EM384: Analytical Methods for Engineering Management

Lesson 19: Introduction to Minimum Cost Network Flow Problems

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Lesson Objectives

Lesson Objectives

- Understand network flow problems as a subset of linear programming problems.
- Define a network flow problem and identify where it may be applicable.
- Understand minimum cost flow network flow problems.
- Generate a diagram for a minimum cost network flow problem.

Network Flow Problems

Network Flow Problems

- The network model describes patterns of flow in a connected system, where the flow might involve material, people, or funds.
- When we construct diagrams to represent such systems, the elements are represented by nodes, or circles, in the diagram. The paths of flow are represented by arcs, or arrows.
- We consider Minimum Cost Network Flow Problems in this course.

Terminology

- The model for any minimum-cost flow problem is represented by a network with flow passing through it.
- The circles in the network are called nodes.
- Each node where the net amount of flow generated (outflow minus inflow) is a fixed positive number is a supply node.
- Each node where the net amount of flow generated is a fixed negative number is a demand node.
- Any node where the net amount of flow generated is fixed at zero is a transshipment node. **Having the amount of flow out of the node equal the amount of flow into the node** is referred to as conservation of flow.
- The arrows in the network are called arcs. (Must have arrows at the end of an arc to indicate network flow)
- The maximum amount of flow allowed through an arc is referred to as the capacity of that arc.

Notation

You may choose your own notation, however it should be consistent throughout your problem. Example:

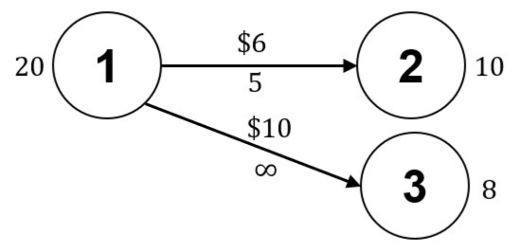
- Let i be a node where $i = 1, \ldots, n$
- Let c_{ij} be the cost of flow for 1 unit of a good from node i to node j (note that if there are 10 nodes or more of any kind we would write $c_{i,j}$ with a comma)
- let x_{ij} be the total units that flow from node i to node j.

Network Flow Diagrams

Network Flow Diagrams

In EM384 we will adopt the following convention for networ flow diagrams:

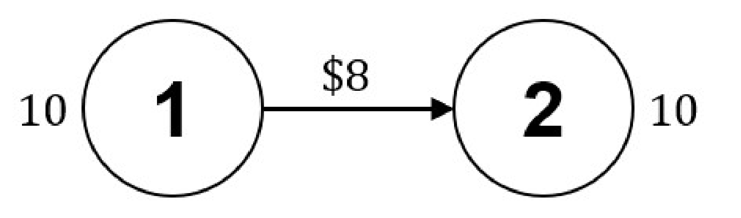
- A number next to a supply node represents the supply.
- A number next to a demand node represents the demand.
- A number *above* an arc represents the cost of one unit of flow on that arc.
- A number *below* an arc represents the capacity of that arc (no number means uncapacitated)
- any additional information you wish to draw on the diagram should be linked to its variable name for clarity (e.g. $x_{12} = 10$).



Network Flow Diagrams

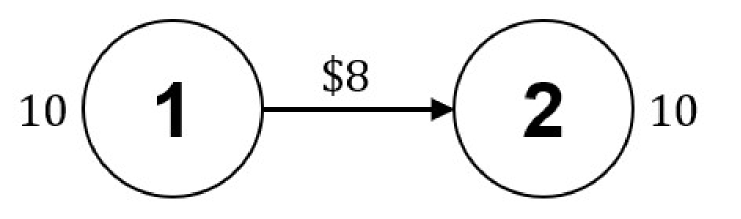
Given the diagram below, Answer the following questions:

1. What nodes are supply nodes? What is their supply?
2. What nodes are demand nodes? What is their demand?
3. What is the flow x_{ij} for each arc?
4. What is the total minimum cost Z?



Given the diagram below, Answer the following questions: This problem is balanced, since supply $=$ demand

1. What nodes are supply nodes? What is their supply? Node 1, with a supply of 10
2. What nodes are demand nodes? What is their demand? Node 2, with a demand of 10
3. What is the flow x_{ij} for each arc? $x_{12} = 10$
4. What is the total minimum cost Z? \$80



Network Flow Diagrams

Given the diagram below, Answer the following questions:

1. What nodes are supply nodes? What is their supply?
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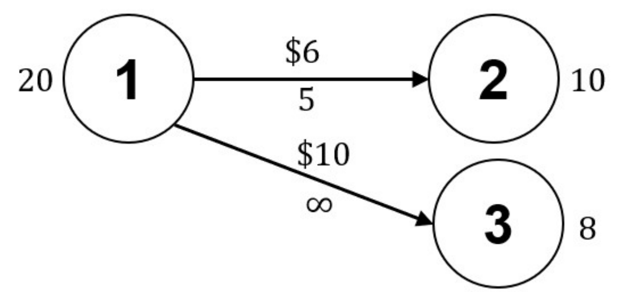
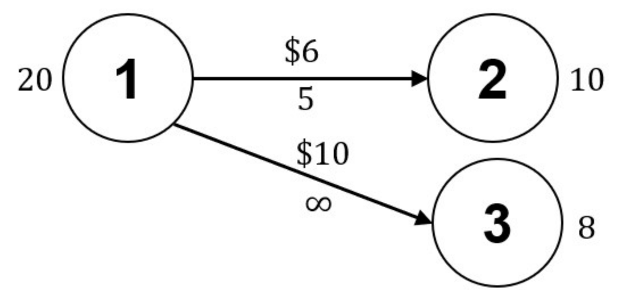


Network Flow Diagrams

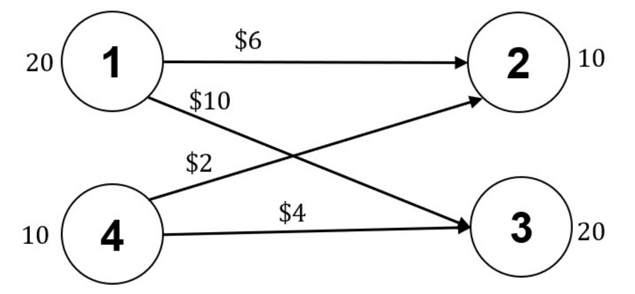
Given the diagram below, Answer the following questions: This problem is unbalanced, since supply $\neq$ demand

1. What nodes are supply nodes? What is their supply? Node 1, with a supply of 20
2. What nodes are demand nodes? What is their demand? Node 2, with a demand of 10, and Node 3, with a demand of 8
3. What is the flow x_{ij} for each arc? $x_{12} = 5$, $x_{13} = 8$
4. What is the total minimum cost Z? $5(6) + 8(10) = \$110$



Given the diagram below, Answer the following questions:

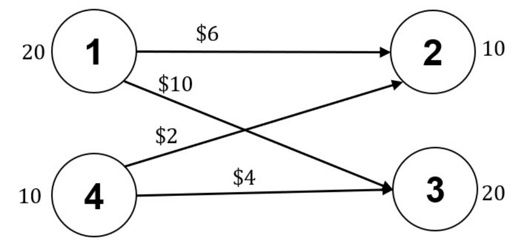
1. What nodes are supply nodes? What is their supply?
2. What nodes are demand nodes? What is their demand?
3. What is the flow x_{ij} for each arc?
4. What is the total minimum cost Z?



Network Flow Diagrams

Given the diagram below, Answer the following questions: This problem is balanced, since supply $=$ demand

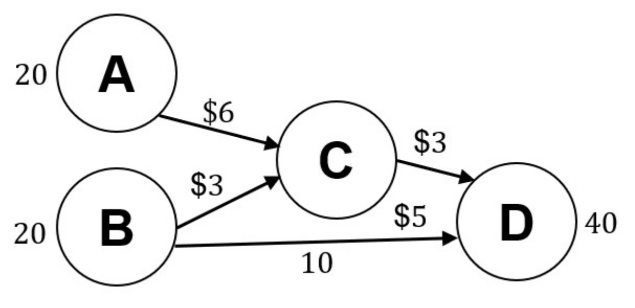
1. What nodes are supply nodes? What is their supply? Node 1, with a supply of 20, and Node 4, with a supply of 10
2. What nodes are demand nodes? What is their demand? Node 2, with a demand of 10, and Node 3, with a demand of 20
3. What is the flow x_{ij} for each arc? $x_{12} = 10$, $x_{13} = 10$, $x_{42} = 0$, $x_{43} = 10$
4. What is the total minimum cost Z? $10(6) + 10(10) + 10(4) = \$200$



Network Flow Diagrams

Given the diagram below, Answer the following questions:

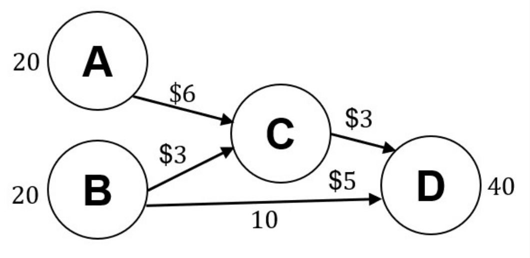
1. What nodes are supply nodes? What is their supply?
2. What nodes are demand nodes? What is their demand?
3. What is the flow x_{ij} for each arc?
4. What is the total minimum cost Z?



Network Flow Diagrams

Given the diagram below, Answer the following questions: This problem is balanced, since supply = demand. Node C is a Transshipment node!

1. What nodes are supply nodes? What is their supply? Node A, with a supply of 20, and Node B, with a supply of 20
2. What nodes are demand nodes? What is their demand? Node D, with a demand of 40.
3. What is the flow x_{ij} for each arc? $x_{AC} = 20$, $x_{BC} = 10$, $x_{CD} = 30$, $x_{BD} = 10$
4. What is the total minimum cost Z? $20(6) + 10(3) + 30(3) + 10(5) = \$290$



Practical Exercise

Practical Exercise

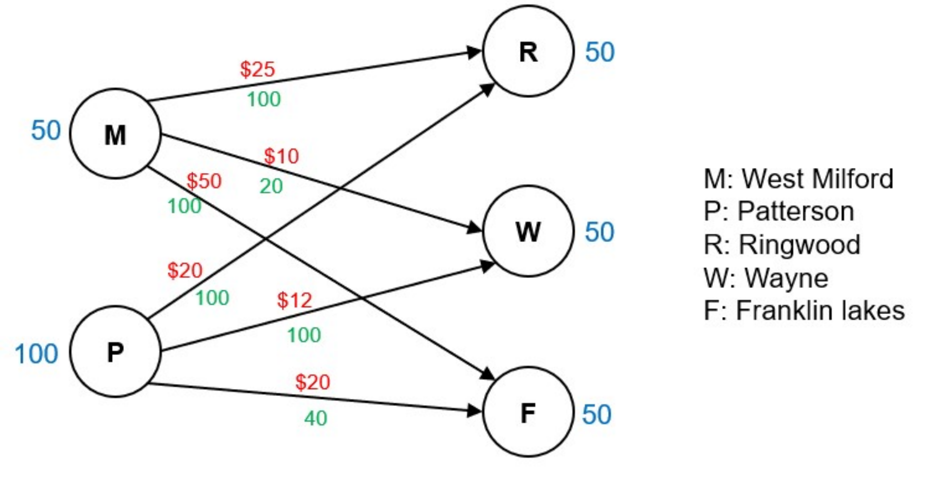
You are the manager of SNES games inc., a vintage video game retailer. You have two warehouses in West Milford and Patterson, and three retail locations in Ringwood, Wayne, and Franklin Lakes. The West Milford warehouse has a supply of 50 boxes and the Patterson warehouse has a supply of 100 boxes. The demand from all three retail locations is the same, 50 per store. The costs to ship one box are outlined below:

From / To	Ringwood	Wayne	Franklin Lakes
West Milford	\$25	\$10	\$50
Patterson	\$20	\$12	\$20

In addition, truck capacity limits the number of boxes that can go between West Milford and Wayne to 20 boxes, and from Patterson and Franklin lakes to 40 boxes. All other routes have a capacity of 100 boxes.

1. Draw a complete Network Flow diagram to represent this problem.
2. Try to find the optimal flow on each arc and total minimum cost without a solver software.

In the next lesson we will learn to model and solve minimum cost network flow problems using Excel.



PE Solution

Let x_{ij} be the number of boxes transported from warehouse i to retail location j.

The solution to the minimum cost network flow problem is given below:

$$x_{MR} = 20$$
$$x_{MW} = 20$$
$$x_{MF} = 10$$
$$x_{PR} = 30$$
$$x_{PW} = 30$$
$$x_{PF} = 40$$

The minimum total cost is $Z = \$2960$

Conclusion

Next Class

Homework:

- Read Chapter 9.1

Next Lesson: Same Lesson Objectives

- Recognize a transportation problem given a network problem.
- Formulate and solve a transportation problem in Excel Solver.
- Interpret the reduced costs for a transportation problem solution.