Zeeman Effect and Peak Deconvolution

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In this paper, I examined the classic quantum mechanic experiment exploring the linear region of Zeeman splitting in a cadmium atom with an eye towards finding the electric dipole moment of an electron. I used a Perot-Fabry etalon to create circular fringes that split when a magnetic field is applied to the cadmium lamp. This work specifically looks at lower magnetic fields, while my experimental set up is in a transverse position. When in this position, I was only able to filter out one of the three possible peaks, and as such, I was left with two overlapping peaks. I was able to deconvolute these peaks by forcing a fit of a double Gaussian equation and using the subsequent two Gaussian peaks for calculations. By doing this, I was able to probe much lower magnetic fields in the transverse position and measure the electric dipole moment to within 2 percent error.

I. INTRODUCTION

An interesting application of atomic shell structure and quantum mechanics is the introduction of the concept of degeneracies in atomic electron shells. The electrons actually can form shells that are each composed of two electrons, but these shells are only distinct when the atom is exposed to a magnetic field. In a zero magnetic field environment, the shells are degenerate, and thus, indistinguishable from each other and so fits the more traditional shell structure and model. This behavior, from a theoretical and model perspective, can be described as the magnetic field acting on the magnetic dipole moment of the electron, thus creating a torque on the electron.

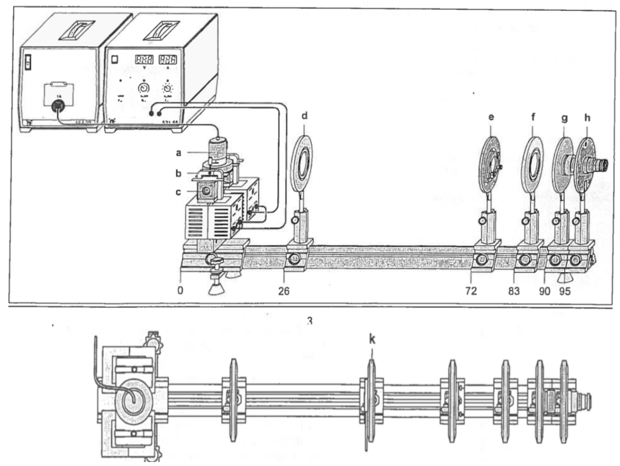
This behavior is indispensable from an application stand-point with interesting implementations in different fields. It allows scientists to observe magnetic fields by observing spectral lines of light which is of particular note for astronomers and astrophysicists as it allows a detailed analysis of the sun's magnetic field in some great depth. It has also been posited that certain proteins in pigeons' eyes are able to shift based on the Zeeman effect caused by the earth's magnetic field.

In this study, I will be looking at low magnetic fields and the light that is being emitted orthogonal to the magnetic field or what is referred to as the "transverse position" of the experimental set up. In the transverse position and a low magnetic field, the experiment is unable to resolve the splitting of two spectral lines, but deformed peaks are visible. As such, I will able to fit a double Gaussian curve to this data to deconvolute the peaks and gain an udersadating of the Zeeman splitting at low magnetic fields.

II. EXPERIMENTAL SETUP

My experimental setup is relatively simple and can be seen in Figure 1.

This setup produces a cross section of the circular fringe that is produced by the etalon and gives many peaks that I can use for my data analysis processing.



Experimental setup for observing the *Zeeman* effect in transverse configuration. The position of the left edge of the optics riders is given in cm.

a	Cadmium lamp with holding plate
b	Clamps
c	Pole pieces
d	Positive lens, f = 150 mm (Condenser lens)
e	*Fabry-Perot* etalon
f	Positive lens, f = 150 mm (imaging lens)
g	Interference filter in holder
h	VideoCom (it is helpful to use an ocular for adjustment)

FIG. 1. Note that k is a linear polarizing filter and the camera is not shown but is placed in the VideoCom optic seen as the end of the set up at h.

Figure 2 gives an example of the type of data I will be processing.

The peaks as shown in Figure 2 are taken with no magnetic field, and as such, have no splitting. Once the magnetic field is turned high enough, the peaks will split, an example of which is shown in Figure 3.

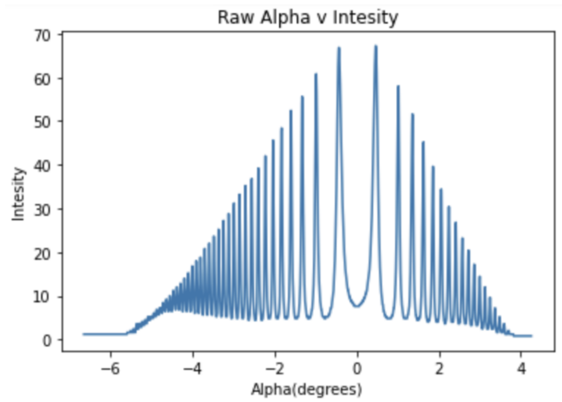


FIG. 2. Alpha is the incident angle of light on the etalon. Additionally, this is the raw data given by the camera and is a horizontal cross section of a classic circular fringe.

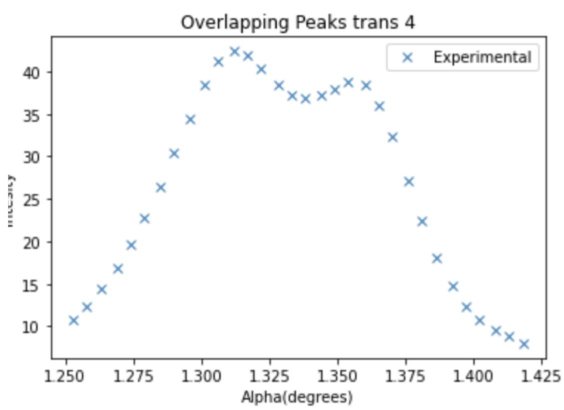


FIG. 3. Double peak splitting seen in a single peak of the circular fringe at a magnetic field made by a 4 Amp current.

III. ENERGY SPLITTING AND PEROT-FABRY ETALON

The splitting of atomic shells at lower energies is roughly linear and can be approximated very well with this formula:

$$\Delta E = \mu_B \cdot B \tag{1}$$

To use this formula, I had to first find the change in energy (Δ E). This was done by using the geometry of the etalon's structure, basic quantum mechanical formulas and ratios of multiple similar measurements:

$$2dcos(\beta) = m\lambda \tag{2}$$

$$\frac{2dcos(\beta_2)}{2dcos(\beta_1)} = \frac{m\lambda_1}{m\lambda_0} \tag{3}$$

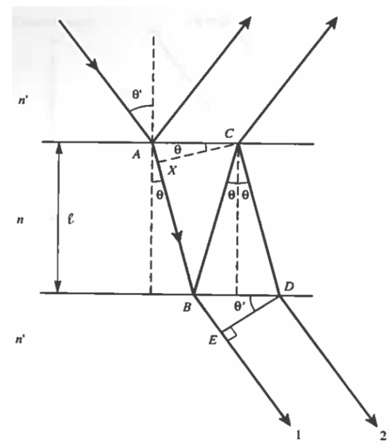


FIG. 4. The geometry of the etalon filter. Note the angle of incidence is denoted as α and the internal angle is denoted as β for the purpose of calculations.

$$\frac{cos(\beta_2)}{cos(\beta_1)} = \frac{\lambda_1}{\lambda_0} \tag{4}$$

$$\frac{cos(\beta_2)}{cos(\beta_1)} - 1 = \frac{-\Delta\lambda_1}{\lambda_0} \tag{5}$$

$$E = \frac{hc}{\lambda} \tag{6}$$

$$\Delta E = -hc\frac{\Delta\lambda_1}{\lambda_0^2} \tag{7}$$

$$\Delta E = \frac{\Delta\lambda_1}{\lambda_0}E \tag{8}$$

$$\Delta E = [\frac{cos(\beta_2)}{cos(\beta_1)} - 1] * E \tag{9}$$

As you can see, I only have a dependence of change in energy based on the internal angle of refraction of the etalon filter. This is slightly problematic, as our camera software only gives us the external angle of incidence, but this was easily solved by using Snell's law:

$$n_1cos(\alpha) = n_2cos(\beta) \tag{10}$$

All measurements for the change in energy levels are averaged for their magnetic field strength to give the best picture of the splitting that is experienced at a given magnetic field.

IV. DECONVOLUTING PEAKS

I now have a solid understanding for what analysis I will do with the peaks once I have them. This still leaves me with the question of how I will try to actually deconvolute the peaks. In this instance, I choose to use python as a data processor and specifically the scipy.curvefit optimizer to fit a double Gaussian equation to our data:

$$f(x) = ae^{\frac{(x-b)^2}{2c^2}} + de^{\frac{(x-f)^2}{2g^2}} \tag{11}$$

This equation was applied to each individual peak in the data set using guess parameters based on the observed data, while not exposed to the magnetic field. The scipy fitting tool uses root mean square error to fit the curves as closely as it could. From this data, I was able to take the center or peak of each Gaussian and treat it as a peak of the circle fringe in our data analysis. An example fitted peak can be seen in Figure 5 below.

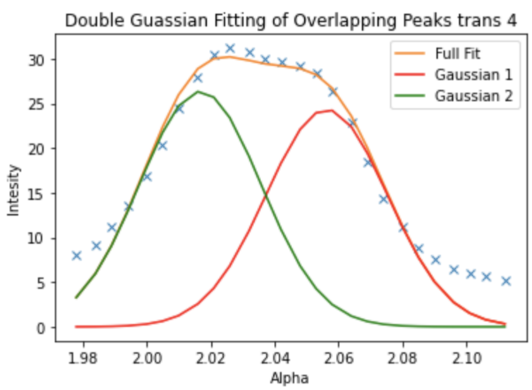


FIG. 5. Double peak fitting is seen here with the two component Gaussians shown in red and green, the combined fit curve in orange, and the actual data in blue X's. Of particular note is that the raw data itself has a fair amount of noise on the sides of the peaks and is thus not fitted.

V. RESULTS

Once I had the change in energy levels for multiple magnetic fields, I was able to plot the change in energy versus the change in magnetic field. Plotting this data, finding the trend line, and using Equation 1 to relate energy splitting to magnetic field strength, I found that the slope of this line is actually the magnetic dipole moment for the electron, also known as the Bohr magneton. Additionally, for a comparison, I had previous data from an earlier project that only had data points for higher magnetic fields, as I could only use normal splitting at the time.

As can be seen in Figures 6 and Table 1, the slope of the line is slightly higher and thus closer to the Bohr magneton. This decreased the error of my estimate of the Bohr magneton from 5.30 percent to 1.46 percent, a significant decrease in error by using this peak deconvolution method.

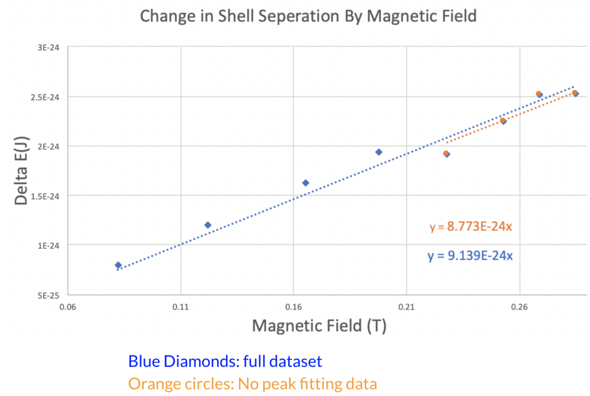


FIG. 6. A plot of the new trend line with additional data points in blue and the old trend line with no additional data in orange.

TABLE I. Results

Measurement	Value (J/T)	Percent Difference
μ_B (actual)	$9.27401E-24$	0
μ_B (peak finding)	$8.773E-24$	5.40
μ_B (peak fitting)	$9.139E-24$	1.46

VI. CONCLUSION

By using this peak deconvolution method, I was able to decrease the percent error of my estimate of the Bohr magneton by about 4 percent, a significant decrease in error. While this method is somewhat complicated and slightly computationally expensive, the size of the data set is small, and the number of processed data points is low, so this is a valid approach to decreasing the error of my estimate by a significant degree.

ACKNOWLEDGMENTS

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