EM384: Analytical Methods for Engineering Management

Lesson 20: Transportation Problems

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Lesson Objectives

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- Recognize a transportation problem given a network problem.
- Formulate and solve a transportation problem in Excel Solver.
- Interpret the reduced costs for a transportation problem solution.

Review

Network Flow Problems

- The network model describes patterns of flow in a connected system, where the flow might involve material, people, or funds.
- When we construct diagrams to represent such systems, the elements are represented by nodes, or circles, in the diagram. The paths of flow are represented by arcs, or arrows.
- We consider Minimum Cost Network Flow Problems in this course.

Types of Minimum Cost Network Flow problems covered in EM384

- Transportation Problems: A minimum cost network flow problem with only supply and demand nodes.

- Assignment Problems: A special type of transportation problem where we want to assign people to tasks, or items to categories. All nodes have a supply or a demand of 1.

- Transshipment Problems: A minimum cost network flow problem with supply, demand, and transshipment nodes.

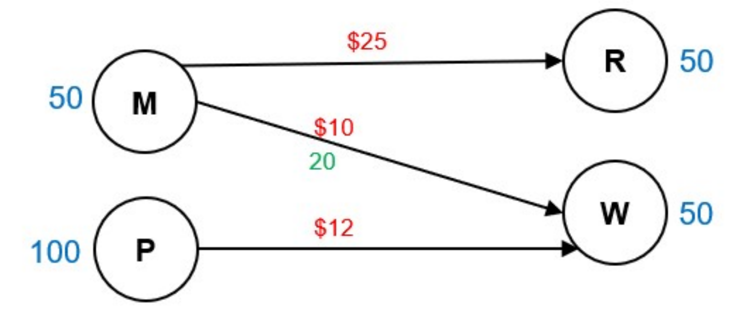
Terminology

- The model for any minimum-cost flow problem is represented by a network with flow passing through it.
- The circles in the network are called nodes.
- Each node where the net amount of flow generated (outflow minus inflow) is a fixed positive number is a supply node.
- Each node where the net amount of flow generated is a fixed negative number is a demand node.
- Any node where the net amount of flow generated is fixed at zero is a transshipment node. **Having the amount of flow out of the node equal the amount of flow into the node** is referred to as conservation of flow.
- The arrows in the network are called arcs. (Must have arrows at the end of an arc to indicate network flow)
- The maximum amount of flow allowed through an arc is referred to as the capacity of that arc.

The Transportation Problem

The Transportation Problem

- The transportation problem is a **minimum cost network flow problem** with only supply nodes and demand nodes (no transshipment nodes!).
- A transportation problem may be capacitated or uncapacitated.
- A transportation problem may be balanced or unbalanced.



Notation

You may choose your own notation, however it should be consistent throughout your problem. Example:

- Let i be a node where $i = 1, ..., n$
- Let c_{ij} be the cost of flow for 1 unit of a good from node i to node j (note that if there are 10 nodes or more of any kind we would write $c_{i,j}$ with a comma)
- let x_{ij} be the total units that flow from node i to node j.

Formulation as a Linear Program

Once you have made a network flow diagram for a problem, you follow these rules to create your algebraic LP formulation:

1. Define your nodes in the context of your problem (can use letters or numbers):
   E.g. Let nodes M, P, R, and W represent the cities of West Milford, Patterson, Ringwood and Wayne, respectively.
2. Define your decision variables, **one per arc**. These represent the flow.
   E.g. Let x_{ij} be the the number of boxes transported from node i to node j.
3. Write your objective function as Minimize $Z = \sum c_{ij}x_{ij}$ for every arc x_{ij} in your network.
4. Define your constraints, **one per node** to ensure flow balance, plus **one for each decision variable that has a capacity constraint**, plus **one per decision variable** to ensure non-negativity.

Formulation for a Balanced Problem

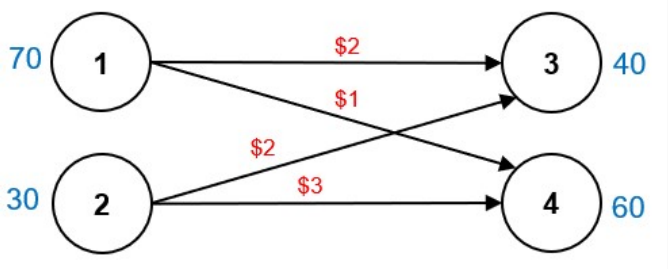
Using the diagram below, we want to ship widgets from locations 1 and 2 to locations 3 and 4 for the lowest total cost.

Decision Variables

Let x_{ij} be the flow of widgets from node i to node j, $\forall i \in \{1,2\}, j \in \{3,4\}$

Objective Function

Minimize $Z =$ $2x_{13} + x_{14} + 2x_{23} + 3x_{24}$



Constraints

Supply Constraints

Node 1: $x_{13} + x_{14} = 70$

Node 2: $x_{23} + x_{24} = 30$

Demand Constraints

Node 3: $x_{13} + x_{23} = 40$

Node 4: $x_{14} + x_{24} = 60$

Non-neg:

$x_{13}, x_{14}, x_{23}, x_{24} \geq 0$

Formulation for an Unbalanced Problem: Excess Supply

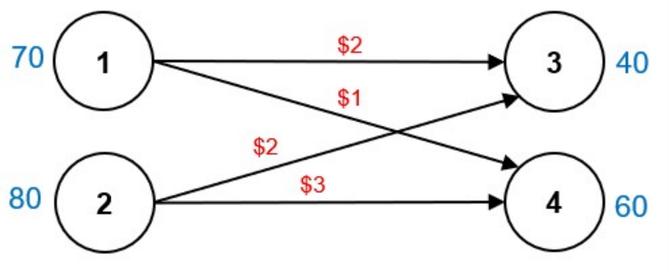
Using the diagram below, we want to ship widgets from locations 1 and 2 to locations 3 and 4 for the lowest total cost.

Decision Variables

Let x_{ij} be the flow of widgets from node i to node j, $\forall i \in \{1,2\}, j \in \{3,4\}$

Objective Function

Minimize $Z =$ $2x_{13} + x_{14} + 2x_{23} + 3x_{24}$



Constraints

Supply Constraints

Node 1: $x_{13} + x_{14} \leq 70$

Node 2: $x_{23} + x_{24} \leq 80$

Demand Constraints

Node 3: $x_{13} + x_{23} = 40$

Node 4: $x_{14} + x_{24} = 60$

Non-neg:

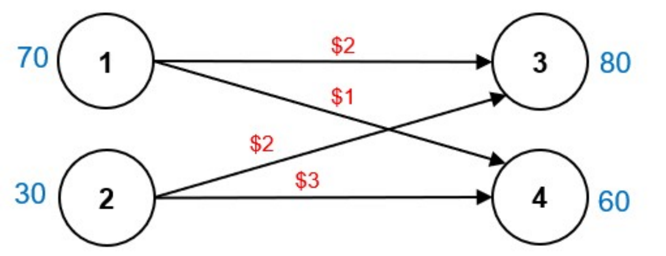
$x_{13}, x_{14}, x_{23}, x_{24} \geq 0$

Formulation for an Unbalanced Problem: Excess Demand

Using the diagram below, we want to ship widgets from locations 1 and 2 to locations 3 and 4 for the lowest total cost.

Decision Variables

Let x_{ij} be the flow of widgets from node i to node j, $\forall i \in \{1, 2\}, j \in \{3, 4\}$



Objective Function

Minimize $Z =$ $2x_{13} + x_{14} + 2x_{23} + 3x_{24}$

Constraints

Supply Constraints

Node 1: $x_{13} + x_{14} = 70$

Node 2: $x_{23} + x_{24} = 30$

Demand Constraints

Node 3: $x_{13} + x_{23} \leq 80$

Node 4: $x_{14} + x_{24} \leq 60$

Non-neg:

$x_{13}, x_{14}, x_{23}, x_{24} \geq 0$

Capacities on Arcs

- When arcs have capacity limits add one constraint per capacitated arc to force the decision variable to be less than or equal to the capacity, e.g. $x_{ij} \leq 20$.

Feasibility

- A feasible minimum cost network flow problem (and thus a transportation problem) must be able to satisfy all of the demand, or if total demand is more than total supply, as much of the total demand as there is total supply. In other words, there must be *at least one* set of constraints (supply or demand) that has all $=$ signs, in order to force the flow through the network.

- An infeasible problem has both left over supply and demand. This could be caused by one of two things:
    1. The structure of the arcs (e.g. if some arcs are not present) can make the problem infeasible.
    2. Adding capacities to arcs can sometimes make the problem . A feasible problem must be able to either: 1) move all supply, 2) receive all demand, or 3) both (a balanced problem).

- There are ways to fix the problem of infeasibility (e.g. adding additional dummy nodes or arcs with zero cost) but this method will depend on the structure of the original problem and where the bottlenecks are. This is beyond the scope of EM384.

Excel Solver

Example Problem

Recall from Lesson 19:

You are the manager of SNES games inc., a vintage video game retailer. You have two warehouses in West Milford and Patterson, and three retail locations in Ringwood, Wayne, and Franklin Lakes. The West Milford warehouse has a supply of 50 boxes and the Patterson warehouse has a supply of 100 boxes. The demand from all three retail locations is the same, 50 per store. The costs to ship one box are outlined below:

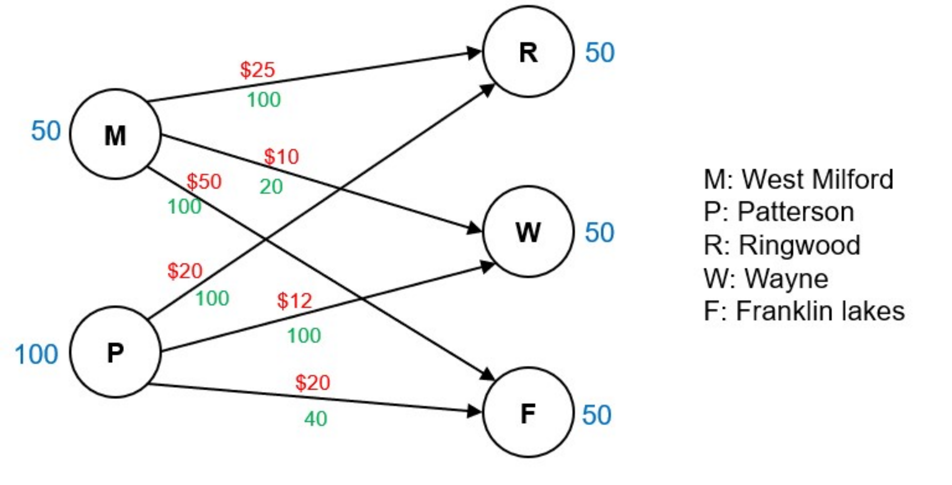
From / To	Ringwood	Wayne	Franklin Lakes
West Milford	\$25	\$10	\$50
Patterson	\$20	\$12	\$20

In addition, truck capacity limits the number of boxes that can go between West Milford and Wayne to 20 boxes, and from Patterson and Franklin lakes to 40 boxes. All other routes have a capacity of 100 boxes.

1. Draw a complete Network Flow diagram to represent this problem.
2. Find the optimal flow and optimal cost using Excel Solver

Example Problem

Network Flow Diagram:



Now let’s model and solve this transportation problem in Excel.

Example Problem

Let x_{ij} be the number of boxes transported from warehouse i to retail location j.

The solution to the minimum cost network flow problem is given below:

$$x_{MR} = 20$$
$$x_{MW} = 20$$
$$x_{MF} = 10$$
$$x_{PR} = 30$$
$$x_{PW} = 30$$
$$x_{PF} = 40$$

The minimum total cost is $Z = \$2960$

Practical Exercise

Some good video resources

Formulating Transportation Problems:
https://www.youtube.com/watch?v=WZIyL6pcItY

Solving Transportation Models with Excel Solver:
https://www.youtube.com/watch?v=C$_v0rlpTEmc$

Conclusion

Next Class

Homework:

- Read Chapter 9.3

Next Lesson:

- Recognize an assignment problem given a network problem.
- Formulate and solve an assignment problem in Excel Solver.
- Interpret the reduced costs for an assignment problem solution.