Middle East Technical University Department of Computer Engineering

CENG 424

Logic For Computer Science

Spring 2022-2023

Assignment 2

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Answer for Q1

1. F(x): x is a formula, V(x): x is valid, S(x): x is satisfiable, Val(x,y): x computes T for valuation y
$\forall x(F(x) \Rightarrow V(x) \Leftrightarrow \forall y(Val(x,y))) \land \forall x(F(x) \Rightarrow S(x) \Leftrightarrow \exists y(Val(x,y)))$

2. Y(x): x is young, S(x): x is superhero, M(x): x is a mentor, G(x,y): x guides y, E(x): x is an enemy, T(x,y): x troubles y
$\forall x(Y(x) \land S(x) \Rightarrow \exists y(M(y) \land G(y,x)) \lor \exists z(E(z) \land T(z,x)))$

3. F(x): x can fly, R(x): x can run, T(x): x can teleport, C(x): x is a creature, G(x): x is Gandalf's unicorn
$\forall x(F(x) \land R(x) \land T(x) \land C(x) \Rightarrow G(x))$

Answer for Q2

1.

   a A(x): x is an atomic fact, O(x): x is an object, C(x,y): x is combination of y
   $\forall x(A(x) \Rightarrow \exists y(O(y) \land C(x,y)))$

   b All variables are bound in the formula. All x's are in the scope of $\forall x$ and all y's are in the scope of $\exists y$. (If we look at the parse tree, we can see that there is an upward path from x and y variables to $\forall x$ and $\exists y$)

2.

   a P(x): x is a picture, S(x): x is a possible situation, L(x): x is in logical space, R(x,y): x represents y
   $\forall x \exists y(P(x) \Rightarrow S(y) \land L(y) \land R(x,y))$

   b All variables are bound in the formula. All x's and y's are in the scope of $\forall x \exists y$. (If we look at the parse tree, we can see that there is an upward path from x and y variables to $\forall x$ $\exists y$)

Answer for Q3

a.

1.	$\forall x \forall y (P(y) \Rightarrow Q(x))$	premise
2.	$\exists y P(y)$	assumption
3.	x_0	
4.	$y_0 \quad P(y_0)$	assumption
5.	$\forall x (P(y_0) \Rightarrow Q(x))$	$\forall y_e \quad 1$
6.	$P(y_0) \Rightarrow Q(x_0)$	$\forall x_e \quad 5$
7.	$Q(x_0)$	$\Rightarrow_e 6, 4$
8.	$Q(x_0)$	$\exists y_e \quad 2, 4-7$
9.	$\forall x Q(x)$	$\forall x_i \quad 3-8$
10.	$\exists y P(y) \Rightarrow \forall x Q(x)$	$\Rightarrow_i \quad 2-9$

b.

1.	$\exists x \exists y (P(x,y) \vee P(y,x))$	premise
2.	$x_0 \quad \exists y (P(x_0,y) \vee P(y,x_0))$	assumption
3.	$y_0 \quad P(x_0,y_0) \vee P(y_0,x_0)$	assumption
4.	$P(x_0,y_0)$	assumption
5.	$\exists y P(x_0,y)$	$\exists y_i \quad 4$
6.	$\exists x \exists y P(x,y)$	$\exists x_i \quad 5$
7.	$P(y_0,x_0)$	assumption
8.	$\exists x P(x,y_0)$	$\exists x_i \quad 7$
9.	$\exists x \exists y P(x,y)$	$\exists y_i \quad 8$
10.	$\exists x \exists y P(x,y)$	$\vee_e \quad 3, 4-6, 7-9$
11.	$\exists x \exists y P(x,y)$	$\exists y_e \quad 2, 3-10$
12.	$\exists x \exists y P(x,y)$	$\exists x_e \quad 1, 2-11$

c.

1.	$\forall x(P(x) \vee Q(x))$	premise
2.	$\exists x \neg Q(x)$	premise
3.	$\forall x(R(x) \Rightarrow \neg P(x))$	premise
4.	$x_0 \quad \neg Q(x_0)$	assumption
5.	$R(x_0) \Rightarrow \neg P(x_0)$	$\forall x_e \quad 3$
6.	$P(x_0) \vee Q(x_0)$	$\forall x_e \quad 1$
7.	$P(x_0)$	assumption
8.	$\neg\neg P(x_0)$	$\neg\neg_i \quad 7$
9.	$\neg R(x_0)$	MT 5,8
10.	$\exists y \neg R(y)$	$\exists y_i \quad 9$
11.	$Q(x_0)$	assumption
12.	$\perp$	$\neg_e \quad 11, 4$
13.	$\neg R(x_0)$	$\perp_e \quad 12$
14.	$\exists y \neg R(y)$	$\exists y_i \quad 13$
15.	$\exists y \neg R(y)$	$\vee_e \quad 6, 7-10, 11-14$
16.	$\exists y \neg R(y)$	$\exists x_e \quad 2, 4-15$

Answer for Q4

a. Consider F = {s} where s is nullery and P = {P, Q} where P is unary and Q is binary. A model M of (F, P):
1. A = {a, b, c}
2. s^M = a
3. P^M = {a, b}
4. Q^M = {(a,b), (b,c)}

Therefore, for the model M, if we choose a for x and b for y, (P(a) $\wedge$ Q(a,b)) $\Rightarrow$ (P(b) $\wedge$ b $\neq$ a) evaluates T. Thus, ϕ is satisfiable.

b. Consider F = {e} where e is nullery and P = {P} where P is unary. A model M of (F, P):
1. A = {a, b}
2. e^M = b
3. P^M = {a}

Therefore, for the model M, if we choose a for x and b for y, (P(a) $\Rightarrow$ P(b)) $\wedge$ (P(b) $\Rightarrow$ P(a)) evaluates F. Thus, ϕ is not valid.

c. Let's convert formula to words: For all x, for all z, for at least one y, (x < y) and if (z < y), then (x < z). $M \models \phi$ does not hold since we can find some numbers x, y, z such that (x < y) and (z < y) evaluate T, but (x < z) evaluates F. Let's choose 5 for x, 6 for y and 4 for z. Then, $P(5, 6) \wedge (P(4, 6) \Rightarrow$ P(5, 4)) evaluates F. Thus, $M \models \phi$ does not hold.

Answer for Q5

1.	$\forall x(LOVES(Jane, x) \Rightarrow TRAVELER(x))$	premise	
2.	$\forall x(PERSON(x) \wedge \neg EARN(x) \Rightarrow \neg TRAVEL(x))$	premise	
3.	$DOCTOR(Jim)$	premise	
4.	$\forall x(DOCTOR(x) \Rightarrow PERSON(x))$	premise	
5.	$\forall x(DOCTOR(x) \wedge \neg WORK(x) \Rightarrow \neg EARN(x))$	premise	
6.	$\forall x(\neg TRAVEL(x) \Rightarrow \neg TRAVELER(x))$	premise	
7.	$DOCTOR(Jim) \Rightarrow PERSON(Jim)$	$\forall x_e$	4
8.	$DOCTOR(Jim) \wedge \neg WORK(Jim) \Rightarrow \neg EARN(Jim)$	$\forall x_e$	5
9.	$PERSON(Jim) \wedge \neg EARN(Jim) \Rightarrow \neg TRAVEL(Jim)$	$\forall x_e$	2
10.	$\neg TRAVEL(Jim) \Rightarrow \neg TRAVELER(Jim)$	$\forall x_e$	6
11.	$LOVES(Jane, Jim) \Rightarrow TRAVELER(Jim)$	$\forall x_e$	1
12.	$\neg WORK(Jim)$	assumption	
13.	$DOCTOR(Jim) \wedge \neg WORK(Jim)$	$\wedge_i$	$3, 12$
14.	$\neg EARN(Jim)$	$\Rightarrow_e$	$8, 13$
15.	$PERSON(Jim)$	$\Rightarrow_e$	$7, 3$
16.	$PERSON(Jim) \wedge \neg EARN(Jim)$	$\wedge_i$	$15, 14$
17.	$\neg TRAVEL(Jim)$	$\Rightarrow_e$	$9, 16$
18.	$\neg TRAVELER(Jim)$	$\Rightarrow_e$	$10, 17$
19.	$LOVES(Jane, Jim)$	assumption	
20.	$TRAVELER(Jim)$	$\Rightarrow_e$	$11, 19$
21.	$\perp$	$\neg_e$	$20, 18$
22.	$\neg LOVES(Jane, Jim)$	$\neg_i$	$19 - 21$
23.	$\neg WORK(Jim) \Rightarrow \neg LOVES(Jane, Jim)$	$\Rightarrow_i$	$12 - 22$