CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 1

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1. e $= \lim_{n\to\infty}(1+\frac{1}{n})^n = \lim_{n\to 0}(1+n)^{\frac{1}{n}}$. By the definition of the derivative;

   $\frac{d}{dt}e^t = \lim_{\Delta t\to 0}\frac{e^{t+\Delta t}-e^t}{\Delta t} = \lim_{\Delta t\to 0}\frac{e^t e^{\Delta t}-e^t}{\Delta t} = e^t\ \lim_{\Delta t\to 0}\frac{e^{\Delta t}-1}{\Delta t} \Rightarrow$

   Let n = $e^{\Delta t}$ - 1. n+1 = $e^{\Delta t}$. $\ln(n+1) = \Delta t$. Thus; when $\Delta t \to 0 \Rightarrow n \to 0$

   $\Rightarrow e^t\ \lim_{n\to 0}\frac{n}{\ln(n+1)} = e^t\ \lim_{n\to 0}\frac{\frac{1}{n}n}{\frac{1}{n}\ln(n+1)} = e^t\ \lim_{n\to 0}\frac{1}{\ln(n+1)^{\frac{1}{n}}} = e^t\ \frac{1}{\ln(\lim_{n\to 0}(1+n)^{\frac{1}{n}})} = e^t\ \frac{1}{\ln e} =$ e^t end of proof

2. (a) z = x + jy $\Rightarrow$ x + jy - 3 = j - 2x + 2jy $\Rightarrow$ x = 1, y = -1, z = 1 - j. Thus; $|z|^2 = 1^2 + (-1)^2 = 2$

   (b) z = $re^{j\theta}$ and z^4 = -81. $z^4 = r^4e^{j4\theta} = r^4\ (cos4\theta + jsin4\theta)$ where $cos4\theta$ = -1 and $sin4\theta$ = 0. Hence, r = 3 and $4\theta = \pi$, $\theta = \frac{\pi}{4}$. Thus; z = $3e^{j\frac{\pi}{4}}$

   (c) z = $\frac{(\frac{1}{2}+\frac{1}{2}j)(1-j)(1+\sqrt{3}j)}{(1-\sqrt{3}j)(1+\sqrt{3}j)} = \frac{1+\sqrt{3}j}{4} \Rightarrow$ z = $\frac{1}{2}e^{j\frac{\pi}{3}}$. Thus; r = $\frac{1}{2}$, $\theta = \frac{\pi}{3}$

   (d) z = $\frac{-3}{j}e^{j\frac{\pi}{2}}$ = 3j$e^{j\frac{\pi}{2}}$. Since j = $e^{j\frac{\pi}{2}} \Rightarrow$ z = $3e^{j\frac{\pi}{2}}e^{j\frac{\pi}{2}} = 3e^{j\pi}$

3.

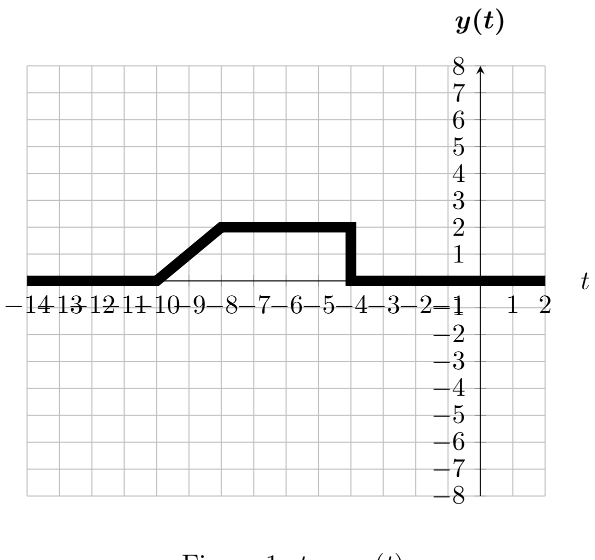

Figure 1: t vs. $y(t)$.

4. (a)

$x[-n] + x[2n+1]$

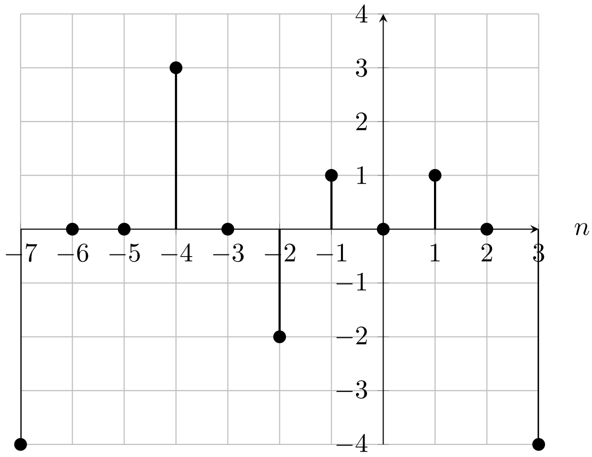

Figure 2: n vs. $x[-n]+x[2n+1]$.

(b) x[-n] + x[2n+1] = -4δ[n+7] + 3δ[n+4] - 2δ[n+2] + δ[n+1] + δ[n-1] - 4δ[n-3]

5. (a) x(t) $= 3\cos(7\pi t - \frac{4\pi}{5})$ = x(t+T_0) $= 3\cos(7\pi t + 7\pi T_0 - \frac{4\pi}{5})$
$\Rightarrow 7\pi T_0 = 2\pi \Rightarrow T_0 = \frac{2}{7}$ It is periodic and its fundamental period is $T_0 = \frac{2}{7}$

(b) x[n] = sin[4n-$\frac{\pi}{2}$] = x[n+N_0] = sin[4n+$4N_0 - \frac{\pi}{2}$]
$\Rightarrow 4N_0 = 2\pi m \Rightarrow N_0 = \frac{2\pi m}{4}$ Since N_0 is not an integer for any integer m, x[n] is not periodic

(c) x[n] = 2cos[$\frac{7\pi}{5}n$] + 7sin[$\frac{5\pi}{2}n - \frac{\pi}{3}$]. Let x[n] = x_1[n] + x_2[n] where x_1[n] = 2cos[$\frac{7\pi}{5}n$], x_2[n] = 7sin[$\frac{5\pi}{2}n - \frac{\pi}{3}$]
For x_1[n] $\Rightarrow$ x_1[n] = 2cos[$\frac{7\pi}{5}n$] $= x_1[n+N_0] =$ 2cos[$\frac{7\pi}{5}n + \frac{7\pi}{5}N_0$] $\Rightarrow \frac{7\pi}{5}N_0 = 2\pi m \Rightarrow N_0 = \frac{10\pi m}{7\pi}$ for m=7, $N_0 = 10$ for x_1[n]
For x_2[n] $\Rightarrow$ x_2[n] = 7sin[$\frac{5\pi}{2}n - \frac{\pi}{3}$] $= x_2[n+N_0] =$ 7sin[$\frac{5\pi}{2}n + \frac{5\pi}{2}N_0 - \frac{\pi}{3}$]
$\Rightarrow \frac{5\pi}{2}N_0 = 2\pi m \Rightarrow N_0 = \frac{4\pi m}{5\pi}$ for m = 5, $N_0 = 4$ for x_2[n]. Thus; fundamental period of x[n] is LCM(10,4) = 20 = N_0

6. (a) A signal is even if and only if x(t) = x(-t). In Figure 1, one can see that x($\frac{3}{2}$) $\neq$ x($\frac{-3}{2}$). x($\frac{3}{2}$) = 0 but x($\frac{-3}{2}$) = $\frac{1}{2}$. Thus; x(t) is not an even signal.
A signal is odd if and only if x(t) = -x(-t). In Figure 1, one can see that x($\frac{1}{2}$) = x($\frac{-1}{2}$) = 1. Thus; x(t) is not an odd signal.

(b) Even part of the signal $\Rightarrow$ x($-4 \le t \le -2$) = x($2 \le t \le 4$) = 0 and x($-1 \le t \le 0$) = x($0 \le t \le 1$) = 1

Even decomposition of $x(t)$

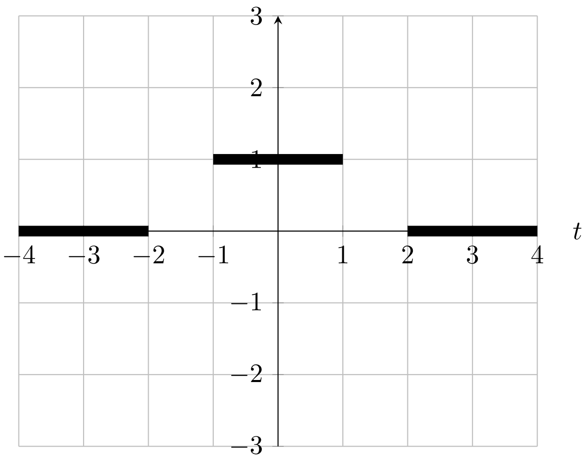

Figure 3: t vs. *Even decomposition of* $x(t)$.

Odd part of the signal $\Rightarrow$ x$(-4 \leq t \leq -2)$ = -x$(2 \leq t \leq 4)$ = 0

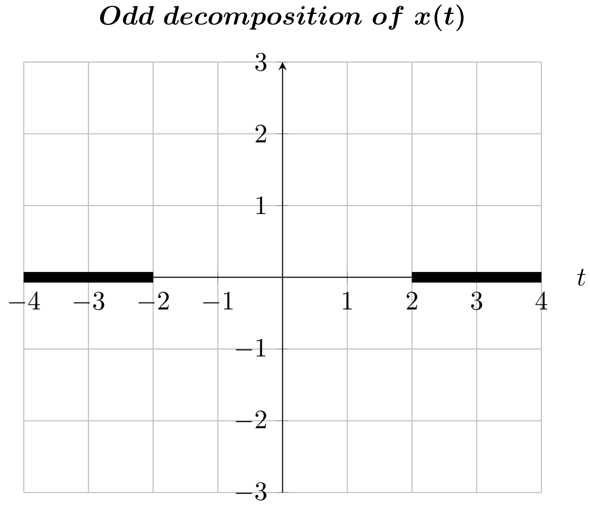

Figure 4: t vs. *Odd decomposition of* $x(t)$.

7. (a) x(t) = -3u(t-2) + 5u(t-3) - 3u(t-5)

   (b) $\dfrac{dx(t)}{dt} = -3\delta(t-2) + 5\delta(t-3)$ - $3\delta(t-5)$

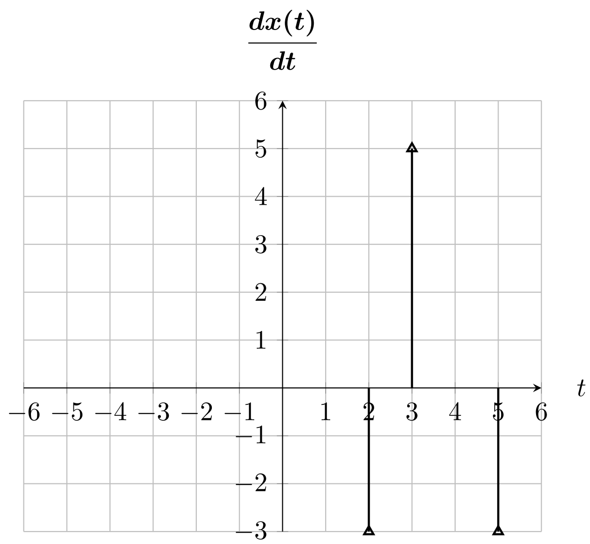

Figure 5: t vs. $\dfrac{dx(t)}{dt}$.

8. (a) y[n] = x[3n-5] $\Rightarrow$ It has memory since y[n] depends on past and future values of input. y[2] = x[1] (past), y[3] = x[4] (future). Since y[n] depends on some future values of input, it is not casual. It is invertible, we can find a unique h^{-1}, x[n] = y$[\frac{n+5}{3}]$ (n+5 = 3k for integer k) It is stable since for bounded input, it has bounded output. It is not time invariant since x$[n-n_0] \rightarrow$ y$[n-n_0]$ = x$[3n-3n_0-5] \neq$ x$[3n-n_0-5]$. It is linear since it has superposition property such that y_1[n] = x_1[3n-5], y_2[n] = x_2[3n-5], thus; y = $a_1y_1 + a_2y_2$ = h$(a_1x_1 + a_2x_2)$

   (b) y(t) = x(3t-5) $\Rightarrow$ It has memory since y(t) depends on past and future values of input. y(2) = x(1) (past), y(3) = x(4) (future). Since y(t) depends on some future values of input, it is not casual. It is invertible, we can find a unique h^{-1}, x(t) = y$(\frac{t+5}{3})$. It is stable since for bounded input, it has bounded output. It is not time invariant since x$(t-t_0) \rightarrow$ y$(t-t_0)$ = x$(3t-3t_0-5) \neq$ x$(3t-t_0-5)$. It is linear since it has superposition property such that y_1(t) = x_1(3t-5), y_2(t) = x_2(3t-5), thus; y = $a_1y_1 + a_2y_2$ = h$(a_1x_1 + a_2x_2)$

   (c) y(t) = tx(t-1) $\Rightarrow$ It has memory since y(t) depends on past values of input, y(1) = x(0). It is casual since y(t) does not depend on future values of input. It is invertible for $t \neq -1$; x(t) = $\dfrac{y(t+1)}{t+1}$. It is not stable since for bounded input, it has unbounded output. Since there is no restrictions over t, it can be any value $-\infty < t < \infty$. It is not time invariant since x$(t-t_0) \rightarrow$ y$(t-t_0)$ = $(t-t_0)$x$(t-t_0-1) \neq$ tx$(t-t_0-1)$. It is linear since it has superposition property such that y_1(t) = t$x_1(t-1)$, y_2(t) = t$x_2(t-1)$, thus; y = $a_1y_1 + a_2y_2$ = h$(a_1x_1 + a_2x_2)$.

(d) y[n] $= \sum_{k=1}^{\infty} x[n-k] \Rightarrow$ It has memory since y[n] depends on past values of input. It is casual since y[n] does not depend on future values of input. It is not stable since it is unbounded. Let x[n] = u[n], a bounded signal. However, y[n] will go to infinity. It is time invariant since x$[n-n_0] \rightarrow$ y$[n-n_0] = \sum_{k=1}^{\infty} x[n-n_0-k] =$ y'[n]. It is linear since it has superposition property such that y_1[n] $= \sum_{k=1}^{\infty} x_1[n-k]$, y_2[n] $= \sum_{k=1}^{\infty} x_2[n-k]$, thus; y = $a_1y_1 + a_2y_2 =$ h$(a_1x_1 + a_2x_2)$. It is invertible since one can obtain x[n] = y[n+1] - y[n].