Set 1

For each sets 1-3, you must correctly solve at least 2 out of the 3 problems to move on. You will receive three points for each correct answer when you choose to turn in the set, but each time you come up to check your answers you will lose one point; thus the maximum number of points you can earn on any one set is 8 points (if you correctly solve all three problems on your first try).

1. Come holiday time, three friends were boasting about how many presents they each had received. Kristina claimed she had gotten 8^{10} presents. Not to be outdone, Saketh said he had 9^9. In order to one-up Saketh, Billy boasted that he had gotten 10^8 presents. Who amongst the Tired Trio truly received the most presents?

2. Lemy Ree was decorating his favorite tree and asked his best friend Warker Pon to hand him some ornaments from a box with 32 ornaments in it: 6 blue, 8 red, 4 yellow, and 14 white. Lemy had severe allergic reactions anytime he saw the color white (like this page) and as a result, he asked Warker to hand him some non-white ornaments. Unfortunately, Warker did not hear Lemy and the unknowing Mr. Pon decided to pull out three ornaments all at once. What is the probability that it was not Lemys Last Christmas?

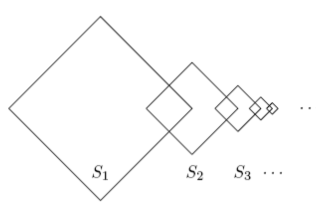
3. Kiffa, who is visiting from China, decided to celebrate the holidays in true VMT style. He called up his trusty sidekick Ritwik and asked him for the most challenging problem Ritwik had ever heard. Ritwik responded with the following question: What is the smallest positive integer k that such that $2 * k$ is a perfect square and $3 * k$ is a perfect cube? What is the answer that Lil Kiffer gave his secondhand sidekick?

Set 2

4. Katie and Julie are baking Holiday cookies for a bake sale. They flatten out a circle of dough with radius 20. One of the cookie cutters is a circle of radius 5 and another is a square with x edge length. The square cookie can be cut out tangent to the circle and has two corners on the edge of the dough. What is the length x of the cookie cutter?

5. Joe Park is skating one day in a private rink. He, feeling very festive, imagines the center of the rink to be $(0,0)$ on a coordinate system. He graphs the equations $y = k$, $y = \sqrt{3} * x + 2 * k$, and $y = -\sqrt{3} * x + 2k$ for $k = -10, -9, -8, \ldots, 9, 10$. How many equilateral triangles of side $\frac{2}{\sqrt{3}}$ did he make during his ice skating adventures?

6. Kristina is hosting a holiday party at her house. She made 5 square $2D$ cookies (S_1, S_2, S_3, S_4, and S_5), the first of which is 1x1, and the subsequent cookies have $\frac{1}{4}$ the area of the preceding cookie. She lays them out so that the two adjacent sides of the square S_i are the perpendicular bisectors of two adjacent sides of square S_{i+1}. What is the total area enclosed by the squares?



Set 3

7. A circular winter storm of 50 miles in diameter starts off on a coordinate plane at $(0, 110)$ and travels SE at a speed of $\frac{1}{2}\sqrt{2}$ miles per minute. Rudolph sees this storm and starts dashing through the snow towards his home far down the positive x-axis (East) at a speed of $\frac{2}{3}$ miles per minute. Unfortunately, Rudolph enters the storm at time t_1 along his way back, but leaves the storm circle later at time t_2. What is $\frac{t_2+t_1}{2}$?

8. After the storm blew past Rudolph, it hit a town of gingerbread houses. Thus it was snowing heavily outside when Johnny was heading home from school. Although Johnny could now barely see his own hand in front of his face, he still has a good sense of where his house is. However, after walking for a certain distance he figures that he has drifted off track, and will stop, turn 90 degrees clockwise, and continue walking until he thinks he is lost again. If Johnny first walks from the origin (school) 1 mile North, then East for a $\frac{1}{2}$ mile, then South for a $\frac{1}{4}$ mile, and so on, going $\frac{1}{2}$ the distance gone before, what coordinate will he end up at relative to the school?

9. The increasing sequence $1, 3, 4, 9, 10, 12, 13,$... consists of all those positive integers which are powers of 3 or sums of distinct powers of 3. Find the 100th term of this sequence.

Set 4

10. Santa Clause has recently called for a round table meeting between 5 elves, 5 reindeer, and 5 trolls. At this meeting, the fifteen individuals will sit in a circular table ordered from 1 to 15 in clockwise order. Santas orders are that (1) An elf must sit in chair 1, (2) A reindeer must sit in chair 15, and (3) No elf can sit immediately to the left of a reindeer, no reindeer can sit immediately to the left of a troll, and no troll can sit immediately to left of an elf. Given that everyone follows Santas orders (lest they be placed on the naughty list) the number of possible arrangements is $N * (5!)^3$. Find N.

11. Frobenius the Snowman (Frostys mathematically inclined cousin) was interested in finding the number of positive integers with three not ncessarily distinct digits, abc, with $a \neq 0$, $c \neq 0$, such that both abc and cba are divisible by 4. Please help Frobenius find the answer to this question.

12. John Jacob Jingleheimer Schmidt recently realized that y, the number of candy canes he will receive this winter break, follows the form $y = \frac{9x^2 sin^2 x+4}{sin x}$. Find the minimum number of candy canes John will receive given that $0 < x < \pi$.

Set 5

13. Define a positive integer n to be a factorial tail if there is some positive integer m such that the decimal representation of $m!$ ends with exactly n zeroes. How many positive integers less than 1992 are not factorial tails?

14. Katie and Sarthak, little elves living in the North Pole, are walking in the same direction, Katie at 3 feet per second and Sarthak at 1 foot per second, on parallel paths that are 200 feet apart. At the North Pole lies a giant circular building 100 feet in diameter and centered midway between the paths. At the instant when the building first blocks the line of sight between Sarthak and Katie, they are 200 feet apart. What is the amount of time, in seconds, before Sarthak and Katie can see each other again?

15. In $\triangle ABC, AB = 360, BC = 507$, and $CA = 780$. Let M be the midpoint of $\overline{CA}$, and let D be the point on $\overline{CA}$ such that $\overline{BD}$ bisects angle ABC. Let F be the point on $\overline{BC}$ such that $\overline{DF} \perp \overline{BD}$. Suppose that $\overline{DF}$ meets $\overline{BM}$ at E. The ratio $DE : EF$ can be written in the form m/n, where m and n are relatively prime positive integers. Find $m + n$.