Middle East Technical University | Department of Computer Engineering

CENG 424

Logic For Computer Science

Spring 2022-2023

Assignment 2

Regulations

1. The homework is due by **23:55 on May 7th, 2023**. Late submission is not allowed.
2. Submissions will be via ODTUClass, do not send your homework via e-mail.
3. You can use any typesetting tool (LaTex, Word, etc.) while writing the homework (but no handwriting). However, you must upload the homework as a **searchable pdf file**. Other formats will not be considered for grading.
4. Send an e-mail to **garipler@metu.edu.tr** if you need to get in contact.
5. **This is an individual homework, which means you have to answer the questions on your own. Any contrary case including but not limited to getting help from automated tools, sharing your answers with each other, extensive collaboration etc. will be considered as cheating and university regulations about cheating will be applied.**

Question 1

Translate the following sentences into predicate logic. For each sentence, state what your predicates mean.

1. A formula is valid iff it computes T for all its valuations; it is satisfiable iff it computes T for at least one of its valuations.
2. For any young superhero, there is a mentor who guides her or an enemy who troubles her.
3. Gandalf's unicorn is the only creature that can fly, run, and teleport.

Question 2

The following sentences are taken from the Tractacus Logico Philosophicus of Wittgenstein:

1. An atomic fact is a combination of objects.
2. A picture represents a possible situation in logical space.

Your are asked to;

a. Translate each sentence into predicate logic. Give definitions of your predicates.

b. Specify which variables are free, which ones are bound. Why? Answer seperately for each sentence.

Question 3

Prove the validity of the following sequents by natural deduction.

a. $\forall x \forall y(P(y) \Rightarrow Q(x)) \vdash \exists y P(y) \Rightarrow \forall x Q(x)$

b. $\exists x \exists y(P(x,y) \vee P(y,x)) \vdash \exists x \exists y P(x,y)$

c. $\forall x(P(x) \vee Q(x)), \exists x \neg Q(x), \forall x(R(x) \Rightarrow \neg P(x)) \vdash \exists y \neg R(y)$

Question 4

a. Given $\phi = \forall x \forall y((P(x) \wedge Q(x,y)) \Rightarrow (P(y) \wedge y \neq x))$ find a model to show that ϕ is satisfiable.

b. Given $\phi = \forall x \forall y((P(x) \Rightarrow P(y)) \wedge (P(y) \Rightarrow P(x)))$ find a model to show that ϕ is not valid.

c. Given $\phi = \forall x \forall z \exists y(P(x,y) \wedge (P(z,y) \Rightarrow P(x,z)))$ and the model $\mathcal{M}$ consisting of the set natural numbers with $P^{\mathcal{M}} = \{(m,n) | m < n\}$, show that $\mathcal{M} \vDash \phi$ does not hold.

Question 5

Given the axioms below;

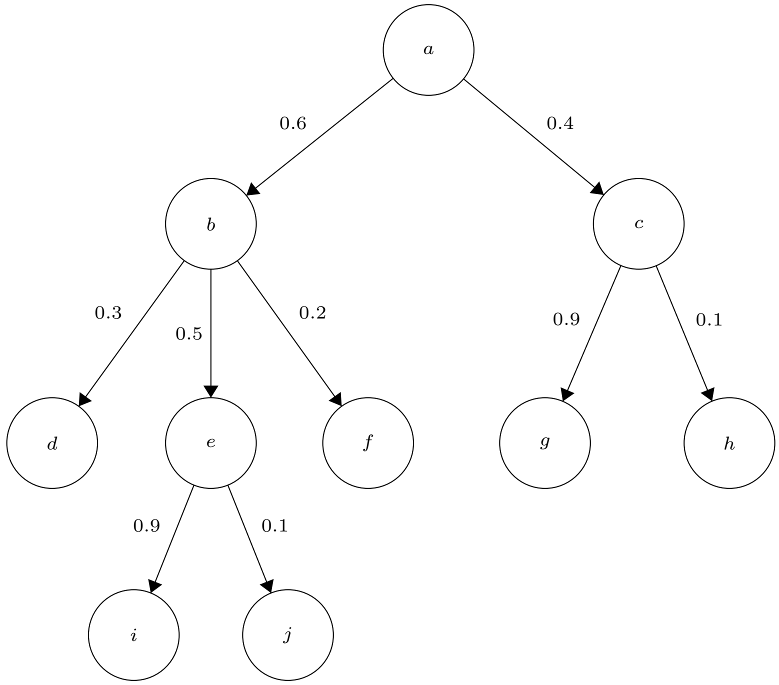
- Everyone whom Jane loves is a traveler.
- Any person who does not earn money, does not travel.
- Jim is a doctor.
- Every doctor is a person.
- Any doctor who does not work, does not earn money.
- Anyone who does not travel, is not a traveler.

Use natural deduction to conclude that;
"If Jim does not work, then Jane does not love Jim."

Use the following relation symbols while writing predicate logic sentences:

LOVES(x,y) (for x loves y),
DOCTOR(x),
EARN(x),
TRAVEL(x),
WORK(x),
PERSON(x),
TRAVELER(x)

Question 6 (Ungraded)



Given the probabilistic computation tree above, use natural deduction to compute the probability of the computation ending at node j. Show the steps of your solution.
Hint1: The answer will be a byproduct of the proof.
Hint2: Begin with creating your small knowledge base (i.e. set of premises) which represents the knowledge depicted by the given tree. You can represent a transition from x to y with probability p with the predicate $P(x, y, p)$.
Hint3: If there is a transition from x to y with probability p_1 and a transition from y to z with probability p_1, then probability of reaching z from x is $p_1 \times p_2$. Formalize and use this fact.