Middle East Technical University | Department of Computer Engineering

CENG 424

Logic For Computer Science

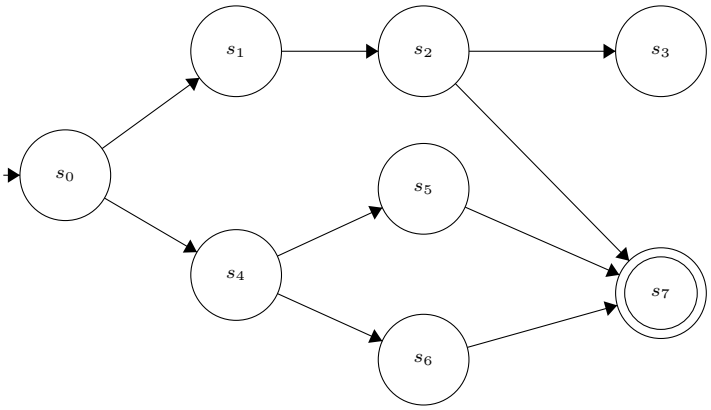
Spring 2022-2023

Assignment 1

Regulations

1. The homework is due by **23:55 on April 9th, 2023**. Late submission is not allowed.
2. Submissions will be via ODTUClass, do not send your homework via e-mail.
3. You can use any typesetting tool (LaTex, Word, etc.) while writing the homework. However, you must upload the homework as a **searchable pdf file**. Other formats will not be considered for grading.
4. Send an e-mail to **garipler@metu.edu.tr** if you need to get in contact.
5. **This is an individual homework, which means you have to answer the questions on your own. Any contrary case including but not limited to getting help from automated tools, sharing your answers with each other, extensive collaboration etc. will be considered as cheating and university regulations about cheating will be applied.**

Question 1



The nondeterministic transition system given above models correct and incorrect executions of a nondeterministic program.

The model structure is similar to finite automata; there are states, an initial state, accepting states (1 accepting state in the given case) and transitions.

However, the concept of transition is defined differently: The given system does neither require nor consume any input to make a transition. If the execution is at a state q and if there is a directed edge from the state q to a state q', then, there is a transition from q to q' without any condition. If there are multiple outgoing transitions from a state q to states q' and q'', one of those transitions is nondeterministically choosen.

A run of the program is a **correct run** if the execution reaches an accepting state as the result of a sequence of transitions.

Another property in which the given structure differs from finite automata is that it **emits propositions at each state** upon reaching that state (including the initial state and final states).

The table below shows propositions emitted by each state of given system:

State	Emitted Proposition
s_0	**p**
s_1	**q**
s_2	**s**
s_3	**u**
s_4	**r**
s_5	nondeterministically chooses one of **u** or **t**
s_6	**q**
s_7	**x** if u is emitted previously **z** if q is emitted previously **y** else

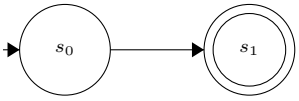
As understood from the table, a state may emit the same symbol each time or the symbol emitted by a state may be totally nondeterministically chosen or it may depend on the symbols previously emitted during the execution.

Lastly, for the given system, **the starting state is s_0 and the accepting state is s_7.**

a.

Give the propositional formula that represents all correct runs of the given system (i.e. the formula specifying propositions emitted during correct runs).

Hint: The following simple system emits q at s_0 and r at s_1. The formula for the accepting runs of that system is $\mathbf{q} \wedge \mathbf{r}$.



b.

Convert the formula you have written at part-a to disjunctive normal form.

c.

Which ones of the following formulae are correct under accepting runs of the given system? In other words, which ones of the following formulae are entailed by the formula you have given at previous parts? Explain your reasoning either formally or semi-formally in 1-2 sentences for each (also for the incorrect ones).

1. $p \Rightarrow s$
2. $r \Rightarrow p$
3. $x \Rightarrow u$
4. $y \Leftrightarrow t$
5. $r \Leftrightarrow z$

Question 2

The natural deduction proof given below has some missing parts:

1.	$\neg s \Rightarrow \neg q$	premise
2.	$p \Rightarrow q$	premise
3.	...	premise
4.	$r \wedge (\neg t \Rightarrow u)$	premise
5.	$p \vee \neg u$	assumption
6.	p	assumption
7.	q	$\Rightarrow_e$ 2,6
8.	$\neg\neg q$	$\neg\neg_i$ 7
9.	...	...
10.	...	...
11.	$q \wedge s$	$\wedge i$ 7, ...
12.	...	$\Rightarrow_e$ 3, ...
13.	...	assumption
14.	$\neg t \Rightarrow u$	$\wedge e_2$ 4
15.	...	...
16.	t	...
17.	t	$\vee e$ 5, 6-12, 13-16
18.	$(p \vee \neg u) \Rightarrow t$	...

a.

Find the missing premise. Briefly explain your reasoning. Then, complete the proof. Do not add or remove steps while completing the proof. (**Hint:** The missing premise includes s.)

b.

Repeat the proof using propositional resolution. Clearly indicate the premises and the goal. Show each step of your solution clearly.