Cyclic Quadrilaterals

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1 Definition of a Cyclic Quadrilateral

Unlike triangles, not all quadrilaterals can be inscribed in a circle. However, those that can be inscribed are called cyclic quadrilaterals. Some examples of cyclic quadrilaterals include rectangles and isosceles trapezoids. The vertices of such quadrilaterals are said to be concyclic.

2 Other Geometry Knowledge to Keep in Mind

Power of a Point - In quadrilateral $ABCD$, where P is the intersection of diagonals AC and BD, $AP \times PC = BP \times PD$ iff $ABCD$ is cyclic.

Central angle vs inscribed angle - It's important to remember that the measure of the central angle is two times the measure of the inscribed angle.

Angle and Arc Chasing - This idea is used quite a bit to solve cyclic problems.

3 Establishing that a Quadrilateral is Cyclic

Given any convex quadrilateral $ABCD$, we know that it is cyclic iff the following properties hold true.

- The sum of opposite angles is 180°,

$$\angle A + \angle C = \angle B + \angle D = 180^\circ.$$

- When diagonals AC and BD are drawn, four pairs of equal angles are created. They are

$$\angle ABD = \angle ACD$$
$$\angle ACB = \angle ADB$$
$$\angle BAC = \angle BDC$$
$$\angle CBD = \angle CAD$$

  Note that these pairs of inscribed angles are equal because the angles subtend the same arc.

4 Useful Theorems and Formulas

These formulas apply to cyclic quadrilaterals only. Given cyclic quadrilateral $ABCD$.

Ptolemy's Theorem -

$$AB * CD + AD * BC = AC * BD$$

Brahmagupta's Theorem -

$$[ABCD] = \sqrt{(s - AB)(s - AD)(s - BC)(s - CD)}$$

where s is the semiperimeter of the $ABCD$.

5 Problems

1. Find the diagonal length of an isosceles trapezoid with legs of 8 and bases of lengths 6 and 10.
2. In cyclic quadrilateral $ABCD$ with diagonals intersecting at E, we have $AB = 5$, $BC = 10$ and $CD = 6$. Find the length of CE.
3. Quadrilateral $ABCD$ has sides $AB = 7$, $BC = 24$ and $CD = 20$. Given that it is inscribed in a circle with circumference 25π, what is the length of the other diagonal of $ABCD$?
4. Prove the previous Power of a Point statement.
5. Prove Ptolemy's Theorem.
6. In quadrilateral $ABCD$ with diagonals AC and BD intersecting at O, $BO = 4$, $OD = 6$, $AO = 8$, $OC = 3$ and $AB = 6$. Find AD. (ASHME)