4. Problems

1. Prove that if n, m are positive integers, then $\frac{m}{n} < \sqrt{2}$ if and only if $\sqrt{2} < \frac{m+2n}{m+n}$.

2. $(IMO, 1960)$ For which real values of x the following inequality holds: $\frac{4x^2}{(1-\sqrt{1+2x})^2} < 2x+9$?

3. If $a, b, c > 0$ satisfy that $abc = 1$, prove that

$$\frac{1+ab}{1+a} + \frac{1+bc}{1+b} + \frac{1+ac}{1+c} \geq 3$$

4. Let a, b, c be positive numbers with $a+b+c=1$, prove that

$$\left(\frac{1}{a}-1\right)\left(\frac{1}{b}-1\right)\left(\frac{1}{c}-1\right) \geq 8$$

5. Let x, y, z be positive real numbers. Prove that

$$\frac{x}{x+2y+3z} + \frac{y}{y+2z+3x} + \frac{z}{z+2x+3y} \geq \frac{1}{2}$$

6. $(Croatia, 2004)$ Let x, y, z be positive real numbers. Prove that

$$\frac{x^2}{(x+y)(y+z)} + \frac{y^2}{(y+z)(z+x)} + \frac{z^2}{(z+x)(z+y)} \geq \frac{3}{4}$$

7. $(India, 2002)$ If a, b, c are positive real numbers, prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq \frac{c+a}{c+b} + \frac{a+b}{a+c} + \frac{b+c}{b+a}$$

8. $(Romania, 2002)$ If a, b, c are real numbers in the interval (0,1), prove that

$$\sqrt{abc} + \sqrt{(1-a)(1-b)(1-c)} < 1$$

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9. $(Kazakhstan, 2008)$ Let x, y, z be positive real numbers such that $xyz = 1$. Prove that

$$\frac{1}{yz+z} + \frac{1}{zx+x} + \frac{1}{xy+y} \geq \frac{3}{2}$$

10. *Let a, b, c be positive real numbers. Prove that

$$\frac{a^2+b^2+c^2+ab+bc+ca}{(a+b)(b+c)(c+a)} + \frac{1}{2 * \sqrt[3]{abc}} \geq \frac{16 * \sqrt[3]{a^2b^2c^2}}{(a+b)(b+c)(c+a)}$$

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