HW 1

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1.5

a

$$\begin{aligned} A(4) &= 1000 \times (1 - 0.06 \times 4) \\ &= 1000 \times (1.24) \\ &= \$1240.00 \end{aligned}$$

b

$$\begin{aligned} A(4) &= 1000 \times (1 - 0.06 \times 4) \\ &= 1000 \times (0.76) \\ &= \$760.00 \end{aligned}$$

c

$$\begin{aligned} A(4) &= 1000 \times (1 + 0.06)^4 \\ &= 1000 \times (1.262) \\ &= \$1262.48 \end{aligned}$$

d

$$\begin{aligned} r^{(4)} &= 0.06 \\ i(t) &= 1 + (\frac{r^{(m)}}{m})^m - 1 \\ &= 1 + \frac{0.06}{4}^4 - 1 \\ &= 1 + 0.015^4 - 1 \\ &= 0.0614 \end{aligned}$$

$$\begin{aligned} A(4) &= 1000 \times (1 + 0.0614)^4 \\ &= 1000 \times (1.269) \\ &= \$1268.99 \end{aligned}$$

e

$$
\begin{aligned}
d^{(12)} &= 0.06 \\
i(t) &= 1 - (\frac{d^{(m)}}{m})^{-m} - 1 \\
&= 1 - \frac{0.06}{12}^{-12} - 1 \\
&= 1 - 0.005^{-12} - 1 \\
&= 0.0620
\end{aligned}
$$

$$
\begin{aligned}
A(4) &= 1000 \times (1 + 0.0620)^4 \\
&= 1000 \times (1.272) \\
&= \$1272.01
\end{aligned}
$$

f

$$
\begin{aligned}
A(4) &= 1000 \times e^{\delta t} \\
&= 1000 \times e^{0.06 \times 4} \\
&= 1000 \times e^{0.24} \\
&= 1000 \times (1.271) \\
&= \$1271.25
\end{aligned}
$$

1.9

Present Value

$$
\begin{aligned}
A(0) &= \sum_{i=1}^{5} C_t \times (1+i)^{-t} \\
&= \sum_{i=1}^{5} C_t \times (1.02)^{-t} \\
&= 950 \times (1.02)^0 + 800 \times (1.02)^{-1} + 150 \times (1.02)^{-2} + 400 \times (1.02)^{-3} + 120 \times (1.02)^{-4} \\
&= 950 \times 1 + 800 \times 0.9804 + 150 \times 0.9612 + 400 \times 0.9423 + 120 \times 0.9239 \\
&= 950 + 784.314 + 144.175 + 376.9289 + 110.8615 \\
&= \$2366.28
\end{aligned}
$$

A(5)

To calculate the value in 5 years, we could begin as above, and accumulate each cash flow individually to $t = 5$ rather thatn $t = 0$. However, it is equivalent to simply accumulate the sum as a whole.

$$
\begin{aligned}
A(5) &= A(0) \times 1.02^5 \\
&= \$2366.28 \times 1.10408 \\
&= \$2612.56
\end{aligned}
$$

1.36

To solve for the interest earned in the fifth year, we can subtract $A(5) - A(4)$

$$
\begin{aligned}
A(5) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt \\
A(4) &= A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
A(5) - A(4) &= A(0) \times \int_0^5 \frac{1}{10(1+t)^3} dt - A(0) \times \int_0^4 \frac{1}{10(1+t)^3} dt \\
&= A(0) \times (\int_0^5 \frac{1}{10(1+t)^3} dt - \int_0^4 \frac{1}{10(1+t)^3} dt) \\
&= A(0) \times (\int_4^5 \frac{1}{10(1+t)^3} dt) \\
&= 100 \times \frac{1}{10} \times \int_4^5 (1+t)^{-3} dt \\
&= 10 \times \frac{-(1+t)^{-2}}{2} |_4^5 \\
&= 10 \times (\frac{-(1+5)^{-2}}{2} - \frac{-(1+4)^{-2}}{2}) \\
&= 5 \times ((1+4)^{-2} - (1+5)^{-2}) \\
&= 5 \times (\frac{1}{25} - \frac{1}{36}) \\
&= 5 \times (0.0122) \\
&= \$0.0611
\end{aligned}
$$

1.42

$$a(t) = e^{\int_0^t \delta(s)ds}$$

To solve for $a_x(t) = a_y(t)$, we must equate:

$$
\begin{aligned}
e^{\int_0^t \delta_x(s)ds} &= e^{\int_0^t \delta_y(s)ds} \\
\int_0^t \delta_x(s)ds &= \int_0^t \delta_y(s)ds
\end{aligned}
$$

Observing the graph, $\delta_x(s) = 0.1t$ for $t \in [0, 10]$ and $\delta_y(s) = 0.025t$ for $t \in [0, 2)$ and 0.05 or $t \in [2, 10]$. Thus, integrating both functions to solve for the t in which they are equal:

$$
\begin{aligned}
\Delta_x(t) &= \int_0^t \delta_x(s)ds \\
&= \int_0^t 0.01(s)ds \\
&= 0.005t^2
\end{aligned}
$$

$$
\begin{aligned}
\Delta_y(t) &= \int_0^t \delta_y(s)ds \\
&= \int_0^t 0.025(s)ds \text{ , } 0 \leq \text{t} \leq 2 \\
&\quad \int_0^2 0.025(s)ds + \int_2^t 0.05ds \text{ , } 2 \leq t \leq 10 \\
&= 0.0125t^2 \text{ , } 0 \leq \text{t} < 2 \\
&\quad 0.05 + 0.05t - 0.1 \text{ , } 2 \leq t \leq 10 \\
&= 0.0125t^2 \text{ , } 0 \leq \text{t} < 2 \\
&\quad -0.05 + 0.05t \text{ , } 2 \leq t \leq 10 \\
&= 0.0125t^2 \text{ , } 0 \leq \text{t} < 2 \\
&\quad 0.05(t-1) \text{ , } 2 \leq t \leq 10
\end{aligned}
$$

Thus, to solve for points where $0 \leq t < 2$, we must equate

$$
\begin{aligned}
0.005t^2 &= 0.0125t^2 \\
t &= 0
\end{aligned}
$$

To solve for points where $2 \leq t \leq 10$, we must equate

$$
\begin{aligned}
0.005t^2 &= -0.05 + 0.05t \\
0.005t^2 - 0.05t + 0.05 = 0& \\
0.005(t^2 - 10 + 10) = 0& \\
t &= 5 \pm \sqrt{15}
\end{aligned}
$$

Given the constraint of $2 \leq t \leq 10$ for this equality, we find $t = 5 + \sqrt{15} = 8.873$. Thus, the two solutions are $t = \{0, 8.873\}$.

2.5

a

$$
\begin{aligned}
AV(750a_{\overline{8}|4\%}) &= 750\sum_{k=0}^{7}(1.04)^k \\
&= 750(9.21423) \\
&= \$6910.67
\end{aligned}
$$

b

$$
\begin{aligned}
i_{effective} &= 1.04^2 - 1 \\
&= 0.0816 \\
AV(750a_{\overline{8}|8.16\%}) &= 750\sum_{k=0}^{3}(1.0816)^k \\
&= 750(3.25146) \\
&= \$2438.59
\end{aligned}
$$

c

$$\begin{aligned} i_{effective} &= 1.04^{\frac{1}{2}} - 1 \\ &= 0.0198 \\ AV(750a_{\overline{8}|1.98\%}) &= 750\sum_{k=0}^{3}(1.08198)^k \\ &= 750(8.57691) \\ &= \$6432.68 \end{aligned}$$

2.47

a

$Rate(18, -1, 11, 0, 0) = 5.791\%$

b

$Rate(18, -1, 11, 0, 1) = 6.656\%$

c

$Rate(28, -1, 18, 0, 0) = 3.343\%$

d

$Rate(28, -1, 18, 0, 1) = 3.645\%$

2.53

$$\begin{aligned} s_{\overline{2n}|} &= s_{\overline{n}|}(1+i)^{-1} + s_{\overline{n}|} \\ y &= x(1+i)^{-1} + x \\ y &= x((1+i)^{-1} + 1) \end{aligned}$$

$$\begin{aligned} (1+i)^{-1} + 1 &= \frac{y}{x} \\ (1+i)^{-1} &= \frac{y}{x} - 1 \end{aligned}$$

$$\begin{aligned} s_{\overline{kn}|} &= s_{\overline{n}|}(1+i)^{-(k-1)} + s_{\overline{n}|} \\ &= x(\frac{y}{x} - 1)^{k-1} + x \\ &= x((\frac{y}{x} - 1)^{k-1} + 1) \end{aligned}$$

Thus, for $k = 1$, $s_{\overline{kn}|} = x$, and for $k \geq 2$, $s_{\overline{kn}|} = x((\frac{y}{x} - 1)^{k-1} + 1)$

3.4

a

$$
\begin{aligned}
i_1^F &= i_1^S \\
&= 2.1\%
\end{aligned}
$$

$$
\begin{aligned}
i_2^S &= (1+i_1^S)(1+i_2^F) - 1 \\
&= (1.021)(1.026) - 1 \\
&= ((1.021)(1.026))^{\frac{1}{2}} - 1 \\
&= 2.35\%
\end{aligned}
$$

$$
\begin{aligned}
i_3^F &= \frac{(1+i_3^S)^3}{(1+i_2^S)^2} - 1 \\
&= \frac{(1.031)^3}{(1.0235)^2} - 1 \\
&= \frac{1.0959}{1.0476} - 1 \\
&= 4.62\%
\end{aligned}
$$

$$
\begin{aligned}
i_4^S &= ((1+i_3^S)^3(1+i_4^F))^{\frac{1}{4}} - 1 \\
&= ((1.031)^3(1.036))^{\frac{1}{4}} - 1 \\
&= 3.22\%
\end{aligned}
$$

b

$$
\begin{aligned}
\ddot{a}_{\overline{4}|} &= \sum_{k=0}^{3}(1+i_k^S)^{-k} \\
&= 1 + (1+i_1^S)^{-1} + (1+i_2^S)^{-2} + (1+i_3^S)^{-3} \\
&= 1 + (1.021)^{-1} + (1.0235)^{-2} + (1.031)^{-3} \\
&= 3.847
\end{aligned}
$$

To calculate $\ddot{s}_{\overline{4}|}$, we could accumulate each cash flow individually, or accumulate the sum as a whole as such:

$$
\begin{aligned}
\ddot{s}_{\overline{4}|} &= \ddot{a}_{\overline{4}|}(1+i_1^S)^3 \\
&= 3.847(1.031)^3 \\
&= 4.215
\end{aligned}
$$

3.8

a

$$
\begin{aligned}
100a_{\overline{4}|} &= 100\sum_{k=1}^{4}(1+i_k^S)^{-k} \\
&= 100((1.06)^{-1} + (1.07)^{-2} + (1.0775)^{-3} + (1.0825)^{-4}) \\
&= 100(3.344469) \\
&= \$334.47
\end{aligned}
$$

b

$Rate(4, -100, 334.447, 0, 0) = 7.565\%$

3.18

a

$$\begin{aligned}
R_S &= \frac{\sum_{t=1}^n m_t i_t^F (1+i_t^S)^{-t}}{\sum_{t=1}^n m_t (1+i_t^S)^{-t}} \\
&= \frac{\sum_{t=1}^n i_t^F (1+i_t^S)^{-t}}{\sum_{t=1}^n (1+i_t^S)^{-t}} \\
&= \frac{i_1^F(1+i_1^S)^{-1} + i_2^F(1+i_2^S)^{-2} + i_3^F(1+i_3^S)^{-3} + i_4^F(1+i_4^S)^{-4} + i_5^F(1+i_5^S)^{-5}}{(1+i_1^S)^{-1} + (1+i_2^S)^{-2} + (1+i_3^S)^{-3} + (1+i_4^S)^{-4} + (1+i_5^S)^{-5}} \\
&= \frac{0.0071(1.0071)^{-1} + 0.0079(1.0143)^{-2} + 0.0042(1.0205)^{-3} + 0.0031(1.0294)^{-4} + 0.0035(1.0355)^{-5}}{(1.0071)^{-1} + (1.0143)^{-2} + (1.0205)^{-3} + (1.0294)^{-4} + (1.0355)^{-5}} \\
&= 0.0052587 \\
&= 0.53\%
\end{aligned}$$

b

$$\begin{aligned}
R_S &= \frac{\sum_{t=1}^n m_t i_t^F (1+i_t^S)^{-t}}{\sum_{t=1}^n m_t (1+i_t^S)^{-t}} \\
&= \frac{m_1 i_1^F(1+i_1^S)^{-1} + m_2 i_2^F(1+i_2^S)^{-2} + m_3 i_3^F(1+i_3^S)^{-3} + m_4 i_4^F(1+i_4^S)^{-4} + m_5 i_5^F(1+i_5^S)^{-5}}{m_1(1+i_1^S)^{-1} + m_2(1+i_2^S)^{-2} + m_3(1+i_3^S)^{-3} + m_4(1+i_4^S)^{-4} + m_5(1+i_5^S)^{-5}} \\
&= \frac{1(0.0071)(1.0071)^{-1} + 3(0.0079)(1.0143)^{-2} + 5(0.0042)(1.0205)^{-3} + 7(0.0031)(1.0294)^{-4} + 9(0.0035)(1.0355)^{-5}}{1(1.0071)^{-1} + 3(1.0143)^{-2} + 5(1.0205)^{-3} + 7(1.0294)^{-4} + 9(1.0355)^{-5}} \\
&= 0.0042678 \\
&= 0.43\%
\end{aligned}$$

c

$$\begin{aligned}
R_S &= \frac{\sum_{t=1}^n m_t i_t^F (1+i_t^S)^{-t}}{\sum_{t=1}^n m_t (1+i_t^S)^{-t}} \\
&= \frac{m_1 i_1^F(1+i_1^S)^{-1} + m_2 i_2^F(1+i_2^S)^{-2} + m_3 i_3^F(1+i_3^S)^{-3} + m_4 i_4^F(1+i_4^S)^{-4} + m_5 i_5^F(1+i_5^S)^{-5}}{m_1(1+i_1^S)^{-1} + m_2(1+i_2^S)^{-2} + m_3(1+i_3^S)^{-3} + m_4(1+i_4^S)^{-4} + m_5(1+i_5^S)^{-5}} \\
&= \frac{5(0.0071)(1.0071)^{-1} + 4(0.0079)(1.0143)^{-2} + 3(0.0042)(1.0205)^{-3} + 2(0.0031)(1.0294)^{-4} + 1(0.0035)(1.0355)^{-5}}{5(1.0071)^{-1} + 4(1.0143)^{-2} + 3(1.0205)^{-3} + 2(1.0294)^{-4} + 1(1.0355)^{-5}} \\
&= 0.0060351 \\
&= 0.60\%
\end{aligned}$$

d

With a level notional contract, the annual fixed rate payment is $m_t R_S$, or $0.00525866(1000000) = \$5258.66$

e

$$\begin{aligned}
MV_3 &= \sum_{k=3}^{5} m_k(R_S - i_k^F) \\
&= m_k \sum_{k=3}^{5} (R_S - i_k^F) \\
&= 1000000 \sum_{k=3}^{5} (R_S - i_k^F) \\
&= 1000000((0.00525866 - 0.0042) + (0.00525866 - 0.0031) + (0.00525866 - 0.0035)) \\
&= 1000000(0.00497598) \\
&= \$4975.98
\end{aligned}$$

f

$$\begin{aligned}
R_S &= \frac{\sum_{t=t^*+1}^{n} m_t i_t^F (1+i_t^S)^{-t}}{\sum_{t^*+1}^{n} m_t (1+i_t^S)^{-t}} \\
&= \frac{\sum_{t=2}^{n} i_t^F (1+i_t^S)^{-t}}{\sum_{t=2}^{n} (1+i_t^S)^{-t}} \\
&= \frac{i_2^F(1+i_2^S)^{-2} + i_3^F(1+i_3^S)^{-3} + i_4^F(1+i_4^S)^{-4} + i_5^F(1+i_5^S)^{-5}}{(1+i_2^S)^{-2} + (1+i_3^S)^{-3} + (1+i_4^S)^{-4} + (1+i_5^S)^{-5}} \\
&= \frac{0.0079(1.0143)^{-2} + 0.0042(1.0205)^{-3} + 0.0031(1.0294)^{-4} + 0.0035(1.0355)^{-5}}{(1.0143)^{-2} + (1.0205)^{-3} + (1.0294)^{-4} + (1.0355)^{-5}} \\
&= 0.0047568 \\
&= 0.48\%
\end{aligned}$$