Bijections Continued
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Practice (from last time):

1. A frog is at the point (0,0). Every second, he can jump one unit either up or right. He can only move to points (x,y) where x and y are not both odd. How many ways can he get to the point (6,12)?

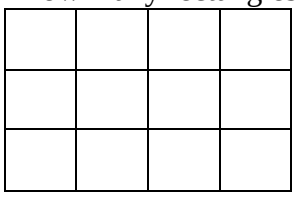
2. Let $(a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9, a_{10}, a_{11}, a_{12})$ be a permutation of $(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12)$ for which $a_1 > a_2 > a_3 > a_4 > a_5 > a_6$ and $a_6 < a_7 < a_8 < a_9 < a_{10} < a_{11} < a_{12}$. An example of such a permutation is $(6, 5, 4, 3, 2, 1, 7, 8, 9, 10, 11, 12)$ Find the number of such permutations.

3. Find the number of 5-element subsets of $\{1,2, \ldots ,30\}$ such that no two pairs of elements differ by less than 3.

Some More Strategies:

- Counting Consecutive Sequences
  How many consecutive subsets are there from the set {1,2,…50}? Examples of consecutive subsets are {2}; {4,5,6,7}; {1,2,3,4,5}.

- Counting Rectangles
  How many rectangles are there in the figure below? Generalize for any *m x n* grid.



- Splitting Sets (this one's really cool!)
  For each positive integer n let S_n denote the set {1,2,…n}. Find the number of triples of subsets A, B, C of S_{2006} (not necessarily nonempty or proper) such that A is a subset of B and S_{2006}-A (elements of S_{2006} not contained in A) is a subset of C.

Examples:

i. Determine the number of subsets S of {1,2,…10} with the following property: there exist integers $a < b < c$ with $a \in S, b \notin S, c \in S$.

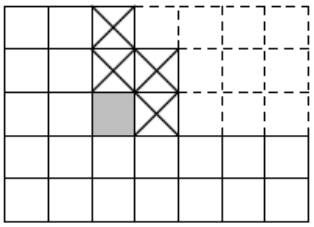
ii. On a 2 x 2003 unit square grid, the two unit squares in the middle of the grid are colored black. Consider the set of all rectangles formed by the unit squares in the grid. If a rectangle is chosen randomly from this set, what is the probability that the rectangle does not include a black square?

iii. Ten Cs are written in a row. Some are upper-case, some are lower-case, and each is written in either green or yellow ink. It is given that there is at least one lower-case C, at least one green C, and at least one C that is both upper-case and yellow. Furthermore, no lower-case C can be followed by an upper-case C, and now yellow C can be followed by a green C. In how many ways can the Cs be written?

Practice:

1. How many ways can the integers from -10 to 10 inclusive be arranged in a sequence such that the absolute value of the numbers in the sequence does not decrease?

2. To temporarily escape the workload of junior year, 2 students decide to play the (very unhealthy) game of Chomp, in which they alternately take bites from a 5" x 7" bar of chocolate. To take a bite, a player chooses one of the remaining squares of chocolate, and then eats all squares in the quadrant defined by the left edge (extended upward) and the lower edge (extended rightward) of the chosen square. For example, the bite determined by the shaded square in the diagram would remove the shaded square and the four squares marked by X. (The squares with two or more dotted edges have been removed form the original board in previous moves.)



The object of the game is to make one's opponent take the last bite. The marked squares in the diagram above form one of the many subsets of the set of 35 unit squares that can occur during the game of Chomp. How many different subsets are there in all? Include the full board and empty board in your count.