EM384: Analytical Methods for Engineering Management

Lesson 9: Introduction to Linear Programs

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Lesson Objectives

Lesson 9 Objectives

- Understand the characteristics of linear programming as a subset of optimization.
- Formulate a linear program algebraically.
- Identify the three parts to a linear program: Objective Function, Decision Variables, and Constraints.

What is Linear Programming?

- Linear programming is a technique for the optimization of a linear objective function, subject to linear equality constraints and/or linear inequality constraints.
- A Linear Program (LP) is a mathematical model which contains three components: Decision Variables, an Objective Function, and Constraints (D.O.C.).

Steps to formulating an Linear Program (LP)

- Define the decision variables: these represent quantities that you are trying to determine.
- Define the objective function: This is a linear function of the decision variables. Is it a minimization or maximization problem?
- Define the constraints: What values can the decision variables take? What equalities or inequalities define these constraints?

Linearity Requirement

- What is the linearity requirement?
- Why might we prefer to solve linear problems?
- What are some examples of **linear** equations or inequalities?

Example Problem

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Kalo Fertilizer Company makes a fertilizer using two ingredients that provide nitrogen, phosphate, and potassium.

A pound of ingredient A contributes 10 pounds of nitrogen and 6 pounds of phosphate, while a pound of ingredient B contributes 2 pounds of nitrogen, 6 pounds of phosphate, and 1 pound of potassium.

Ingredient A costs $3 per pound, and ingredient B costs $5 per pound.

The company want to know how many pounds of each chemical ingredient to put into each bag of fertilizer to meet the minimum requirement of 20 pounds of nitrogen, 36 pounds of phosphate, and 1 pound of potassium while minimizing cost.

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Example Problem

Decision Variables:

x_1: Amount of ingredient A to put in each bag of fertilizer (in pounds)

x_2: Amount of ingredient B to put in each bag of fertilizer (in pounds)

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Objective Function

Minimize $Z = 3x_1 + 5x_2$

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Example Problem

What is the relationship between ingredients and their content? This helps you build the constraints.

	Nitrogen	Phosphate	Potassium
Ingredient A	10	6	0
Ingredient B	2	6	1

Recall that x_1 is the amount of ingredient A (in pounds) and x_2 is the amount of ingredient B (in pounds).

The problem states that there is a minimum requirement of 20 pounds of nitrogen, 36 pounds of phosphate, and 2 pounds of potassium.

Linear Program Formulation

Decision Variables:

x_1: Amount of ingredient A to put in each bag of fertilizer (in pounds)

x_2: Amount of ingredient B to put in each bag of fertilizer (in pounds)

Objective Function

Minimize $Z = 3x_1 + 5x_2$

Constraints

$10x_1 + 2x_2 \geq 20$ (Nitrogen constraint)

$6x_1 + 6x_2 \geq 36$ (Phosphate constraint)

$x_2 \geq 2$ (Potassium constraint)

$x_1, x_2 \geq 0$ (Non-negativity constraint)

(note that $x_2 \geq 0$ is less restrictive than $x_2 \geq 1$ and can be omitted)

Practical Exercises

Practical Exercise 1

You run New Jersey's finest bagel shop in Ringwood, and you want to maximize the daily profits you make from selling both plain and 'everything' bagels. You are experimenting with a new conveyor belt oven that can cook bagels one at a time but much faster than traditional methods.

You make \$0.30 profit on each plain bagel sold, and \$0.40 on each 'everything' bagel sold. Every day you receive 50 pounds of bagel dough. Each plain bagel takes 0.3 pounds of dough and 1 minutes of cook time, while each 'everything' bagel takes 0.25 pounds of dough and 1.5 minutes of of cook time. You only have 4 hours of cook time each day before the shop opens.

Formulate a linear program to optimize the number of plain and 'everything' bagels that you should make each day.

Practical Exercise 1 LP Formulation

Decision Variables:

x_1: Number of plain bagels cooked

x_2: Number of 'everything' bagels cooked

Objective Function:

Maximize $Z = 0.3x_1 + 0.4x_2$

Constraints:

$0.3x_1 + 0.25x_2 \leq 50$ (Dough)

$x_1 + 1.5x_2 \leq 240$ (Cook time)

$x_1, x_2 \geq 0$ (Non-negativity)

Practical Exercise 1 Solution

$x_1 = 75$

$x_2 = 110$

$Z = 0.3(75) + 0.4(110) = 22.5 + 44 = \$66.5$

PARAMETERS						
	Plain	Everything				
Dough	0.3	0.25		Total Dough		50
Cook Time	1	1.5		Total Cook Time		240
Profit	0.3	0.4				
DECISION VARIABLES						
Plain Bagels	75					
Everything Bagels	110					
OBJECTIVE						
Maximize	66.5					
CONSTRAINTS						
	Plain	Everything	LHS		RHS	
Dough	0.3	0.25	50	<=	50	
Cook Time	1	1.5	240	<=	240	

(After WPR1, we will introduce Excel solver, and learn this type of spreadsheet model setup for linear programs)

Practical Exercise 2

The B&W Leather Company wants to add handmade belts and wallets to its product line. Each batch of belts nets the company $1800 in profit, and each batch of wallets nets $1200. Both belts and wallets require cutting and sewing. A batch of Belts requires 2 hours of cutting time and 6 hours of sewing time. A batch of Wallets requires 3 hours of cutting time and 3 hours of sewing time. If the cutting machine is available 12 hours a week and the sewing machine is available 18 hours a week, what number of batches of belts and wallets will produce the most profit?

Practical Exercise 2 LP Formulation

Decision Variables:

x_1: Number of batches of belts to produce

x_2: Number of batches of wallets to produce

Objective Function:

Maximize $Z = 1800x_1 + 1200x_2$

Constraints:

$2x_1 + 3x_2 \leq 12$ (Cutting)

$6x_1 + 3x_2 \leq 18$ (Sewing)

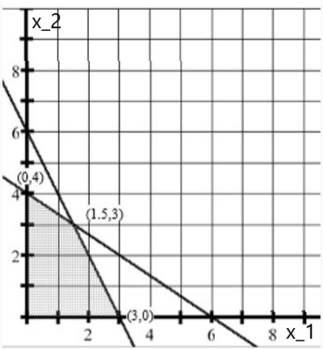
$x_1, x_2 \geq 0$ (Non-negativity)

Practical Exercise 2 Solution

$x_1 = 1.5$

$x_2 = 3$

$Z = 1800(1.5) + 1200(3) = 2700 + 3600 = \$6300$



(In the next two classes, we will be talking about this type of graphical solution method)

Conclusion

Next Class

Homework:

- Read Chapter 3 (Pages 35-end of 3.2)

Next Lesson:

- Solve a linear program with two decision variables graphically by enumerating extreme points or using level curves.
- Understand and apply the following terminology for Linear Programs: Feasible region, Infeasible region, Feasible solution, Infeasible solution, Optimal solution, Boundary, Extreme Points.

WPR1 is rapidly approaching! (Lesson 12). It will cover all content from lesson 1-11, and be conducted in the lab (The room will be put out by your instructor beforehand). You will be expected to do problems on paper, as well as in Excel on the lab computer. You may only use your issued scientific calculator (no graphing calculators). No outside references are allowed.