Geometry Performance Contest

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1. Given regular pentagon $ABCDE$, point F is inside the pentagon such that ABF is an equilateral triangle. Find the measure of $\angle FCD$ in degrees.

2. In right triangle ABC, $\angle B = 90°$, M is the midpoint of AC, and points X and Y are on sides AB and BC respectively. Given that $\angle XMY = 90°$, if $BX > AX$, $AX = 6$, and $CY = 9$, find XY.

3. In $\triangle ABC$, $AB = 13, BC = 14$, and $CA = 15$. Point D lies on side BC such that triangles ABD and ACD have equal inradii of length $6 - 2\sqrt{3}$. Find the length of AD.

4. Let $[ABC]$ denote the area of $\triangle ABC$. Given rectangle $MNPQ$, X and Y are on PQ and NP, respectively, such that $[MNY] = 12$ and $[YPX] = [XQM] = 8$. If $MN = x$ where $x = \frac{7\pi\sqrt{3}}{5}$, find $[MXY]$.

5. Quadrilateral $ABCD$ is inscribed in a circle such that $AB = BC = 6$, $AD = 4$, and $\angle CDA = 120°$. Find BD.

6. In triangle PQR, the circle with diameter PR intersects *segments* PQ and QR at M and N respectively, such that $PM < RN$. If $[PQR] = 4[QMN]$ and it is given that one of the angles in $\triangle PQR$ is 1.4 times another angle, find the maximum possible value of $\angle QPR$ in degrees.