Middle East Technical University Department of Computer Engineering

CENG 424

Logic For Computer Science

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Assignment 1

Name Surname: Andaç Berkay Seval
Student ID: 2235521

Answer for Q1

a. **The formula**: $p \wedge ((q \wedge s \wedge z) \vee (r \wedge (((u \wedge x) \vee (t \wedge y)) \vee (q \wedge z))))$

b. **The formula in DNF**: $(p \wedge q \wedge s \wedge z) \vee (p \wedge r \wedge u \wedge x) \vee (p \wedge r \wedge t \wedge y) \vee (p \wedge r \wedge q \wedge z)$

c. **List of correct formula**:$[2, 3, 4]$

Reasonings:

1. p is emitted for all the correct runs of the system since it is emitted in the initial state. However, s is not has to be true for all the correct runs of the system. Hence, $p \Rightarrow s$ is not entailed by the formula in b.

2. p is emitted for all the correct runs of the system since it is emitted in the initial state. Therefore, it is always true in all the correct runs of the system. Hence, $r \Rightarrow p$ is entailed by the formula in b.

3. x is emitted in the accepting state if u is emitted previously. Therefore, if u is emitted (true) in a correct run, then x will be emitted (true) in the accepting state. Hence, if u is true, x will be true. Thus, there is not an option when x is true and u is false since x is dependent on u, $x \Rightarrow u$ is entailed by the formula in b.

4. For all the correct runs of the system, if t is emitted (true), y will be emitted (true) in the accepting state since other propositions x, z are dependent on u and q in the previous states respectively. Thus, if t is emitted (true), y is emitted (true). Also, for all the correct runs, if y is emitted (true) in the accepting state, t has to be emitted (true). There is not an option when y is emitted and t is not emitted. Hence, $y \Leftrightarrow t$ is entailed by the formula in b.

5. If r is emitted (true) in a correct run, z is not has to be emitted (true). Also, if z is emitted (true) in a correct run, r is not has to be emitted (true). z is dependent on q for the accepting state. Hence $r \Leftrightarrow z$ is not entailed by the formula in b.

Answer for Q2

a. **The missing premise**: $(q \wedge s) \Rightarrow t$

The completed proof:

1.	$\neg s \Rightarrow \neg q$	premise
2.	$p \Rightarrow q$	premise
3.	$(q \wedge s) \Rightarrow t$	premise
4.	$r \wedge (\neg t \Rightarrow u)$	premise
5.	$p \vee \neg u$	assumption
6.	p	assumption
7.	q	$\Rightarrow_e$ 2,6
8.	$\neg\neg q$	$\neg\neg_i$ 7
9.	$\neg\neg s$	MT 1,8
10.	s	$\neg\neg_e$ 9
11.	$q \wedge s$	$\wedge i$ 7, 10
12.	t	$\Rightarrow_e$ 3, 11
13.	$\neg u$	assumption
14.	$\neg t \Rightarrow u$	$\wedge e_2$ 4
15.	$\neg\neg t$	MT 13,14
16.	t	$\neg\neg_e$ 15
17.	t	$\vee e$ 5, 6-12, 13-16
18.	$(p \vee \neg u) \Rightarrow t$	$\Rightarrow_i$ 5-17

Reasoning (about the missing premise): We can deduce that after looking at the lines 5 and 17, there is an or elimination regarding the propositions p, $\neg u$. Therefore, line 12 should be t. Moreover, in line 12, there is an implication elimination with the line 3 (missing premise). Thus, in the missing premise, there is an implication with right hand side of the implication is t. If we look at the assumption box starting with the assumption of p (line 6), after line 8, we can deduce $\neg\neg s$ from the lines 1,8 using modus tollens. Then in line 10, we can use $\neg\neg_e$ using the previous line 9. Finally, in line 11, there is and introduction with using the lines 7 and 10. In order to obtain t from implication elimination with using the missing premise in line 12, left hand side of the missing premise should be $(q \wedge s)$. Hence, implication elimination of the line 12 uses the lines 3 and 11.

b. **Proof by Propositional Resolution** Firsly, convert premises to CNF, then write them in clausal form:
i. $\neg s \Rightarrow \neg q$. CNF: $(s \vee \neg q)$. Clausal form: $\{\{s, \neg q\}\}$
ii. $p \Rightarrow q$. CNF: $(\neg p \vee q)$. Clausal form: $\{\{\neg p, q\}\}$
iii. $(q \wedge s) \Rightarrow t$. CNF: $(\neg q \vee \neg s \vee t)$. Clausal form: $\{\{\neg q, \neg s, t\}\}$
iv. $r \wedge (\neg t \Rightarrow u)$. CNF: $r \wedge (t \vee u)$. Clausal form: $\{\{r\}, \{t, u\}\}$

The whole formula in clausal form: $\{\{s, \neg q\}, \{\neg p, q\}, \{\neg q, \neg s, t\}, \{r\}, \{t, u\}\}$. The goal is to show that $(p \vee \neg u) \Rightarrow t$. Thus, put the negated goal $(p \vee \neg u) \wedge \neg t$ into the formula and apply resolution:

$\{\{s, \neg q\}, \{\neg p, q\}, \{\neg q, \neg s, t\}, \{r\}, \{t, u\}, \{p, \neg u\}, \{\neg t\}\}$

(1) $\{s, \neg q\}$
(2) $\{\neg p, q\}$
(3) $\{\neg q, \neg s, t\}$
(4) $\{r\}$
(5) $\{t, u\}$
(6) $\{p, \neg u\}$
(7) $\{\neg t\}$
(8) $\{\neg q, t\}$ by 1 and 3
(9) $\{\neg q\}$ by 7 and 8
(10) $\{p, t\}$ by 5 and 6
(11) $\{p\}$ by 7 and 10
(12) $\{q\}$ by 2 and 11
(13) $\{\}$ by 9 and 12

Thus, we reach a contradiction. Hence, the goal $(p \vee \neg u) \Rightarrow t$ is entailed by the premises.