CENG 424

Logic For Computer Science

Spring 2022-2023

Assignment 3

Regulations

1. The homework is due by **23:55 on June 14th, 2023**. Late submission is not allowed.

2. Submissions will be via ODTUClass, do not send your homework via e-mail.

3. You can use any typesetting tool (LaTex, Word, etc.) while writing the homework (but no handwriting). However, you must upload the homework as a **searchable pdf file**. Other formats will not be considered for grading.

4. Send an e-mail to **garipler@metu.edu.tr** if you need to get in contact.

5. **This is an individual homework, which means you have to answer the questions on your own. Any contrary case including but not limited to getting help from automated tools, sharing your answers with each other, extensive collaboration etc. will be considered as cheating and university regulations about cheating will be applied.**

Question 1

Express each of the following properties as an LTL formula over the set of atomic propositions $\{a, b, c\}$:

1. Between the events a and b, event c never occurs.

2. Two successive c events never occur.

3. Whenever a is immediately followed by b, c holds until a and b simultaneously hold.

Question 2

For the following LTL equivalences, either prove the equivalence or show that it does not hold via an example (e.g. a transition system that satisfies only one of the formulas from the equivalence).

1. $\mathbf{G}(a \vee b) \equiv \mathbf{G}a \vee \mathbf{G}b$

2. $\mathbf{F}(\neg a \wedge \neg b) \equiv \neg\mathbf{G}a \wedge \neg\mathbf{G}b$

3. $\mathbf{G}a \wedge \mathbf{X}\mathbf{F}a \equiv \mathbf{G}a$

4. $\mathbf{G}a \Rightarrow \mathbf{F}b \equiv a\mathbf{U}(\neg a \vee b)$

Question 3

Given the transition system TS over the set of atomic propositions $AP = \{a, b, c\}$ below;
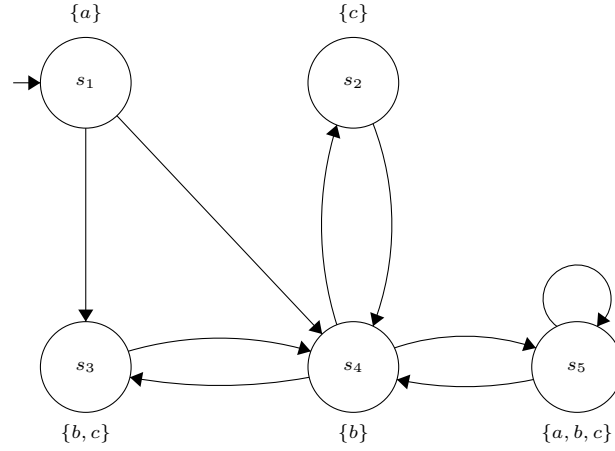
For each of the LTL formulae ϕ_i below, decide whether $TS \vDash \phi_i$ holds. Justify your answers! If $TS \nvDash \phi_i$, provide a path $\pi \in Paths(TS)$ such that $\pi \nvDash \phi_i$.
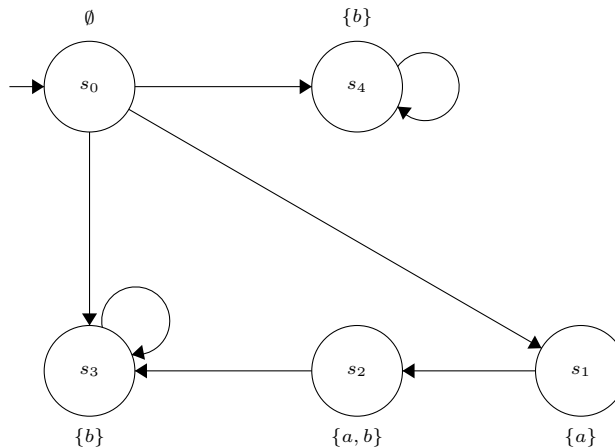
1. $\phi_1 = \mathbf{FG}c$

2. $\phi_2 = \mathbf{GF}c$

3. $\phi_3 = \mathbf{G}a$

4. $\phi_4 = \mathbf{X}\neg c \Rightarrow \mathbf{XX}c$

5. $\phi_5 = a\mathbf{U}\mathbf{G}(b \vee c)$

Question 4

Express each of the following properties as a CTL formula over the set of atomic propositions $\{a, b\}$:

1. a and b simultaneously holds infinitely many times along any path.

2. a and b cannot be true at the same time.

3. Whenever b is satisfied, it is possible to reach a state where a is satisfied.

Question 5

Given the transition system TS' above and CTL formulae ϕ_i below, determine the satisfaction set $Sat(\phi_i)$ for each formula ϕ_i an decide whether $TS' \vDash \phi_i$ holds. Justify your answers!

1. $\phi_1 = \forall\mathbf{G}\forall\mathbf{F}b$

2. $\phi_2 = \forall\mathbf{G}\exists\mathbf{F}a$

3. $\phi_3 = \forall(a\mathbf{U}b)$

4. $\phi_4 = \exists\mathbf{X}(\forall\mathbf{G}b)$

5. $\phi_5 = \forall\mathbf{G}\forall(a\mathbf{U}b)$