CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 4

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1. (a) $\int[\int(x(t)-6y(t))dt+4x(t)-5y(t)]dt=y(t) \Rightarrow y'(t)=\int(x(t)-6y(t))dt+4x(t)-5y(t)$
$\Rightarrow y''(t)=x(t)-6y(t)+4x'(t)-5y'(t) \Rightarrow y''(t)+5y'(t)+6y(t)=4x'(t)+x(t)$

   (b) $x(t)=e^{jwt}, x'(t)=jwe^{jwt}, y(t)=H(jw)e^{jwt}, y'(t)=jwH(jw)e^{jwt}, y''(t)=-w^2H(jw)e^{jwt}$. Inserting them into the differential equation; $-w^2H(jw)e^{jwt}+5jwH(jw)e^{jwt}+6H(jw)e^{jwt}=4jwe^{jwt}+e^{jwt}$
$(-w^2+5jw+6)H(jw)e^{jwt}=(4jw+1)e^{jwt} \Rightarrow H(jw)=\dfrac{4jw+1}{-w^2+5jw+6}$

   (c) $H(jw)=\dfrac{4jw+1}{(jw)^2+5jw+6}=\dfrac{4jw+1}{(jw+3)(jw+2)}=\dfrac{A}{jw+3}+\dfrac{B}{jw+2}$
$\Rightarrow A+B=4, 2A+3B=1 \Rightarrow A=11, B=-7 \Rightarrow H(jw)=\dfrac{11}{jw+3}-\dfrac{7}{jw+2}$
$F^{-1}\{H(jw)\}=h(t)=11e^{-3t}u(t)-7e^{-2t}u(t)$ from table 4.2

   (d) $X(jw)=\dfrac{1}{1+4jw}$ from table 4.2. $y(t)=x(t)*h(t) \longleftrightarrow Y(jw)=X(jw)H(jw)=\left(\dfrac{1}{1+4jw}\right)\left(\dfrac{1+4jw}{(jw+3)(jw+2)}\right)$
$=\dfrac{1}{(jw+3)(jw+2)}$. $F^{-1}\{Y(jw)\}=y(t)=\dfrac{1}{2-3}(e^{-3t}-e^{-2t})u(t)=(e^{-2t}-e^{-3t})u(t)$ from table from lecture notes

2. (a) $H(jw)=\dfrac{Y(jw)}{X(jw)}=\dfrac{jw+4}{-w^2+5jw+6} \Rightarrow (jw)^2Y(jw)+5jwY(jw)+6Y(jw)=jwX(jw)+4X(jw)$.
Thus; the differential equation is $y''(t)+5y'(t)+6y(t)=x'(t)+4x(t)$

   (b) $H(jw)=\dfrac{jw+4}{(jw+3)(jw+2)}=\dfrac{A}{jw+3}+\dfrac{B}{jw+2} \Rightarrow A+B=1, 2A+3B=4 \Rightarrow A=-1, B=2 \Rightarrow$
$H(jw)=\dfrac{2}{jw+2}-\dfrac{1}{jw+3}$. $F^{-1}\{H(jw)\}=h(t)=2e^{-2t}u(t)-e^{-3t}u(t)$ from table 4.2

   (c) $X(jw)=\dfrac{1}{jw+4}-\dfrac{1}{(jw+4)^2}$ from table 4.2. $Y(jw)=X(jw)H(jw)=\left(\dfrac{1}{jw+4}-\dfrac{1}{(jw+4)^2}\right)\left(\dfrac{jw+4}{(jw+3)(jw+2)}\right)$
$=\dfrac{1}{(jw+4)(jw+2)}$

   (d) $F^{-1}\{Y(jw)\}=y(t)=\dfrac{1}{2-4}(e^{-4t}-e^{-2t})u(t)=\dfrac{1}{2}(e^{-2t}-e^{-4t})u(t)$ from table from lecture notes

3. (a) $X(jw)=\int_{-\infty}^{\infty}x(t)e^{-jwt}dt \Rightarrow X(jw)=\int_{-\infty}^{0}e^{t}e^{-jwt}dt+\int_{0}^{\infty}e^{-t}e^{-jwt}dt=\int_{-\infty}^{0}e^{t(1-jw)}dt+\int_{0}^{\infty}e^{t(-1-jw)}dt$
$=\dfrac{1}{1-jw}-\dfrac{1}{-1-jw}=\dfrac{2}{1+w^2}$

   (b) $y(t)=te^{-|t|}=tx(t) \longleftrightarrow Y(jw)=j\dfrac{dX(jw)}{dw}=\dfrac{-4jw}{(1+w^2)^2}$

   (c) The duality property; $g(t) \longleftrightarrow G(jw)$, then $G(t) \longleftrightarrow 2\pi g(jw)$. Since $te^{-|t|} \longleftrightarrow \dfrac{-4jw}{(1+w^2)^2}$ by part(b), use duality $\dfrac{-4jt}{(1+t^2)^2} \longleftrightarrow 2\pi we^{-|w|}$. Multiplying both sides with j; $\dfrac{4t}{(1+t^2)^2} \longleftrightarrow 2\pi jwe^{-|w|}$

4. (a) $2x[n]+\dfrac{3}{4}y[n-1]-\dfrac{1}{8}y[n-2]=y[n] \Rightarrow y[n]-\dfrac{3}{4}y[n-1]+\dfrac{1}{8}y[n-2]=2x[n]$

   (b) $x[n]=e^{jwn}, y[n]=H(e^{jw})e^{jwn}$. Inserting them into the difference equation; $8H(e^{jw})e^{jwn}-6e^{-jw}H(e^{jw})e^{jwn}+e^{-2jw}H(e^{jw})e^{jwn}=16e^{jwn} \Rightarrow H(e^{jw})=\dfrac{16}{e^{-2jw}-6e^{-jw}+8}$

(c) $H(e^{jw}) = \dfrac{16}{(e^{-jw}-4)(e^{-jw}-2)} = \dfrac{A}{e^{-jw}-4} + \dfrac{B}{e^{-jw}-2} \Rightarrow A+B=0, -2A-4B=16 \Rightarrow A=8, B=-8 \Rightarrow$
$H(e^{jw}) = \dfrac{4}{1-\frac{1}{2}e^{-jw}} - \dfrac{2}{1-\frac{1}{4}e^{-jw}}$. $F^{-1}\{H(e^{jw})\} = h[n] = \left[4\left(\dfrac{1}{2}\right)^n - 2\left(\dfrac{1}{4}\right)^n\right]u[n]$ from table 5.2

(d) $X(e^{jw}) = \dfrac{1}{1-\frac{1}{4}e^{-jw}}$ from table 5.2.

$y[n] = x[n] * h[n] \longleftrightarrow Y(e^{jw}) = X(e^{jw})H(e^{jw}) = \left(\dfrac{1}{1-\frac{1}{4}e^{-jw}}\right)\left(\dfrac{4}{1-\frac{1}{2}e^{-jw}} - \dfrac{2}{1-\frac{1}{4}e^{-jw}}\right)$

$Y(e^{jw}) = \dfrac{4}{(1-\frac{1}{4}e^{-jw})(1-\frac{1}{2}e^{-jw})} - \dfrac{2}{(1-\frac{1}{4}e^{-jw})^2} = \dfrac{A}{1-\frac{1}{4}e^{-jw}} + \dfrac{B}{1-\frac{1}{2}e^{-jw}} - \dfrac{2}{(1-\frac{1}{4}e^{-jw})^2} \Rightarrow A=-4, B=8$

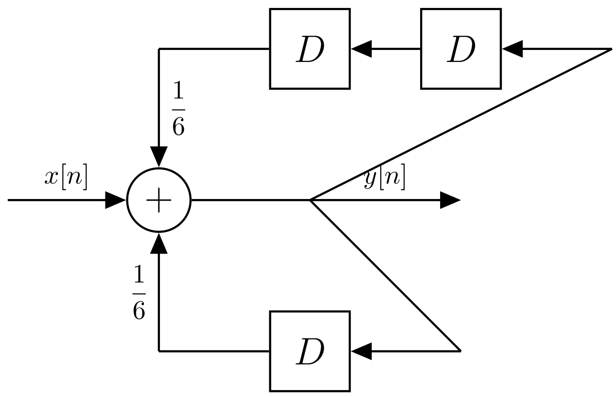
$F^{-1}\{Y(e^{jw})\} = y[n] = \left[8\left(\dfrac{1}{2}\right)^n - 4\left(\dfrac{1}{4}\right)^n - 2(n+1)\left(\dfrac{1}{4}\right)^n\right]u[n] = \left[8\left(\dfrac{1}{2}\right)^n - 2(n+3)\left(\dfrac{1}{4}\right)^n\right]u[n]$

5. Impulse response of the combined system is $h[n] = h_1[n] + h_2[n]$. $H(e^{jw}) = \dfrac{5e^{-jw}-12}{(e^{-jw}-4)(e^{-jw}-3)} = \dfrac{A}{e^{-jw}-4} + \dfrac{B}{e^{-jw}-3} \Rightarrow A+B=5, -3A-4B=-12 \Rightarrow A=8, B=-3$. $H(e^{jw}) = \dfrac{8}{e^{-jw}-4} - \dfrac{3}{e^{-jw}-3} = \dfrac{1}{1-\frac{1}{3}e^{-jw}} - \dfrac{2}{1-\frac{1}{4}e^{-jw}}$
$F^{-1}\{H(e^{jw})\} = h[n] = \left[\left(\dfrac{1}{3}\right)^n - 2\left(\dfrac{1}{4}\right)^n\right]u[n] = h_1[n] + h_2[n]$. Thus; $h_2[n] = -2\left(\dfrac{1}{4}\right)^n u[n]$ from table 5.2

6. (a) $H(e^{jw}) = \dfrac{Y(e^{jw})}{X(e^{jw})} = \dfrac{1}{1-\frac{1}{6}e^{-jw}-\frac{1}{6}e^{-2jw}} = -\dfrac{1}{6}e^{-2jw}Y(e^{jw}) - \dfrac{1}{6}e^{-jw}Y(e^{jw}) + Y(e^{jw}) = X(e^{jw})$

Thus; the difference equation is $y[n] - \dfrac{1}{6}y[n-1] - \dfrac{1}{6}y[n-2] = x[n]$

(b)



(c) $H(e^{jw}) = \dfrac{6}{-e^{-2jw}-e^{-jw}+6} = \dfrac{-6}{(e^{-jw}+3)(e^{-jw}-2)} = \dfrac{A}{e^{-jw}+3} + \dfrac{B}{e^{-jw}-2} \Rightarrow A+B=0, -2A+3B=-6 \Rightarrow$
$A = \dfrac{6}{5}, B = \dfrac{-6}{5} \Rightarrow H(e^{jw}) = \dfrac{6}{5}\left(\dfrac{\frac{1}{3}}{1+\frac{1}{3}e^{-jw}} + \dfrac{\frac{1}{2}}{1-\frac{1}{2}e^{-jw}}\right)$

$F^{-1}\{H(e^{jw})\} = h[n] = \dfrac{6}{5}\left[\dfrac{1}{3}\left(\dfrac{-1}{3}\right)^n + \left(\dfrac{1}{2}\right)^{n+1}\right]u[n]$