EM384: Analytical Methods for Engineering Management

Lesson 22: Transshipment Problems

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Lesson Objectives

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- Recognize a transshipment problem given a network problem.
- Formulate and solve a transshipment problem in Excel Solver.

Transshipment Problems

Transshipment Problems

- A transshipment problem is a special case of the transportation problem where there are intermediate nodes that are not supply or demand nodes, but rather where goods flow ‘through’ them on to their final destination.
- The objective function for transshipment problems, like transportation problems, is the sum of $flow * unit\,cost$.
- Like transportation problems, transshipment problems can be balanced or unbalanced (excess supply or demand). The objective function remains the same for balanced and unbalanced problems.

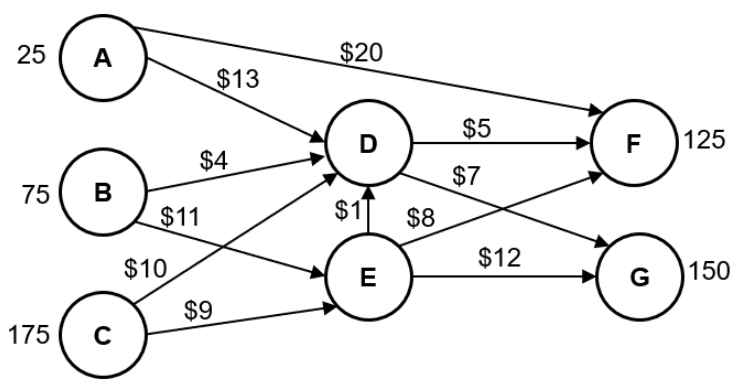
Constraint Formulation

For transshipment problems, identify your supply nodes, transshipment nodes, and demand nodes. Then write your constraints as follows:

- For each supply node, there is one supply constraint:
  Sum of flow out $=$ supply (if the problem is balanced) OR
  Sum of flow out $\leq$ supply (if there is excess supply)
- For each demand node, there is one demand constraint:
  Sum of flow in $=$ demand (if the problem is balanced) OR
  Sum of flow in $\leq$ demand (if there is excess demand)
- For each transshipment node, there is one transshipment constraint:
  Sum of flow in $=$ sum of flow out (always an equality)
- A non-negativity constraint for each decision variable.

Problem Formulation (Balanced Problem)

Network Flow Diagram (Balanced Problem)



The above network flow diagram is a transshipment problem because there are intermediate nodes (D, E), in addition to the supply nodes (A,B,C) and demand nodes (F,G). The problem is balanced since *supply* = *demand* $(25 + 75 + 175 = 125 + 100)$.

Decision Variables and Objective Function

After clearly defining your node names (using letter, numbers, etc) and a set of nodes, you can define your decision variables:

Decision Variables: x_{ij} is the flow of goods from node i to node j, where $i, j \in N$ and $i \neq j$. If there is no arc between nodes i and j then $x_{ij} = 0$.

Objective Function:

$$\textit{Minimize} \qquad Z = 20x_{AF} + 13x_{AD} + 2x_{AE} + 4x_{BD} + 11x_{BE} + 10x_{CD} + 9x_{CE} + x_{ED} + 5x_{DF} + 7x_{DG} + 8x_{EF} + 12x_{EG}$$

Constraint Formulation (Balanced Problem)

Constraints (balanced problem):

Supply constraints:

$x_{AF} + x_{AD} = 25$

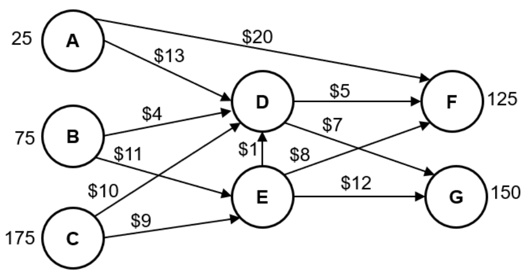
$x_{BD} + x_{BE} = 75$

$x_{CD} + x_{CE} = 175$

Demand constraints:

$x_{AF} + x_{DF} + x_{EF} = 125$

$x_{DG} + x_{EG} = 150$



Flow balance constraints:

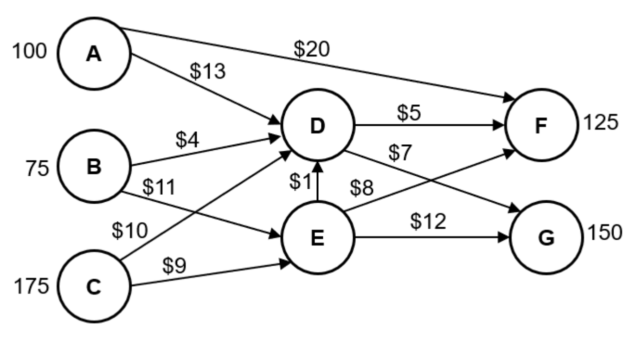
$x_{AD} + x_{BD} + x_{CD} + x_{ED} = x_{DF} + x_{DG}$

$x_{BE} + x_{CE} = x_{ED} + x_{EF} + x_{EG}$

Non-negativity Constraints:

$x_{ij} \geq 0 \qquad \forall i \in \{A, B, C, D, E\}, j \in \{D, E, F, G\}$, and $i \neq j$

Network Flow Diagram (Unbalanced Problem: Excess Supply)



The above network flow diagram is a transshipment problem because there are intermediate nodes (D, E), in addition to the supply nodes (A,B,C) and demand nodes (F,G). The problem is unbalanced since *supply* $>$ *demand* $(100 + 75 + 175 > 125 + 100)$.

Constraint Formulation (Excess Supply)

Constraints (Excess Supply):

Supply constraints:

$x_{AF} + x_{AD} \leq 100$

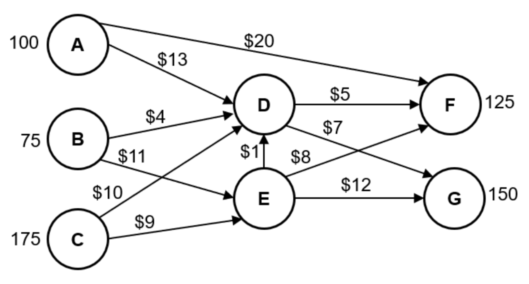
$x_{BD} + x_{BE} \leq 75$

$x_{CD} + x_{CE} \leq 175$

Demand constraints:

$x_{AF} + x_{DF} + x_{EF} = 125$

$x_{DG} + x_{EG} = 150$



Flow balance constraints:

$x_{AD} + x_{BD} + x_{CD} + x_{ED} = x_{DF} + x_{DG}$

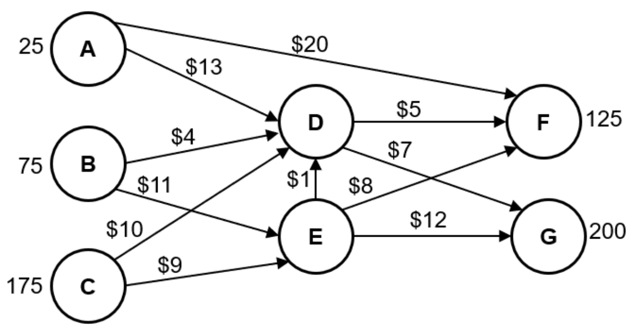
$x_{BE} + x_{CE} = x_{ED} + x_{EF} + x_{EG}$

Non-negativity Constraints:

$x_{ij} \geq 0 \qquad \forall i \in \{A, B, C, D, E\}, j \in \{D, E, F, G\}$, and $i \neq j$

Problem Formulation (Unbalanced Problem: Excess Demand)

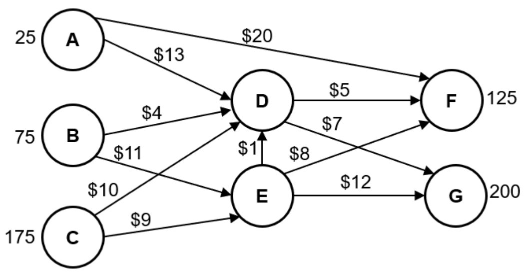
Network Flow Diagram (Unbalanced Problem: Excess Demand)



The above network flow diagram is a transshipment problem because there are intermediate nodes (D, E), in addition to the supply nodes (A,B,C) and demand nodes (F,G). The problem is unbalanced since *supply* < *demand* ($25 + 75 + 175 < 125 + 200$).

Constraint Formulation (Excess Demand)

Constraints (Excess Demand):



Supply constraints:

$x_{AF} + x_{AD} = 100$

$x_{BD} + x_{BE} = 75$

$x_{CD} + x_{CE} = 175$

Demand constraints:

$x_{AF} + x_{DF} + x_{EF} \leq 125$

$x_{DG} + x_{EG} \leq 200$

Flow balance constraints:

$x_{AD} + x_{BD} + x_{CD} + x_{ED} = x_{DF} + x_{DG}$

$x_{BE} + x_{CE} = x_{ED} + x_{EF} + x_{EG}$

Non-negativity Constraints:

$x_{ij} \geq 0 \qquad \forall i \in \{A, B, C, D, E\}, j \in \{D, E, F, G\}$, and $i \neq j$

Formulating the Transshipment Problem in Excel

Some Useful Videos

Solving Transshipment Problems with Excel Solver:
https://www.youtube.com/watch?v=vVzvdP57MFQ

https://www.youtube.com/watch?v=ci2nbrQQib8

Conclusion

Next Class

Homework:

- Read Chapter 23.1 in PDF (Uploaded to Teams)

Next Lesson:

- Recognize a transshipment problem given a network problem.
- Formulate and solve a transshipment problem in Excel Solver.
- Interpret the reduced costs for a transshipment problem solution.