ST 501 HW 2

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1.14) Splitting 52 cards into 4 groups of 13 is creating a partition for the deck with $n = 52$, $n_1 = n_2 = n_3 = n_4 = 13$

Therefore, the number of ways to do this would be

$$\binom{52}{13\ 13\ 13\ 13} = \frac{52!}{13!\,13!\,13!\,13!} = \boxed{\frac{52!}{(13!)^4}}$$

1.17)

curve((1-x/100)^4, from =0, to =100, xlab = "% defective in lot", ylab = "Probability of Acceptance")

1.32) $P(4 \text{ Right}) = P(\text{1st Right} \cap \text{2nd Right} \cap \text{3rd Right} \cap \text{4th Right})$

$= P(\text{1st Right}) \cdot P(\text{2nd} \mid \text{1st Right}) \cdot P(\text{3rd Right} \mid \text{1st \& 2nd})$
$\cdot P(4^{th} \mid 1^{st}, 2^{nd}, 3^{rd} \text{ Right})$

$= \frac{1}{4} \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{1} = \boxed{\frac{1}{24}}$

1.33) $P(5 \text{ on different floors}) = \dfrac{\text{\# of ways to pick 5 distinct floors}}{\text{\# of ways to pick 5 floors}}$

$= \dfrac{{}_7P_5}{7^5} = \dfrac{\frac{7!}{2!}}{7^5} = \dfrac{360}{2401} \approx .1499$

1.41) a) $P(\text{Matching socks}) = P(\text{Black match} \cup \text{blue match} \cup \text{green match})$

$= P(\text{Black match}) + P(\text{Blue match}) + P(\text{Green match})$
$= P(1^{st} \text{ Black} \cap 2^{nd} \text{ Black}) + P(1^{st} \text{ Blue} \cap 2^{nd} \text{ Blue}) + P(\text{1st G} \cap \text{2nd Green})$
$= P(\text{1st Black}) \cdot P(\text{2nd Black} \mid \text{1st Black}) + P(\text{1st Blue}) \cdot P(\text{2nd Blue} \mid \text{1st Blue})$
$+ P(\text{1st Green}) \cdot P(\text{2nd Green} \mid \text{1st Green})$

$= \frac{7}{24} \cdot \frac{6}{23} + \frac{8}{24} \cdot \frac{7}{23} + \frac{9}{24} \cdot \frac{8}{23} = \boxed{\frac{85}{276} \approx .308}$

b) $P(\text{1st Black}) \cdot P(\text{2nd Black} \mid \text{1st Black}) = \frac{7}{24} \cdot \frac{6}{23} = \boxed{\frac{42}{552} \approx .0761}$

1.46) a) $P(\text{Red}) = P(\text{Heads}) \cdot P(\text{Red} \mid \text{Heads}) + P(\text{Tails}) \cdot P(\text{Red} \mid \text{Tails})$

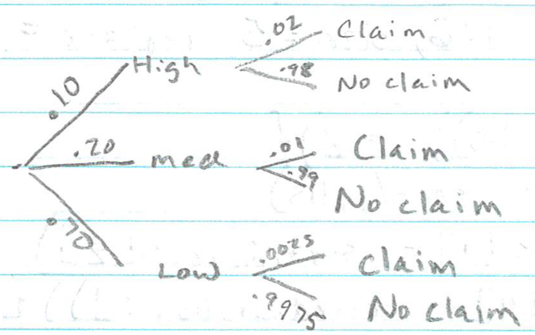
$$= \frac{1}{2} \cdot \frac{3}{5} + \frac{1}{2} \cdot \frac{2}{7}$$

$$\boxed{= \frac{31}{70} \approx .4429}$$

b) $$P(\text{Heads} \mid \text{Red}) = \frac{P(\text{Heads} \cap \text{Red})}{P(\text{Red})} = \frac{P(\text{Red} \mid \text{Heads}) \cdot P(\text{Heads})}{P(\text{Red})}$$

$$= \frac{3/5 \cdot 1/2}{31/70} = \boxed{21/31 \approx .6774}$$

1.53)



$$P(\text{High Risk} \mid \text{Claim}) = \frac{P(\text{High Risk} \cap \text{Claim})}{P(\text{Claim})} = \frac{P(\text{Claim} \mid \text{High}) \cdot P(\text{High})}{P(\text{Claim})}$$

$$= \frac{(.02)(.10)}{(.02)(.10) + (.01)(.20) + (.0025)(.70)} = \boxed{\frac{8}{23} \approx .3478}$$

1.68) We will flip a coin and roll a die simultaneously.
Let A be the event of rolling an even #
B be the event of coin landing on heads
C be the event of rolling an odd #.

Because the die + coin do not impact the others result, $A \perp B$ + $B \perp C$. However, A and C are mutually exclusive events and cannot be independent.

Other Problems

① # of total IDs = $26^2 \cdot 10^3 = 676{,}000$

②
```
a <- replicate(10000, runif(3, min=0, max=3))
b <- colSums(a)
sum(b<2)/10000
```

Simulation produced a probability of 0.0492.

③
```
Results <- replicate(100000, {
rolls <- sample(1:6, size=5, replace=TRUE)

if(length(unique(rolls))==2){
  sorted_rolls <- sort(rolls)
  if((sorted_rolls[1]==sorted_rolls[2]) +
    (sorted_rolls[4]==sorted_rolls[5])){
      TRUE
  } else {
      FALSE
  }
} else {
    FALSE
}
})

mean(Results)
```