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Capacitated Arc Routing Problem and Its Extensions in Waste Collection

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Abstract. Capacitated arc routing problem (CARP) is the youngest generation of graph theory that focuses on solving the edge/arc routing for optimality. Since many years, operational research devoted to CARP counterpart, known as vehicle routing problem (VRP), which does not fit to several real cases such like waste collection problem and road maintenance. In this paper, we highlighted several extensions of capacitated arc routing problem (CARP) that represents the real-life problem of vehicle operation in waste collection. By purpose, CARP is designed to find a set of routes for vehicles that satisfies all pre-setting constraints in such that all vehicles must start and end at a depot, service a set of demands on edges (or arcs) exactly once without exceeding the capacity, thus the total fleet cost is minimized. We also addressed the differentiation between CARP and VRP in waste collection. Several issues have been discussed including stochastic demands and time window problems in order to show the complexity and importance of CARP in the related industry. A mathematical model of CARP and its new version is presented by considering several factors such like delivery cost, lateness penalty and delivery time.

Keywords: Arc routing problem; vehicle routing problem; waste collection.
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INTRODUCTION

Capacitated arc routing problem (CARP) was proposed by Golden and Wong [1], as a respond to the lacked of graph theory which focuses on arc or edge solving for optimality. On the other hand, node routing or vehicle routing problem (VRP) received more attention in research when compared to CARP, which is neglected comparatively. By purpose, CARP network is designed to find a set of routes for vehicles which satisfies all pre-setting constraints in such that all vehicles must start and end at a depot, service a set of demands on edges exactly once without exceeding the capacity, thus the total fleet cost is minimized. CARP arises from many street services, for examples in residential waste collection, road maintenance, pipeline inspection, winter gritting and snow plough. However in this paper, we emphasize the CARP modelling in waste collection environment.

Both CARP and VRP are NP-hard (see [1] and [2]), but the following statements differentiate these two network models in brief. In VRP, the vehicles are required to service the required nodes (vertices) in the network graph exactly once without exceeding their fixed capacity. Bus passenger, retail distributions and money delivery via security van are some typical logistics operations in VRP. In contrary, CARP is intent to service the required edges or arcs in the network graph exactly once without violating vehicle capacity. This explanation thus separates both models in theory and methodology and very rich in research. The design of the CARP model and its solution techniques purposefully is to find minimum routing cost for a fleet of vehicles involved in operation. In other words, finding minimum-cost routing is paramount in order to reduce overall transportation cost that related to vehicles.

Undirected CARP is defined on the connected graph $G = (V, E)$; where V is a set of vertices (or nodes) and E is a set of edges. A fleet of vehicles with fixed capacity Q is stationed at a depot V_0 and they are required to service all edges $E_{ij} \in E$ exactly once without exceeding its capacity. In deadheading circumstance, the vehicle is allowed to traverse each edge more than once without servicing it. Note that undirected CARP on edges is used to model the two-way street while directed CARP is used for modelling the one-way street. Moreover, for urban streets networks mixed CARP (undirected and directed) frequently applied to the problem. Unknown and non-negative demands, $q_{ij} > 0$ and non-negative traversal cost, $c_{ij} > 0$ are assigned to each edge, so that $q_{ij} \subseteq E$ and $c_{ij} \subseteq E$. The goal of CARP is to determine a minimum routing cost and reduce the vehicle trips, such that each trip starts and ends at the depot.

CARP AND VRP IN WASTE COLLECTION

Waste collection problem is often viewed as environmental and pollution issues thus many approaches have been carried out on development of policies and improving of waste management at administration level. On the other hand, lack study has been done on its operational.

Vehicle routing in waste collection can be categorized into two segments. First is node routing when a fleet of trucks are required to pick-up some amount of commercial waste without exceeding its capacity and dispose them at a specific dumpsite. In this problem, a set of customers such like shopping malls, hypermarkets, restaurants, schools, factories and commercial buildings are represent by a set of nodes in the network graph. Usually, the problem is modelled as VRP since the vehicle performs node to node collection. The waste is put in large containers, usually at the back of the building or at one specific corner for easy loading. The pick-up task is done two or three times in a week and also can have special request from the customers. VRP for municipal waste collection was firstly been studied by Beltrami and Bodin [3] for municipality of New York. VRP in commercial waste collection problem can be found in Kim et al. [4] where VRP with time windows (VRPTW) is designed to minimize the number of vehicles and travel time, besides balancing the workload among the vehicles. They also considered lunch break for drivers as a special stop that has a time window. Tung and Pinnoi [5] studied waste collection problem in Hanoi, Vietnam. In addition to the considerations of the standard VRPTW, they considered a landfill operation and inter-arrival time constraints between two consecutive visits at a stop.

Since VRP in waste collection is a node to node problem, it also requires the vehicles to collect the waste several days in a week. This problem then can be seen as periodic VRP (PVRP) since the vehicles do not service their customers every day. To our knowledge, periodic problem in VRP was firstly highlighted by Beltrami and Bodin [3]. In their study, some nodes required 3-days services per week while some other nodes need 6-days services. Angelelli and Speranza [6] formulated the periodic VRP with intermediate facilities (PVRP-IF) by choosing a visiting schedule randomly to each customer and constructing vehicle routes on each day. Another periodic VRP for the separate collection of recycle wastes (glass, paper and plastic/metal) can be seen in Texiera et al. [7]. Recyclable waste problem was modelled as pick-up and delivery VRP (PDVRP) in Aringhieri et al. [8], where several containers need to be collected at each collection point and bring them to an appropriate disposal plant. A survey on vehicle routing in the solid waste industry in North America can be found in Golden et al. [9].

The second segment exists in residential waste collection when a fleet of trucks perform door to door collection when they are on routes. This problem definitely is CARP where the service is given to customers (houses) located on the edges. When routing, the capacity of vehicles is increased until it is nearly full or fully loaded, thus an emptying process need to be done at a dumpsite before the vehicles can resume to the next collection.

Ghiani et al. [10] considered CARP with intermediate facilities (CARPIF), where a vehicle starts from one depot but has a set of recharge nodes. In real applications, the intermediate facilities can be the dumpsites for waste collection or salt storage halls for winter gritting. Sniezek et al. [11] introduced CARP with vehicle-site dependency (CARPVSD) that prohibits a vehicle of a certain class from servicing or traversing the street because of some facility limitations. Each vehicle class has a restriction on the streets that it can or cannot service. For example, some streets are too narrow or some bridges can only support to a limited weight. These conditions force the service provider to use a certain class of vehicle in solid waste collection in Philadelphia.

Very few studies are considered external factors such like temperature and weather that have influences onto CARP model. To our knowledge, Amponsah and Salhi [12] considered environmental aspects and hot weather affects onto garbage collection in Ghana in order to minimize the cost and environmental impact. A paper by Ismail and Ramli [13] is taking into account of rainy factor that affected the waste weight for modelling CARP with stochastic demands (CARPSD). In the paper, household refusals react as a big sponge that absorbs rain drops thus increased the total weight. This parameter consequently burdened the vehicle capacity and extra trips are needed to complete all collections.

Inspired by household refuse collection problem in Lisbon, Mourão and Amado [14] aimed to design the tours for vehicles which passed through mixed (directed and undirected) network. In their MCARP, the vehicles start from a depot, move to crew base to pick-up the crew then to collecting quarter before dumping at a dumpsite. Chu et al. [15] provide cost solutions in weekly basis since road networks in residential area are not service everyday in their periodic CARP (PCARP) model. Based on real case of refuse collection in Troyes, France, Lacomme et al. [16] minimized the bi-objective CARP, the total duration of the trips and duration of the longest trip. Bautista et al. [17] proposed mixed CARP with turn constraints (MCARPTC) then applied in modelling street network of Sant Boi de Llobregat, Barcelona, Spain which taking into account the traffic regulations. In their study, some traversals are forbidden for some links (e.g.: one-way street and too narrow street). A recent MCARP solution can also been found

in Gouveia et al. [18] that model the real street networks of waste collection in Lisbon, Portugal. Ghiani et al. [19] applied capacity and length constraint for arc routing problem with intermediate facilities (CLARPIF). In the model, dumpsites or incinerators are considered as intermediate facilities for emptying process and viewed as delivery process instead of collections.

Stochastic Demand

In residential waste collection, demands on edges are unknown and randomly distributed to a certain distribution. In consequence, typical CARP can be extended to its dynamic demand version namely CARP with *stochastic demands* (CARPSD). Waste operators or managers usually find it is hard to send the correct number of vehicles in order to pickup all demands in smoothly manner due to difficulty in guessing the waste weight. This variable affected the vehicle capacity and trips thus could increase the total cost if improper planning of vehicles occur.

Stochastic demand in waste collection is considered more complicated than other CARP since the vehicles perform pickup process. In waste collection, the amount of waste to be picked-up is unknown. Comparatively, delivery CARP such in road maintenance or winter gritting, the amount of bitumen or salt to be spread on roads is easy to measure since they are depend on the length of roads that need to be covered. Hence, stochastic demand for CARP receives very little attention and considerably a new problem which firstly appeared in Fleury et al. [20]. In the paper, random demand is assumed distributed to Gaussian and vehicle's capacity is reduced 10% to improve the solution cost and trips. Christiansen et al. [21] modelled Poisson distribution for demand on edges for CARPSD model but not directly applied in waste collection. More recently, Ismail and Ramli [13] presented CARPSD in waste collection when external factor like rain drops increased the waste weight and caused more trips and prolonged the routing time. It is possible to have different models based on some behavior in waste collection in order to modelling them accurately (see Ismail and Ramli [22]).

Time Window

In many transportation systems each customer specifies a period of time, called a *time window* in which the service must occur. The objective is to find a set of routes for the vehicles, where each route begins and ends at a depot, serves a subset of customers without violating the vehicle capacity and time window constraints, while minimizing total traversal cost. In CARP with time window (CARPTW), arriving earlier at one location induces a waiting time while arriving later is penalized.

Time window constraint is considered one important factor in logistics systems and street services. In some countries, vehicle crews must satisfy several rules that related with time provided by union contract or company regulations. Thus, time window inherits several components such like working period, maximum duration of working and driving periods, number and duration of breaks during service cycle and overtime [23].

CARPTW received limited attention in the literature when compared to its VRPTW counterpart. Indeed, time window constraint in waste collection is very sparse in literature. Eglese [24] firstly highlighted CARPTW in winter gritting. In his paper, streets are categorized into two segments; streets must be treat within 2 hours are put in category-1 while category-2 for streets that can be treat within 4 hours after call out. Obviously, street treatment (salt spreading) is more crucial in winter gritting problem due to traffic and safety reasons. CARPTW also appeared in PhD dissertation by Wøhlk [25]; where two mathematical models are presented for the problem. She proposed one model based on constructing a node duplicated on which the integer linear programming was built. Another model based on a transformation into equivalent vehicle routing problem, the VRPTW. In addition, Reghioui et al. [26] also studied CARPTW in which the objective is to minimize the total distance.

MATHEMATICAL MODEL FOR CARP

A CARP model is designed to seek for minimization of routing cost for a fleet of vehicles or trips. In this paper, the CARP model is given as follows:

Minimize
$$\sum_{(i,j)\in E}\sum_{k=1}^{K}c_{ij}x_{ijk}$$

Subject to:

$$\sum_{p\in V}x_{ipk}=\sum_{p\in V}x_{pik}\;;\forall i\in V,\;k=1,2,...,K \tag{1}$$

$$\sum_{k=1}^{K}y_{ijk}=1;\forall(i,j)\in R \tag{2}$$

$$x_{ijk}\geq y_{ijk}\;;\forall(i,j)\in R,\;k=1,2,...,K \tag{3}$$

$$\sum_{(i,j)\in E}q_{ij}\,y_{ijk}\leq W;\;k=1,2,...,K\;; \tag{4}$$

$$M\sum_{(i,j)\in V[S]}x_{ijk}\geq\sum_{(j,p)\in S}x_{jpk} \tag{5}$$

$$y_{ijk}\in\{0,1\};\forall(i,j)\in R,\;k=1,2,...,K \tag{6}$$

$$x_{ijk}\in Z^{+};\forall(i,j)\in R,\;k=1,2,...,K \tag{7}$$

Equation (1) ensures route discrete continuity. The equation (2) states that each edge with positive demand is serviced exactly once. The equation (3) guarantees that the traversal circuit k covers the edge $(i, j) \in R$ more than once for deadheading paths. Vehicle capacity is not violated on account of equation (4). The equation (5) prohibits the formation of (infeasible) subtours. Linear formulations are given in equations (6) and (7).

$$y_{ijk}=\begin{cases} 1 \text{ if the edge (arc) } (i,\,j)\in R \text{ is covered in trip } k; \\ \\ 0 \text{ otherwise.} \end{cases}$$

M is a large constant greater than or equal to the sum of traversals of edges and arcs in a given $S\subseteq R$, $V[S]$ is the set of nodes incident to the arc set S, k denotes a trip, and K is the maximum number of trips allowed.

CARP with Delivery Time Window

Incorporating time element into CARP is difficult, obviously because it needs many time variables to consider. In waste collection, total operation time including service time, routing time and delivery time. Service time is the time taken by the vehicle to collect, load and compress the garbage. In real-life operation and especially within a dynamic setting the on-site service time is subjected to stochasticity. In most distribution contexts, the on-site service time in municipal waste collection involves picking-up and compressing the household refusals. Routing time is the time taken by the vehicle to move from one arc to another arc. Lastly, delivery time is the time taken by the vehicle from the last customer to the dumpsite for disposal activity. Due to the randomness of routing time (or travel time) on links, arrival time at each customer location is also characterized as a random variable. Since the real-time traffic conditions for every link is unknown before departure, arrival time at each customer is difficult to predict. An approximate CARPTW model in waste collection is shown in Ismail and Ramli [27].

For waste collection problem in Malaysia, delivery time from the last pickup customer to a dumpsite can be categorized into soft or hard time constraints. Usually, the dumpsite operates from 7 am until 7 pm and closed at night due to safety reason. Time [7 am, 5 pm] is considered as normal time window when the disposal activity is

usually carried out. Meanwhile, time (5 pm, 7 pm] is considered as soft time window where a vehicle is permitted to enter the dumpsite but with a lateness penalty. Moreover, a vehicle is totally forbidden to enter the dumpsite if it is too late (after 7 pm). In this case, the vehicle is said to violate the hard time window and too late penalty will be imposed. FIGURE 1 depicts the relationship between time windows and their penalty costs of being late.
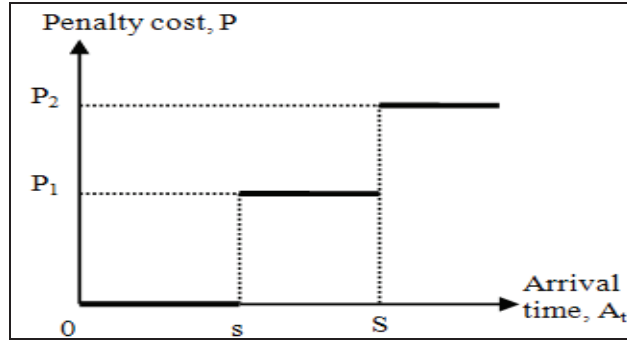

FIGURE 1. Relationship between time windows and penalty costs.

In FIGURE 1, arrival within normal time window is depicted as [0, s]. However, time window is violated within acceptable delay is shown in (s, S]. Vehicle operators will be charge a penalty cost of P_1 if the arrival time is within this soft time window. Violations of hard time window and penalty cost is shown by P_2 if arrival is too late with unacceptable delay, $A_t > S$. The penalty cost, due to violating the upper bound of the time window can be formulated as

$$P = \begin{cases} 0 \text{ if } A_t = [0, s] \\ P_1 \text{ if } A_t = (s, S] \\ P_2 \text{ if } A_t > S. \end{cases}$$

In order to determine delivery time, let t_d denotes the delivery time on the link (v_n, v_o). Let A and A' denotes the set of links without traffic congestion and with probability of traffic congestion respectively, $A \subseteq A'$. For every link $(v_n, v_o) \in A$, t_d on link (v_n, v_o) can be expressed as $t_{d(n,o)} = \beta_{(n,o)}$, where β is a time parameter. Assume some links of A' have the probability p of being congested. For every link $(v_n, v_o) \in A'$, delivery time $t_{d(n,o)} = p\beta_{(n,o)} + \beta_{(n,o)}$. Thus,

$$t_{d(n,o)} = \begin{cases} \beta_{(n,o)} \text{ if } (v_n, v_o) \in A \\ p\beta_{(n,o)} + \beta_{(n,o)} \text{ if } (v_n, v_o) \in A' \end{cases}$$

We extended this formula to calculate delivery cost, c_d from node v_n (last collected customer on link) to node v_o (dumpsite). For every link $(v_n, v_o) \in A$, c_d on link (v_n, v_o) can be expressed as $c_{d(n,o)} = \alpha_{(n,o)}$, where α is a cost parameter. Delivery cost, $c_{d(n,o)}$ which affected by traffic congestion is given by $c_{d(n,o)} = p\alpha_{(n,o)} + \alpha_{(n,o)}$. Thus,

$$c_{d(n,o)} = \begin{cases} \alpha_{(n,o)} \text{ if } (v_n, v_o) \in A \\ (p\alpha_{(n,o)} + \alpha_{(n,o)}) \text{ if } (v_n, v_o) \in A' \end{cases}$$

Overall, the penalty costs incurred into delivery cost is given by

$$
c_{p(n,o)} = \begin{cases}
c_{d(n,o)} + 0 \text{ if } 0 \leq t_{d(n,o)} \leq s \text{ for } (v_n, v_o) \in A \\[6pt]
c_{d(n,o)} + P_1 \text{ if } s < t_{d(n,o)} \leq S \text{ for } (v_n, v_o) \in A \\[6pt]
c_{d(n,o)} + P_2 \text{ if } t_{d(n,o)} > S \text{ for } (v_n, v_o) \in A \\[6pt]
(p\beta_{(n,o)} + \beta_{(n,o)})\, \alpha_{(n,o)} + 0 \text{ if } 0 \leq t_{d(n,o)} \leq s \text{ for } (v_n, v_o) \in A' \\[6pt]
(p\beta_{(n,o)} + \beta_{(n,o)})\, \alpha_{(n,o)} + P_1 \text{ if } s < t_{d(n,o)} \leq S \text{ for } (v_n, v_o) \in A' \\[6pt]
(p\beta_{(n,o)} + \beta_{(n,o)})\, \alpha_{(n,o)} + P_2 \text{ if } t_{d(n,o)} > S \text{ for } (v_n, v_o) \in A'
\end{cases}
$$

Notation $c_{d(n,o)}$ is the delivery cost per unit time from last customer v_n to the dumpsite v_o. $\beta_{(n,o)}$ is the time taken by the vehicle to deliver the loads from last customer to the dumpsite. Both $\alpha_{(n,o)}$ and $\beta_{(n,o)}$ are fixed parameters determined by the vehicle's operator. A and A' represent the traffic conditions without and with congestions respectively. Meanwhile p is the probability for traffic of being congested in A'.

DISCUSSION AND CONCLUSION

In this paper, literatures on several routing problem (namely CARP and VRP) is reviewed. In waste collection for instance, the routing problem usually is formulated into CARP. This methodology is caused by the customers' demands in the street networks of urban waste collection is located on the edges (or arcs), not at the nodes. Meanwhile VRP in waste collection is limited to roll-on-roll-off collection (commercial) where the operation is performed at certain locations such like wet market, school, shopping mall, restaurants and commercial outlets. For these reasons, waste collection routing problem in residential area is more suited to CARP.

In CARP context, this domain received little attention and leaves many areas to be explored, compared to VRP. CARP with stochastic demands is very sparse in literature and considerably a new extension. So far, the demands in CARPSD are assumed to be stochastic according to two different distributions as described in this paper. Thus, it is possible to explore the possibility of having another random distribution for dynamic demands.

CARP with time window is also considered a young variant in waste collection. CARPTW is neglected and only appeared in PhD dissertation by Wøhlk [25] thus make it has large research area to be explored. Since the first work in winter gritting by Eglese [24], not much development on CARPTW has been done especially in waste collection problem. Waste collection problem can be modelled as delivery problem where vehicle has delivery time to travel from customer nodes to a dumpsite. Because of this reason, it is possible to study delivery factors such like traffic congestion, delivery time and lateness penalty which have explicit cost on vehicles. To this extend, it is also possible to modelling another spectrum of time window for CARP which we called CARP with delivery time window (CARPDTW). In conclusion, we believed that CARPDTW represents the real-case of waste collection.

ACKNOWLEDGMENTS

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