

EM384: Analytical Methods for Engineering Management

Lesson 13: Cost-Benefit Trade-off Problems

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Lesson Objectives

Lesson 13 Objectives

- Understand and recognize cost-benefit trade-off problems.
- Formulate cost-benefit trade-off problems algebraically.
- Solve cost-benefit trade-off problems using Excel Solver.

Cost-Benefit Trade-Off Problems

Cost-Benefit Trade-Off Problems

Linear programming problems involving meeting some fixed requirement(s), or benefit, for the lowest possible cost.

Identifying feature: Requirement constraints!

- Category or benefit \geq requirement
- Objective: Minimize

Example Exercise

Cake A provides 10g of sugar, and 50g of carbs per serving, and costs \$12. Cake B provides 15g of sugar and 40g of carbs per serving and costs \$10. Each cake provides 8 servings. Each dessert spread that we make needs to provide at least 1000g of sugar and 5000g of carbs to be delicious and nutritious. For variety, we also want to ensure that each cake type makes up at least $\frac{1}{3}$ of the spread. How many of each cake should be made to meet the requirements while minimizing cost?

1. Formulate your linear program algebraically.
2. Design an Excel model to solve your linear program.

Problem Formulation

Decision variables:

x_1 : Number of cakes of type A that are made

x_2 : Number of cakes of type B that are made

Objective function:

Minimize $Z = 12x_1 + 10x_2$ (cost)

Constraints:

$80x_1 + 120x_2 \geq 1000$ (sugar)

$400x_1 + 320x_2 \geq 5000$ (carbs)

$x_1 \geq 1/3(x_1 + x_2)$ (Cake A min amount)

$x_2 \geq 1/3(x_1 + x_2)$ (Cake B min amount)

$x_1, x_2 \geq 0$ (non-negativity)

Excel Solution

	A	B	C	D	E	F
1	Parameters					
2						
3		Cake A	Cake B	Requirement		
4	Sugar	80	120	1000		
5	Carbs	400	320	5000		
6	Cost	\$12	\$10			
7	Min ratio	0.33	0.33			
8						
9	Decision Variables					
10						
11		Cake A	Cake B			
12		8.928571	4.464286			
13						
14	Objective Function					
15						
16	Minimize Cost	\$151.79				
17						
18	Constraints					
19		Cake A	Cake B	LHS		RHS
20	Sugar	80	120	1250	>=	1000
21	Carbs	400	320	5000	>=	5000
22	Cake A min	1	0	8.928571429	>=	4.464286
23	Cake B min	0	1	4.464285714	>=	4.464286
24						

Practical Exercise

Plan a Spaghetti Dinner for the Corps of Cadets that minimizes cost while meeting nutritional requirements.

	Pasta	Meatballs	Sauce	Garlic Bread	Requirement
Calories	600	500	100	300	2000
Protein	0	10	0	3	30
Carbs	10	2	2	20	60
Cost/Serving	\$0.75	\$1.75	\$1.25	\$1.00	

REQUIREMENT: Formulate the LP (Objective Function, Decision Variables, and Constraints) and solve using Excel Solver.

Algebraic Formulation

Decision variables:

x_1 : Servings of Pasta

x_2 : Servings of Meatballs

x_3 : Servings of Sauce

x_4 : Servings of Garlic Bread

Objective function:

Maximize $Z = 0.75x_1 + 1.75x_2 + 1.25x_3 + x_4$ (Cost, in \$)

Constraints:

$600x_1 + 500x_2 + 100x_3 + 300x_4 \geq 2000$ (Calories)

$10x_2 + 3x_4 \geq 30$ (Protein)

$10x_1 + 2x_2 + 2x_3 + 20x_4 \geq 60$ (Carbs)

$x_1, x_2 \geq 0$ (non-negativity)

Excel Solution

	A	B	C	D	E	F	G	H	I
1	Cadet Mess		Spaghetti and Meatballs						
2									
3	Parameters								
4									
5			Pasta	Meatballs	Sauce	Garlic Bread	Requirement		
6		Calories	600	500	100	300	2000		
7		Protein	0	10	0	3	30		
8		Carbs	10	2	2	20	60		
9		Cost / Serving	\$0.75	\$1.75	\$1.25	\$1.00			
10									
11	Decision Variables								
12			Pasta	Meatballs	Sauce	Garlic Bread			
13			0.2	2.2	0.0	2.7			
14									
15									
16	Objective Function								
17									
18		Minimize Cost	\$ 6.65						
19									
20	Constraints								
21			Pasta	Meatballs	Sauce	Garlic Bread	LHS		RHS
22		Calories	600	500	100	300	2000	>=	2000
23		Protein	0	10	0	3	30	>=	30
24		Carbs	10	2	2	20	60	>=	60
25									

Conclusion

Homework:

- Finish Homework Set 4
- Review Chapter 3.5

Next Lesson:

- Understand and recognize mixed problems.
- Formulate mixed problems algebraically.
- Solve mixed problems using Excel Solver.