December 2011

★ National Level ★

The Mandelbrot Competition
Round Two Test

Time Limit:
40 minutes

Name: _____

1. Determine the smallest positive integer, all of whose digits are even, that is a multiple of 9.

①

2. The goal of this puzzle is to return the letters to alphabetical order: A, B then C, D, E then F, G in the rows from top to bottom. On each move you may rotate any triangular block of circles, such as the highlighted block, by one or two places. What is the least number of moves required?

①

3. Farmer Theo has three straight, rigid lengths of fencing, each 20 feet long. He wishes to create a chicken coop in the shape of a quadrilateral by attaching the fence pieces to one another and to the side of his 100-foot barn. What is the largest area, in square feet, that he can enclose?

②

4. Rounded to the nearest hundredth, the positive real number x satisfying the equation $3^x + 6x = 99$ is given by $x = 3.93$. Find the solution to the equation $3^x + 2x = 31$, rounding your answer to the nearest hundredth.

②

5. A semicircle rests on the negative x-axis and is tangent to the y-axis. A line intersects both axes and the semicircle, as shown. Suppose that the points of intersection create three segments of equal length. What is the slope of the line?

②

6. Let $p(x)$ be a quadratic of the form $x^2 + bx + c$, with b and c real, such that $|p(x)| \leq 100$ whenever $|x| \leq 1$. Find the maximum possible value of $|b^2 - 4c|$.

③

7. Suppose that each face of a $4 \times 4 \times 4$ cube is divided into unit squares. These form twelve bands around the cube; four horizontal and eight vertical bands, one of which is highlighted. How many ways are there to color some of the unit squares so that each band contains exactly one colored square?

③

SCORE: