

# HW 1

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**1.5**

**a**

$$\begin{aligned}A(4) &= 1000 \times (1 - 0.06 \times 4) \\&= 1000 \times (1.24) \\&= \$1240.00\end{aligned}$$

**b**

$$\begin{aligned}A(4) &= 1000 \times (1 - 0.06 \times 4) \\&= 1000 \times (0.76) \\&= \$760.00\end{aligned}$$

**c**

$$\begin{aligned}A(4) &= 1000 \times (1 + 0.06)^4 \\&= 1000 \times (1.262) \\&= \$1262.48\end{aligned}$$

**d**

$$\begin{aligned}r^{(4)} &= 0.06i(t) &= 1 + \frac{r^{(m)}{}^m}{m} - 1 \\&= 1 + \frac{0.06^4}{4} - 1 \\&= 1 + 0.015^4 - 1 \\&= 0.0614A(4) &= 1000 \times (1 + 0.0614)^4 \\&= 1000 \times (1.269) \\&= \$1268.99\end{aligned}$$

**e**

$$\begin{aligned}d^{(12)} &= 0.06i(t) &= 1 - \frac{d^{(m)}{}^{-m}}{m} - 1 \\&= 1 - \frac{0.06^{-12}}{12} - 1 \\&= 1 - 0.005^{-12} - 1 \\&= 0.0620A(4) &= 1000 \times (1 + 0.0620)^4 \\&= 1000 \times (1.272) \\&= \$1272.01\end{aligned}$$

**f**

$$\begin{aligned}A(4) &= 1000 \times e^{\delta t} \\&= 1000 \times e^{0.06 \times 4} \\&= 1000 \times e^{0.24} \\&= 1000 \times (1.271) \\&= \$1271.25\end{aligned}$$

## 1.9

**Present Value**

$$\begin{aligned}A(0) &= \sum_{i=1}^5 C_t \times (1+i)^{-t} \\&= \sum_{i=1}^5 C_t \times (1.02)^{-t} \\&= 950 \times (1.02)^0 + 800 \times (1.02)^{-1} + 150 \times (1.02)^{-2} + 400 \times (1.02)^{-3} + 120 \times (1.02)^{-4} \\&= 950 \times 1 + 800 \times 0.9804 + 150 \times 0.9612 + 400 \times 0.9423 + 120 \times 0.9239 \\&= 950 + 784.314 + 144.175 + 376.9289 + 110.8615 \\&= \$2366.28\end{aligned}$$

## A(5)

To calculate the value in 5 years, we could begin as above, and accumulate each cash flow individually to  $t = 5$  rather than  $t = 0$ . However, it is equivalent to simply accumulate the sum as a whole.

$$\begin{aligned}A(5) &= A(0) \times 1.02^5 \\&= \$2366.28 \times 1.10408 \\&= \$2612.56\end{aligned}$$

## 1.36

To solve for the interest earned in the fifth year, we can subtract  $A(5) - A(4)$

$$\begin{aligned}
A(5) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt \\
A(4) &= A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
A(5) - A(4) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt - A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
&= A(0) \times \left( \int_0^5 \frac{1}{10(1+t)^3} dt - \int_0^4 \frac{1}{10(1+t)^3} dt \right) \\
&= A(0) \times \left( \int_4^5 \frac{1}{10(1+t)^3} dt \right) \\
&= 100 \times \frac{1}{10} \times \int_4^5 (1+t)^{-3} dt \\
&= 10 \times \left. \frac{-(1+t)^{-2}}{2} \right|_4^5 \\
&= 10 \times \left( \frac{-(1+5)^{-2}}{2} - \frac{-(1+4)^{-2}}{2} \right) \\
&= 5 \times ((1+4)^{-2} - (1+5)^{-2}) \\
&= 5 \times \left( \frac{1}{25} - \frac{1}{36} \right) \\
&= 5 \times (0.0122) \\
&= \$0.0611
\end{aligned}$$

**1.42**

$$a(t) = e^{\int_0^t \delta(s) ds}$$

To solve for  $a_x(t) = a_y(t)$ , we must equate:

$$\begin{aligned}
e^{\int_0^t \delta_x(s) ds} &= e^{\int_0^t \delta_y(s) ds} \\
\int_0^t \delta_x(s) ds &= \int_0^t \delta_y(s) ds
\end{aligned}$$

From looking at the graph,  $\delta_x(s) = 0.1t$  and  $\delta_y(s) = 0.025t$  from  $t \in (0, 2)$  and  $0.05$  when  $t > 2$

Thus, integrating both functions to solve for the  $t$  in which they are equal:

$$\begin{aligned}
\Delta_x(t) &= \int_0^t \delta_x(s) ds \\
&= \int_0^t 0.01(s) ds \\
&= 0.005t^2
\end{aligned}$$

$$\begin{aligned}
\Delta_y(t) &= \int_0^t \delta_y(s) ds \\
&= \int_0^t 0.025(s) ds, \quad 0 \leq t \leq 2 \\
&\quad \int_0^2 0.025(s) ds + \int_2^t 0.05 ds, \quad 2 \leq t \leq 10 \\
&= 0.0125t^2, \quad 0 \leq t \leq 2 \\
&\quad 0.05 + 0.05t - 0.1, \quad 2 \leq t \leq 10 \\
&= 0.0125t^2, \quad 0 \leq t \leq 2 \\
&\quad -0.05 + 0.05t, \quad 2 \leq t \leq 10 \\
&= 0.0125t^2, \quad 0 \leq t \leq 2 \\
&\quad 0.05(t - 1), \quad 2 \leq t \leq 10
\end{aligned}$$

Thus, to solve for points where  $0 \leq t \leq 2$ , we must equate

$$\begin{aligned}
0.005t^2 &= 0.0125t^2 \\
t &= 0
\end{aligned}$$

To solve for points where  $2 \leq t \leq 10$ , we must equate

$$\begin{aligned}
0.005t^2 &= -0.05 + 0.05t \\
0.005t^2 - 0.05t + 0.05 &= 0 \\
0.005(t^2 - 10t + 10) &= 0 \\
t &= 5 \pm \sqrt{15}
\end{aligned}$$

Given the constraint of  $2 \leq t \leq 10$  for this equality, we find  $t = 5 + \sqrt{15} = 8.873$ . Thus, the two solutions are  $t = \{0, 8.873\}$ .

## 2.5

**a**

$$\begin{aligned}
AV(750a_{\bar{8}|4\%}) &= 750 \sum_{k=0}^7 (1.04)^k \\
&= 750(9.21423) \\
&= \$6910.67
\end{aligned}$$

**b**

$$\begin{aligned}
i_{effective} &= 1.04^2 - 1 \\
&= 0.0816 \\
AV(750a_{\bar{8}|8.16\%}) &= 750 \sum_{k=0}^3 (1.0816)^k \\
&= 750(3.25146) \\
&= \$2438.59
\end{aligned}$$

**c**

$$\begin{aligned}i_{effective} &= 1.04^{\frac{1}{2}} - 1 \\ &= 0.0198\end{aligned}$$

$$\begin{aligned}AV(750a_{\overline{8}|1.98\%}) &= 750 \sum_{k=0}^3 (1.08198)^k \\ &= 750(8.57691) \\ &= \$6432.68\end{aligned}$$