

2011-12 TJUSAMO Practice Olympiad 1

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1. Let u , v , and w be positive reals. Given that $u\sqrt{vw} + v\sqrt{wu} + w\sqrt{uv} \geq 1$, find, with proof, the minimum possible value of $u + v + w$.
2. In parallelogram $ABCD$, $AD > AB$. The angle bisector of $\angle ABC$ intersects segment AD and the extension of CD at points P and Q , respectively. Let O be the circumcenter of $\triangle DPQ$. Prove that $CDOA$ is cyclic.
3. Two people are playing a game in which they start with a positive integer N . Each turn, the player whose turn it is may subtract any positive perfect square less than or equal to N from N (letting the new number be the new N) and end his turn. The game ends when N becomes zero, and the last player to remove a number wins. If the two players alternate turns, prove that there are infinitely many starting numbers N such that the second player will win if he plays with the optimal strategy.