Recursion

Joseph Park

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1 Basic Problems

1. For how many nonnegative integer values of n less than 1000 is a_n divisible by 3 if $a_0 = 1, a_1 = 1$, and $a_{n+2} = a_{n+1} + a_n$ for $n \geq 0$.

2. What is the units digit of a_{100} if $a_1 = 8, a_2 = 4$, and $a_{n+2} = 12a_{n+1} - 32a_n$?

3. What is a_n (in terms of a_k only- no integers) if $a_0 = 6, a_1 = 12$, and $a_n = a_{n-1} + 2a_{n-2} + 6$ for $n \geq 2$?

4. What is a_n (in terms of n) if $a_0 = 4, a_1 = 32$, and $a_{n+2}a_n^4 = a_{n+1}^5$ for $n \geq 0$?

5. What is a_n (in a nice exponential recurrence) if $a_n = a_{n-1}a_{n-2} + 2a_{n-1} + 2a_{n-2} + 2$?

2 Word Problems

1. Victoria is playing with legos. She is trying to exactly fill a row with 15 slots. She has an unlimted number of three types of blocks: red miniblocks, blue miniblocks, and green megablocks. Miniblocks fill up one slot and megablocks fill up two slots. How many ways can she fill up the row of 15 slots with blocks?

2. How many different strings of ones and zeros are there of length 12 such that no two ones are adjacent?

3. How many different strings of ones, twos, and threes are there of length 6 such that no number is repeated consecutively.

4. How many 10-bit binary strings can be made so that no three consecutive bits are the same?

5. I have green beads and blue beads and I want to put them on a string such that nowhere on the string are there either two consecutive green beads or three consecutive blue beads. How many ways can I construct a string of beads of length 10 given this condition.

6. (Yuqing Zhang) Warker Pon and Genny Jong are in a long-distance relationship, and they can only see each other three times every year: at Duke, at HMMT, and at ARML. They go on a date every time they see each other. On every single date, Warker and Genny play a coin-flipping game to determine who will pay for the dinner. They take turns flipping until a heads is flipped, and the person who flips heads pays. They agree that whoever pays for a date will flip second on the next date. Warker knows that Genny is planning to go to Led Robster on their 6th date, and he wants to avoid paying for that night. Should Warker choose to flip first or second on their first date? Warker decides to do as you told him earlier (flip first or second). At ARML 2012 (their 6th date) Warker squanders away all his money buying presents for Genny, and he doesn't have any money left for their Led Robster date. What is the probability that Genny won't find out that Warker is irresponsible?

7. A paperboy delivers newspapers to 10 houses along Main Street. Wishing to save effort, he doesn't always deliver to every house, but to avoid being fired he never misses three consecutive houses. Compute the number of ways the paperboy could deliver papers in this manner.

8. You have 8 cubes of size 1 through 8. You must build a tower with the constraint that the cube on top of a cube of size k must have a size of at most $k + 2$. How many different towers can be constructed?