$$(P\ddot{a}_{[55]:\overline{10}|} + 0.5P\ {}_{10|}\ddot{a}_{[55]:\overline{10}|})(1 - 0.03) - P(0.25 - 0.03)$$

$$= 0.5P(\ddot{a}_{[55]:\overline{10}|} + \ddot{a}_{[55]})(0.97) - 0.22P$$

$$= 11.4625P$$

Note that we took $0.5P\ddot{a}_{[55]:\overline{10|}}$ and 'filled' in the 10 deferred years in $19\ddot{a}_{[55]:\overline{10|}}$ to achieve the above equality

EPV(benefits):

$$50000\big(\underbrace{A_{[55]}}_{\text{constant \$50k component}} + \underbrace{A^{1}_{[55]:\overline{10|}}}_{\text{additional early \$50k}}\big) = 50000(0.2593)$$

$$= \$12965.63$$

Equating: $P = \dfrac{\$12965.63}{11.4625}$

$$= \boxed{\$1,131.13}$$

7.1 First, set present value ${}_0V = 0$. to determine P (premium amount). Then, a in the numbers in $A_{\overline{\ }|}$ and $\ddot{a}_{\overline{\ }|}$ to calculate ${}_{10}V$.

$${}_0V = 0 = \underbrace{(1.05)^{-20}}_{\text{discount 20 years}}\ \underbrace{{}_{20}p_{[40]}}_{\text{prob making it to then}}\ \underbrace{100000A_{60}}_{\text{when 100k starts, life will be 60.}} \quad +$$

$$\underbrace{50000\,A^{1}_{[40]:\overline{20|}}}_{\text{first component of benefits}} \quad -$$

$$P\ddot{a}_{[40]:\overline{20|}} \quad \text{(only paying benefits for 20 years.}$$

like 6.11, we took half the deferred 100k and "filled in the missing deferred part from the endowment.

$$P = \frac{\overbrace{50000}\left(A_{[40]} + (1.05)^{-20}\,{}_{20}p_{[40]}A_{60}\right)}{\ddot{a}_{[40]:20}}$$

$$= 50000\left(\frac{0.12097 + 0.36667(0.29028)}{12.995}\right)$$

$$= 50000(0.0174996)$$

$$P = \$874.98$$

Then, we write out the analogous RHS-side of the equation for ${}_{10}V$:

$${}_{10}V = \underbrace{{}_{10}p_{50}\, 100000\, A_{60}}_{\text{no discounting, but 10 years left or look}} + \underbrace{50000\, A^{1}_{[50]:\overline{10|}}}_{\text{10 years left}} - (874.98)\underbrace{\ddot{a}_{50:\overline{10|}}}_{\text{10 years left}}$$

$$= 50000(A_{50} + {}_{10}E_{50}\, A_{60}) - 874.98\, \ddot{a}_{50:\overline{10|}}$$

$$= 50000(0.18931 + 0.60182(0.29028)) - 874.98(8.0555)$$

$$= 50000(0.36400) - 7048.38$$

$$= \boxed{\$11,151.94}$$

7.2 Equation 7.4:

$$({}_0V + P)(1+i) = S q_{[x]} + p_{[x]}\, {}_1V$$

Rearrange for ${}_1V = \dfrac{({}_0V + P)(1+i) - S q_{[x]}}{p_{[x]}}$

Substitute with ${}_0V = 0$:

$$\frac{134(1.045) - 10000(0.00106)}{(1 - 0.00106)}$$

Solve for ${}_1V$: $= \dfrac{129.43}{.99894} = \boxed{\$129.57}$