

2012-13 TJUSAMO Practice Olympiad 1 – Hints

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November 12th, 2012

1. There are many ways to go about solving this problem; one of the more intuitive involves primes. What does it mean for that fraction to be an integer?
2. The orthocenter of triangle POQ is a somewhat awkward point that we don't know the most about. Instead let's define a new point H that is the intersection of AC and the height of triangle POQ through O , and try to show that QH and OP are perpendicular. This is an extremely common technique, so make sure you understand why this is the same problem.
3. For (a), try being the librarian with a few different starting positions, and convince yourself that the process must eventually end; then rigorize what you discover for the reader.

For (b), n is a natural number hmmmmm. Let's induct! Try small cases to come up with a conjecture as to what the answer is.