Middle East Technical University Department of Computer Engineering

CENG 424

Logic For Computer Science

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Assignment 3

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Answer for Q1

1. G((a U b) $\Rightarrow$ ¬c)

2. G¬(c $\wedge$ Xc)

3. G(a $\Rightarrow$ (Xb $\Rightarrow$ (c U (a $\wedge$ b))))

Answer for Q2

1. Consider a transition system consists of two states s1 and s2 such that just a holds in s1 and just b holds in s2. s1 has a transition to s2 and s2 has a transition to s1 (s1 $\rightarrow$ s2 and s2 $\rightarrow$ s1). If we take a path s1 $\rightarrow$ s2 $\rightarrow$ s1 $\rightarrow$ s2 $\rightarrow$ ... we can see that G(a $\vee$ b) holds. However, Ga does not hold since a does not hold in every state, just in s1. Also, Gb does not hold since b does not hold in every state, just in s2. Thus, Ga $\vee$ Gb does not hold. Hence, equivalence does not hold.

2. We can write the right hand side of the equivalence as ¬Ga $\wedge$ ¬Gb $\equiv$ F¬a $\wedge$ F¬b. Also, we know that F does not distribute over $\wedge$. Consider a transition system consists of two states s1 and s2. Let atomic propositions of the transition system be x and y such that x $\equiv$ ¬a and y $\equiv$ ¬b. Just x holds in s1 and just y holds in s2. s1 has a transition to s2 and s2 has a transition to s1 (s1 $\rightarrow$ s2 and s2 $\rightarrow$ s1). If we take a path s1 $\rightarrow$ s2 $\rightarrow$ s1 $\rightarrow$ s2 $\rightarrow$ ... we can see that the path satisfies Fx $\wedge$ Fy $\equiv$ F¬a $\wedge$ F¬b but not F(x $\wedge$ y) $\equiv$ F(¬a $\wedge$ ¬b). Hence, equivalence does not hold.

3. Let $\mathcal{M}$ = (S, $\rightarrow$, L) be a model. Ga means that for all i $\geq$ 1, $\pi^i \models$ a for all paths π in the model $\mathcal{M}$. XFa means that $\pi^2 \models$ Fa for all paths π in the model $\mathcal{M}$, which means there is some i $\geq$ 2 such that $\pi^i \models$ a for all paths π in the model $\mathcal{M}$. Since Ga covers XFa, Ga $\wedge$ XFa $\equiv$ Ga.

4. Let $\mathcal{M}$ = (S, $\rightarrow$, L) be a model. Ga means that for all i $\geq$ 1, $\pi^i \models$ a for all paths π in the model $\mathcal{M}$. Fb means that there is some j $\geq$ 1 such that $\pi^j \models$ b for all paths π in the model $\mathcal{M}$. a U ($\neg$a $\vee$ b) means that there is some k $\geq$ 1 such that $\pi^k \models$ ($\neg$a $\vee$ b) and for all l = 1, ..., k-1 $\pi^l \models$ a. If we look at the left hand side of the equivalence, if a holds in every state, then b will hold eventually. In the right hand side, in some state in the future, $\neg$a $\vee$ b will hold and a holds until that state. For the left hand side, since b will hold eventually, $\neg$a $\vee$ b will hold too even $\neg$a is false since Ga holds. Also, until $\neg$a $\vee$ b will hold, a holds until that state. Hence Ga $\Rightarrow$ Fb $\equiv$ a U ($\neg$a $\vee$ b).

Answer for Q3

1. It does not hold. Consider the path s1 $\rightarrow$ s4 $\rightarrow$ s2 $\rightarrow$ s4 $\rightarrow$ s2 $\rightarrow$ ... According to this path, c will not eventually hold in all states (a $\rightarrow$ b $\rightarrow$ c $\rightarrow$ b $\rightarrow$ c $\rightarrow$ ...)

2. It holds. Since for all paths c holds infinitely often. After the starting state s1 there is just one state where c does not hold which is s4. Also, s4 does not have a transition to itself. Thus, it has to go to other states continually which include c. Hence GFc holds.

3. It does not hold since a does not hold in every state. Consider the path s1 $\rightarrow$ s3 $\rightarrow$ s4 $\rightarrow$ s3 $\rightarrow$ s4 $\rightarrow$ ... According to this path, a holds just in the first state s1.

4. It holds. In order to X$\neg$c be true, s4 is the only state where s1 has a transition to since c holds in s3 but not in s4. c holds in all the next states of s4 (s2, s3, s5). Thus, XXc also true. Hence, X$\neg$c $\Rightarrow$ XXc holds. (I assume s1 is the only initial state)

5. It holds. The formula states that there is some i $\geq$ 1 such that $\pi^i \models$ G(b $\vee$ c) and j = 1, ..., i-1 $\pi^j \models$ a. After the initial state s1, b or c hold in every state (b and c hold in s3, b holds in s4). c holds in s2, b and c hold in s5 which are reachable from s4. We can say that after the initial state s1, b $\vee$ c becomes true in all states of all paths and a holds in s1. Hence, a U G(b $\vee$ c) holds. (I assume s1 is the only initial state)

Answer for Q4

1. $\forall$G$\forall$F(a $\wedge$ b)

2. $\forall$G$\neg$(a $\wedge$ b)

3. $\forall$G(b $\Rightarrow$ $\exists$ Fa)

Answer for Q5

1. Sat(ϕ_1) = {s0, s1, s2, s3, s4}. $\forall$G$\forall$Fb states that for all paths globally, eventually b will hold for all paths, so b holds infinitely often in any path along the path. If we look at TS', we can see that for every individual state (we choose them to be initial state), ϕ_1 holds. Hence, TS' $\models \phi_1$ holds since s0 (initial state) is in Sat(ϕ_1).

2. Sat(ϕ_2) = {}. ∀G∃Fa states that for all paths globally, eventually a will hold for some path, so a holds infinitely often in some path along all paths. If we look at TS', we can see that none of the states holds ϕ_2. After reaching states s3 or s4, TS' continues without changing its state and in these states just b holds. Thus, TS' $\models \phi_2$ does not hold.

3. Sat(ϕ_3) = {s1, s2}. ∀(a U b) states that for all paths, a holds until b holds. If we look at the TS', we can see that for the states s1 and s2, a holds in both. s1 has just one transition which is to s2 and b holds in s2. s2 has just one transition which is to s3 and b holds in s3. However, TS' $\models \phi_3$ does not hold since s0 (initial state) is not in Sat(ϕ_3).

4. Sat(ϕ_4) = {s0, s1, s2, s3, s4}. ∃X(∀Gb) states that there is some path in the next state, b globally hold for all paths along the path. If we look at the TS', we can see that for all states as initial states, ϕ_4 holds. Therefore, TS' $\models \phi_4$ holds since s0 (initial state) is in Sat(ϕ_4).

5. Sat(ϕ_5) = {s1, s2}. ∀G∀(a U b) states that for all paths globally, a holds until b holds for all paths along the path. We can see that for states s1 and s2 as initial states, a holds in both. s1 has just one transition which is to s2 and b holds in s2 and s2 has just one transition which is to s3 and b holds in s3 and s3 has just one transition to itself so b holds continuously in s3. However, TS' $\models \phi_5$ does not hold since s0 (initial state) is not in Sat(ϕ_5).