EM384: Analytical Methods for Engineering Management

Lesson 20: Assignment Problems

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Lesson Objectives

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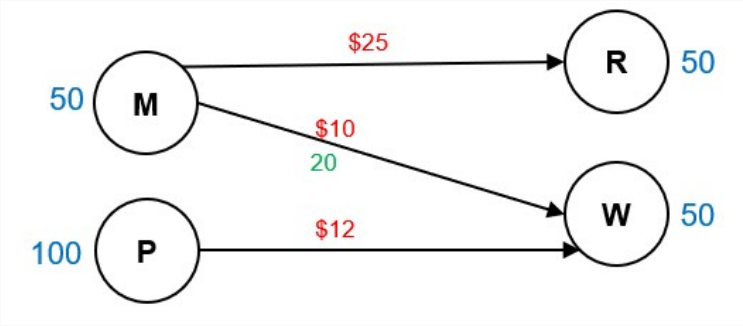
- Recognize an assignment problem given a network problem.
- Formulate and solve an assignment problem in Excel Solver.
- Interpret the reduced costs for an assignment problem solution.

Review

The Transportation Problem

- The transportation problem is a **minimum cost network flow problem** with only supply nodes and demand nodes (no transshipment nodes!).
- A transportation problem may be capacitated or uncapacitated.
- A transportation problem may be balanced or unbalanced.



Assignment Problems

Assignment Problems

- An assignment problem is a network flow problem where we assign people to tasks.
- It is a special case of the transportation where the supply nodes have a supply of 1, and the demand nodes (usually) have a supply of 1.
- Example: assigning cadets to jobs, drivers to clients, etc.

Formulation for an Assignment Problem

Decision Variables

Let x_{ij} be the assignment of person i to job j,

$\forall i \in \{1,2,3\}, j \in \{4,5,6\}$

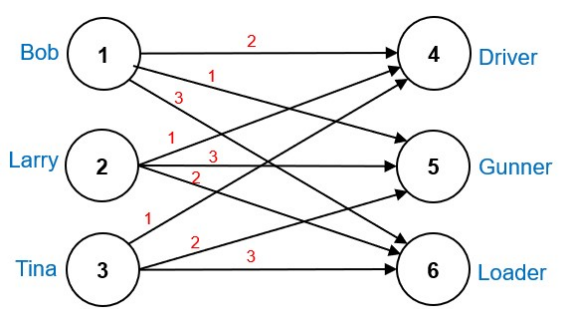
Objective Function

Minimize $Z =$

$2x_{14} + x_{15} + 3x_{16} +$

$x_{24} + 3x_{25} + 2x_{26} +$

$x_{34} + 2x_{35} + 3x_{36}$



Constraints

Supply Constraints

$x_{14} + x_{15} + x_{16} = 1$

$x_{24} + x_{25} + x_{26} = 1$

$x_{34} + x_{35} + x_{36} = 1$

Demand Constraints

$x_{14} + x_{24} + x_{34} = 1$

$x_{15} + x_{25} + x_{35} = 1$

$x_{16} + x_{26} + x_{36} = 1$

Non-neg:

$x_{ij} \geq 0$

$\forall i \in \{1,2,3\}, j \in \{4,5,6\}$

Alternate Formulation for an Assignment Problem (Sigma notation)

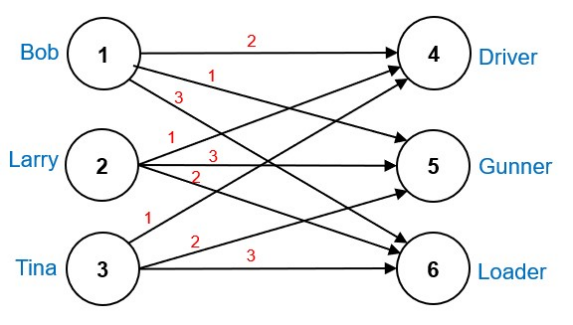
Decision Variables

Let x_{ij} be the assignment of person i to job j,
$\forall i \in \{1,2,3\}, j \in \{4,5,6\}$

Objective Function

Let c_{ij} be the cost of assigning person i to job j.
Minimize

$$Z = \sum_{i=1}^{3} \sum_{j=4}^{6} c_{ij} x_{ij}$$



Constraints

Supply Constraints

$$\sum_{j=4}^{6} x_{ij} = 1 \; \forall \; i \in \{1,2,3\}$$

Demand Constraints

$$\sum_{i=1}^{3} x_{ij} = 1 \; \forall \; j \in \{4,5,6\}$$

Non-neg:

$x_{ij} \geq 0$
$\forall i \in \{1,2,3\}, j \in \{4,5,6\}$

Additional considerations / variations

- An assignment problem is just a special case of the transportation problem where each node has a supply or demand of 1.
- An assignment problem can be modified to allow several people to one task, or one person to several tasks by changing the network flow diagram (and supply or demand quantities), and following the rules for unbalanced problems (see transportation problems).
- In an assignment problem we minimize the total cost of assignment. If a problem gives you benefit instead of cost, then you can maximize the total benefit (or minimize the negative total benefit) to achieve the same result.

Practical Exercise

Branch Night!

- Create a branch night assignment in Excel from cadets to branches, given the branch preference survey results (Teams link), and the following available slots (17 slots given, excess slots needed assigned to Infantry).
- How many cadets got their first choice? How many got their 4th or worse choice?
- Could you ‘game’ this problem for your own assignment? What would you need to know beforehand?

Branch	Slots
Aviation	2
Infantry	1 + excess
Armor	2
Field Artillery	2
Air Defense Artillery	1
Medical Service	1
Chemical	1
Cyber	1
Adjutant General	1
Transportation	1
Quartermaster	1
Signal	1

Some good video resources

Solving Assignment Problems with Excel Solver:
https://www.youtube.com/watch?v=APTweXuMw3w

Conclusion

Next Class

Homework:

- Read Chapter 23.1 in PDF (Uploaded to Teams)

Next Lesson:

- Recognize a transshipment problem given a network problem.
- Formulate and solve a transshipment problem in Excel Solver.
- Interpret the reduced costs for a transshipment problem solution.