November 2012

⋆ National Level ⋆

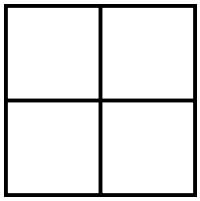
The Mandelbrot Competition

Round One Test

Time Limit: 40 minutes

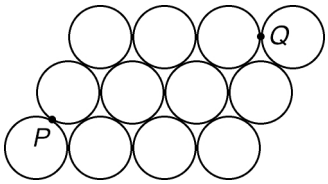
Name: _____

1. It is possible to place a single digit from 0 to 9 in each of the four boxes so that the two-digit numbers reading across have a greatest common divisor of 14, while the two-digit numbers reading down have a greatest common divisor of 3. When this is done, what is the sum of all four digits?



(1)

2. The twelve tangent circles shown all have radius equal to 1. What is the length of the shortest path from point P to point Q that does not pass through the interior of any of the circles?



(1)

3. There is a real number x in the interval $0 < x < 1$ satisfying the equation $\sqrt{1-x} + \sqrt{1+x} = \sqrt{2.012}$. Determine the value of x^2 as a decimal.

(2)

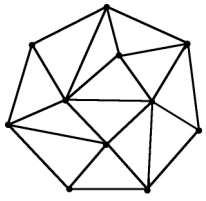
4. Suppose that in quadrilateral $ABCD$ we have $m\angle ABC = m\angle ACD = 90°$ and $m\angle CBD = m\angle CDB$. Label $m\angle DAC = \alpha$ and $m\angle ACB = \beta$. It follows that one of $\sin\alpha$, $\cos\alpha$, or $\tan\alpha$ must always be equal to one of $\sin\beta$, $\cos\beta$, or $\tan\beta$. Which two values are necessarily the same?

(2)

5. Light red paint is made by mixing white paint and red paint in a $1:4$ ratio, while pink is obtained by using a $4:3$ ratio. What ratio of light red to pink paint, in that order, will yield a $5:6$ ratio of white to red paint? (Write your answer as $m:n$, where m and n are relatively prime positive integers.)

(2)

6. To triangulate a polygon is to plot extra points inside, then to connect points with nonintersecting segments until all the inner regions are triangles. Determine the largest number of triangles possible in a triangulation of a regular 100-gon if the average number of segments extending outward from each point (interior and boundary) is at most 5.



Triangulation of a heptagon

(3)

7. Chamran flips twenty-eight fair coins. What is the probability that the number of heads that he obtains is a multiple of 4?

(3)

SCORE: _____