EM384: Analytical Methods for Engineering Management

Lesson 25: Site Selection Problems

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Table of contents

Lesson Objectives

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- Recognize a site selection problem given a transportation problem.
- Formulate and solve in Excel a site selection problem using binary decision variables.

Site Selection Problems

Site Selection Formulation

- Binary decision variables can be used as a ”site selection” variable.
- This arises in the case where sites (either supply, demand, or transshipment nodes) have a fixed cost that is incurred when they are used.
- The mathematical linear programming model (transportation or transshipment) is modified in the following way:
  - We define a binary decision variable y_i for each site i that has a fixed cost F_i (assuming n total sites that have a fixed cost).
  - We add $\sum_{i=1}^{n} F_i y_i$ to the objective function.
  - We modify the RHS constraint of supply/demand nodes with a fixed cost by multiplying the capacity by y_i.
  - if there are fixed costs associated with the use of a transshipment node, we add a new constraint $total\_flow\_in \leq y_i M$ where M is some large number(it is called ”big M” in the literature).

Example Transportation Problem without Fixed Costs

Consider the following regular transportation problem with three supply nodes A, B, and C, and a single demand node D. The cost of unit flow between nodes is $c_{AD} = \$1$, $c_{BD} = \$2$, and $c_{CD} = \$3$. Assume a supply of 100 units at each supply node and a demand of 100 units at the demand node.

Decision Variables:

x_{AD}, x_{BD}, x_{CD}

Objective Function:

Minimize $Z = x_{AD} + 2x_{BD} + 3x_{CD}$

Constraints:

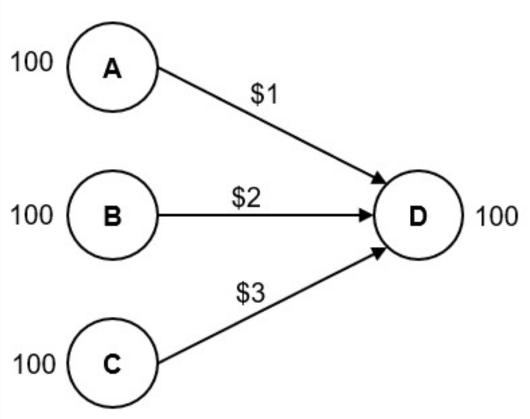
$x_{AD} \leq 100$

$x_{BD} \leq 100$

$x_{CD} \leq 100$

$x_{AD} + x_{BD} + x_{CD} = 100$

$x_{AD}, x_{BD}, x_{CD} \geq 0$



Example Transportation Problem WITH Fixed Costs

Now consider the same transportation problem, but with a fixed cost F_i for each supply site, where F_A=1000, $F_B = 500$, and $F_C = 500$. We modify our original formulation to incorporate these fixed site costs, and reformulate the problem.

y_A, y_B, y_C are binary decision variables where $y_i = 0$ means the site is not selected, and

$y_i = 1$ means the site is selected. A fixed cost for a site is only incurred if the site is selected. Additionally, we only allow flow out of a site if it is selected. Thus, we need to change our supply constraints to also be *linking* constraints.

New Formulation

Decision Variables:

x_{AD}, x_{BD}, x_{CD}

y_A, y_B, y_C are binary decision variables where $y_i = 0$ means site i is not selected, and $y_i = 1$ means site i is selected, $\forall i \in \{A, B, C\}$

Objective Function:

Minimize

$$Z = x_{AD} + 2x_{BD} + 3x_{CD} + F_A y_a + F_B y_B + F_c y_c$$

Constraints:

$$x_{AD} \leq 100\, y_A$$

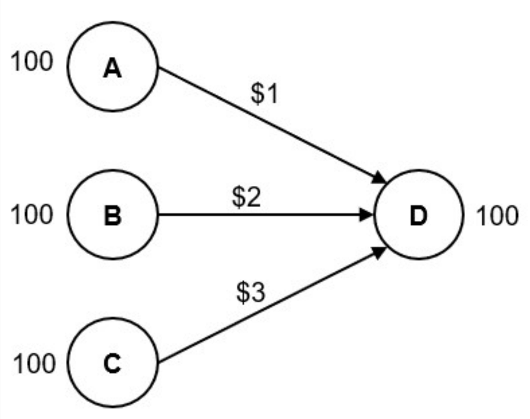
$$x_{BD} \leq 100\, y_B$$

$$x_{CD} \leq 100\, y_C$$

$$x_{AD} + x_{BD} + x_{CD} = 100$$

$$x_{AD}, x_{BD}, x_{CD} \geq 0$$

$$y_A, y_B, y_C \in \{0, 1\}$$



In-Class Exercise

Let there be four supply nodes $\{1, 2, 3, 4\}$ and four demand nodes $\{5, 6, 7, 8\}$. Supply and demand quantities, as well as fixed location costs for the supply nodes are indicated in the table below.

1. Formulate a Binary Integer Program to minimize the cost of meeting the demand in this problem.
2. Model and solve your model in Excel and report the facilities (supply nodes) that are active in your optimal solution.

Parameters

From/To	5	6	7	8	Supply (Mi)	Fixed Cost (Fi)
1	5	5	5	5	1000	$2,000,000.00
2	10	10	10	10	1000	$1,000,000.00
3	10	10	10	10	1000	$1,000,000.00
4	5	5	5	5	1000	$2,000,000.00
Demand	500	500	500	500		

Decision Variables:

Let x_{ij} be the flow from node i to node j, $\forall i \in \{1,2,3,4\}, j \in \{5,6,7,8\}$
Let y_i be a binary decision variable, $\forall i \in \{1,2,3,4\}$, where $y_i = 0$ means site i is not selected, and $y_i = 1$ means site i is selected.

Objective Function:

Let c_{ij} be the cost of transporting one good from node i to node j and F_i be the fixed cost of site i (See table).

$$\text{Minimize } Z = \sum_{i=1}^{4} \sum_{j=5}^{8} c_{ij} x_{ij} + \sum_{i=1}^{4} F_i y_i$$

Constraints :

$$\sum_{j=5}^{8} x_{ij} \leq 1000 y_i \; \forall\, i \in \{1,2,3,4\}$$

$$\sum_{i=1}^{4} x_{ij} = 500 \; \forall\, j \in \{5,6,7,8\}$$

$$x_{ij} \geq 0 \; \forall i \in \{1,2,3,4\} \text{ and } j \in \{5,6,7,8\}$$

$$y_i \in \{0,1\} \; \forall i \in \{1,2,3,4\}$$

In-Class Exercise Solution

Parameters

From/To	5	6	7	8	Supply (Fi)	Fixed Cost (Fi)
1	5	5	5	5	1000	$2,000,000.00
2	10	10	10	10	1000	$1,000,000.00
3	10	10	10	10	1000	$1,000,000.00
4	5	5	5	5	1000	$2,000,000.00
Demand	500	500	500	500		

Objective Function $2,020,000.00

Decision Variables / Constraints

From/To	5	6	7	8	LHS		RHS		
1	0	0	0	0	0	<=	0	y1	0
2	0	0	500	500	1000	<=	1000	y2	1
3	500	500	0	0	1000	<=	1000	y3	1
4	0	0	0	0	0	<=	0	y4	0
LHS	500	500	500	500					
	=	=	=	=					
RHS	500	500	500	500					

Locations 2 and 3 are active in the optimal solution.

Conclusion

Next Class

Homework:

- Read Chapter 12.2 - 12.3

Next Lesson:

- Recognize a site selection problem given a transportation problem.
- Formulate and solve in Excel a site selection problem using binary decision variables.