

CENG 371 SCIENTIFIC COMPUTING

HW2

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Q1)

The functions Sherman's March, Pickett's Charge are implemented. The implementation can be seen from the corresponding files "shermans.m" and "picketts.m".

The implementation of Sherman's March is pretty straightforward. I chose the partition in the matrix A , such that A_k is the first $(n-1) \times (n-1)$ portion of the matrix A , a_{1k} is the $(n-1) \times 1$ portion of the matrix A for the last column, $(a_k)^T$ is the $1 \times (n-1)$ portion of the matrix for the last row and a_{kk} is the 1×1 last element of the matrix A . Therefore, L_{11} and U_{11} are $(n-1) \times (n-1)$ matrices, l_{k1} and u_{1k} are $(n-1) \times 1$ vectors, u_{kk} is a 1×1 element. Thus, I created L and U matrices with the corresponding values and places of L_{11} , U_{11} , l_{k1} , u_{1k} and u_{kk} .

The implementation of Pickett's Charge is a little bit trickier. Since there are 2 block matrices A_k and A_{k1} in the A , and $A_k = L_{11} * U_{11}$ and $A_{k1} = L_{k1} * U_{11}$, I thought the dimensions of matrices L_{11} , L_{k1} and U_{11} should be the same in order to be multipliable with each other. Thus, I partitioned A_k , A_{k1} and L_{11} , L_{k1} such a way that they are interleaved, they have intersections. Thus, the dimension of A_k , A_{k1} , L_{11} , L_{k1} and U_{11} is $(n-1) \times (n-1)$. For example, A_k is the first $(n-1) \times (n-1)$ portion of the A , A_{k1} is the $(n-1) \times (n-1)$ portion of the A for the rows and columns $A(2:end, 1:end-1)$, L_{11} is the first $(n-1) \times (n-1)$ portion of the L , L_{k1} is the $(n-1) \times (n-1)$ portion of the L for the rows and columns $L(2:end, 1:end-1)$, U_{11} is the first $(n-1) \times (n-1)$ portion of the U . u_{1k} is a vector of size $(n-1) \times 1$ for the last column of U . l_{kk} is the vector of size $(n-1) \times 1$ for the rows and columns of $L(2:end, end)$ and u_{kk} is the last 1×1 element of U . Thus, I created L and U matrices with the corresponding values and places of L_{11} , L_{k1} , l_{kk} , U_{11} , u_{1k} , u_{kk} .

Q2)

First of all, I did my homework on Octave Online. Thus, there are some constraints in terms of testing. I got errors like "PAYLOAD TOO LARGE!!!" and although I am pushing it to continue computing, it eventually terminate the program. Therefore, I could not test it with $n = 1:300$, instead, I could test it with $n = 1:10$. Unfortunately, it terminated program even $n = 1:15$.

- a) Comparing in terms of run-time: Of course, run-time depends on the implementation. For my case, for the function:

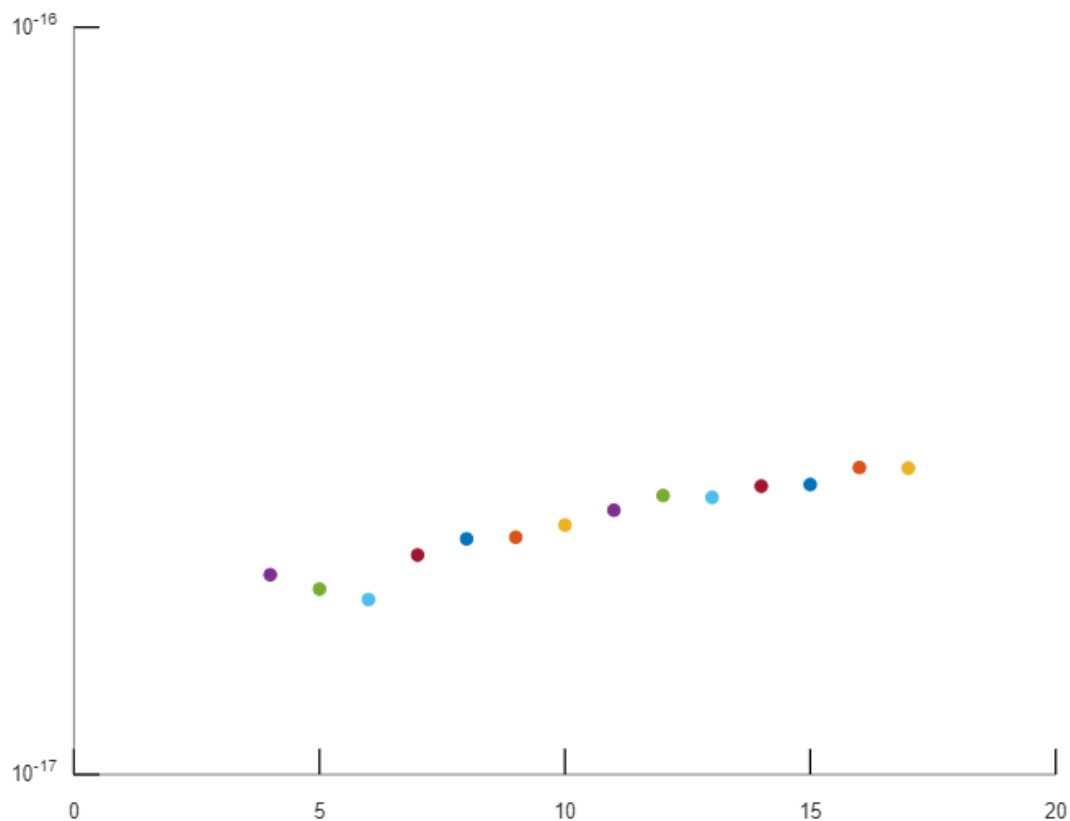
```
tic
for x = 1:10
    h = hilb(x)
    [L,U] = lu_decomposer(h)
end
toc
```

it gives 4.00696 seconds for Sherman's March and 4.3938 seconds for Pickett's Charge. In my Pickett's Charge implementation, matrix blocks have intersections and this might not be accurate to the expected result. However, in an ideal world, total-run time can be found with the function:

```
tic
for x = 1:300
    h = hilb(x)
    [L, U] = lu_decomposer(h)
end
toc
```

b) Comparing in terms of the plots of their relative errors:

!!! PAYLOAD TOO LARGE !!!



NOTICE: Execution paused due to large payload

Figure 1: Sherman's March relative error graph

!!! PAYLOAD TOO LARGE !!!

NOTICE: Execution paused due to large payload

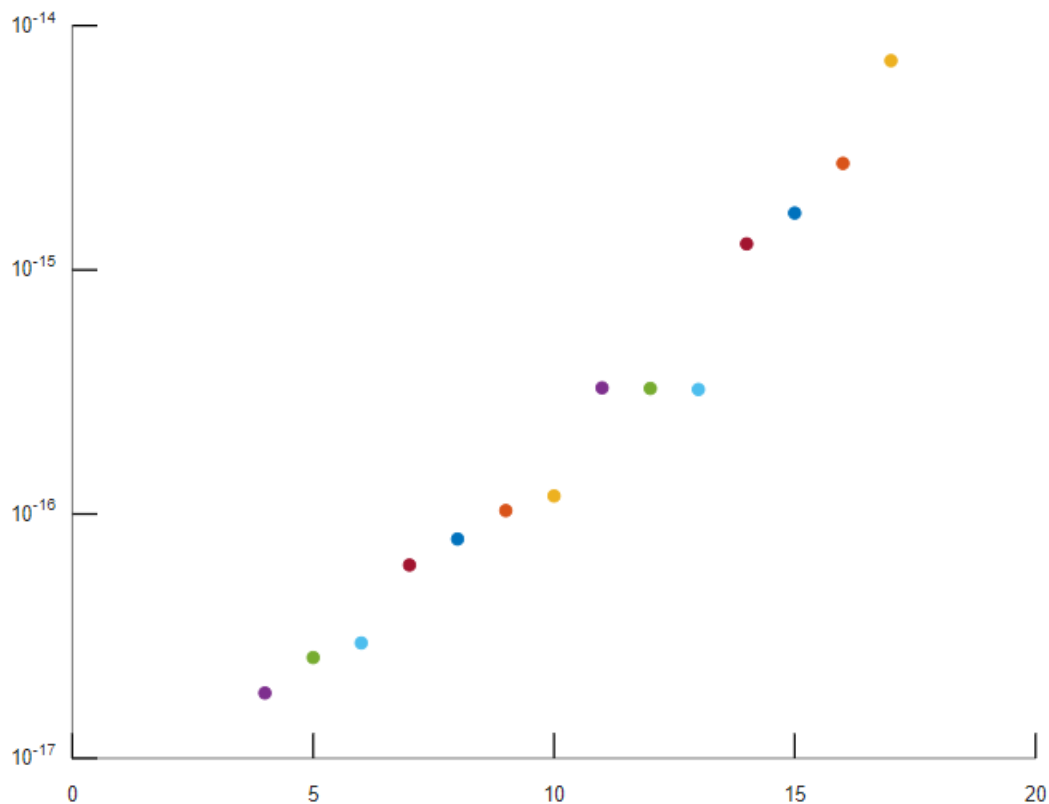


Figure 2: Pickett's Charge relative error graph

Because of Octave Online, I could not test it with $n = 1:300$, instead, I could test it with $n = 1:20$. Therefore, for my implementations, while n is increasing, the relative error of Pickett's Charge is increasing more than relative error of Sherman's March. I used the function for graphs:

```
hold on
for x = 1:20 (*)
    h = hilb(x)
    [L, U] = lu_decomposer(h)
    y = norm(h - L*U)/norm(h)
    semilogy(x, y)
end
```

However, in an ideal world, instead of the line (*), for $x = 1:300$ could be written.

- c) No, it can not factorize any square matrix. If a matrix A cannot be reduced to row echelon form by a Gaussian elimination method without interchanging two rows, then the matrix A does not have a LU decomposition. Thus, the functions that I implemented could not factorize these matrices. However, for this matrices, there exists a permutation matrix P such that $PA = LU$ has always a LU decomposition.