CENG 384 - Signals and Systems for Computer Engineers
Spring 2021
Homework 2

Seval, Andaç Berkay
e2235521@ceng.metu.edu.tr

January 31, 2024

---

1. (a) $y'(t) = x(t) - 6\int_{-\infty}^{t} y(t) - 5y(t) \Rightarrow y"(t) + 5y'(t) + 6y(t) = x'(t)$

   (b) The input of the signal is now x'(t) = $(-e^{-t} - 4e^{-4t})$u(t)
   For particular solution ⇒ y_p(t) = Kx'(t) = K$(-e^{-t} - 4e^{-4t})$u(t), y'_p(t) = K$(e^{-t} + 16e^{-4t})$u(t), $y"_p$(t) = K$(-e^{-t} - 64e^{-4t})$u(t). Put them in the differential equation ⇒ $-Ke^{-t} - 64Ke^{-4t} + 5Ke^{-t} + 80Ke^{-4t} - 6Ke^{-t} - 24Ke^{-4t} = -e^{-t} - 4e^{-4t} \Rightarrow K = \frac{1}{2}$. Thus; y_p(t) = $\frac{1}{2}(-e^{-t} - 4e^{-4t})$u(t).
   For homogenous solution ⇒ y_h(t) = $Ce^{\alpha t}$, $y'_h(t) = \alpha Ce^{\alpha t}$, $y"_h(t) = \alpha^2 Ce^{\alpha t}$. Put them in the differential equation ⇒ $\alpha^2 Ce^{\alpha t} + 5\alpha Ce^{\alpha t} + 6Ce^{\alpha t} = 0$. ⇒ $\alpha^2 C + 5\alpha C + 6C = 0$. ⇒ $\alpha_1 = -3$, $\alpha_2 = -2$. Thus; y_h(t) = $C_1 e^{-3t} + C_2 e^{-2t}$.
   General solution is y(t) = $y_p(t) + y_h(t) = [\frac{1}{2}(-e^{-t} - 4e^{-4t}) + C_1 e^{-3t} + C_2 e^{-2t}]$u(t). Since the system is initially at rest, $y(0) = y'(0) = y"(0) = 0$. y(0) = $\frac{-5}{2} + C_1 + C_2 = 0$, $y'(0) = \frac{17}{2} - 3C_1 - 2C_2 = 0$. ⇒ $C_1 = \frac{7}{2}, C_2 = -1$.
   Therefore; y(t) = $[\frac{1}{2}(-e^{-t} - 4e^{-4t}) + \frac{7}{2}e^{-3t} - e^{-2t}]$u(t)

2. (a) x_1[n] = x[n] - x[n-2]. Thus; by using superposition and time invariance properties, y_1[n] = y[n] - y[n-2] = $\delta[n-1] - \delta[n-3]$.

   (b) y[n] = $\delta[n-1]$, x[n] = $\delta[n] + \delta[n-1]$. y[n+1] + y[n] = x[n]. Thus; y[n] + y[n-1] = x[n-1]. Set y[n] = h[n] and x[n] = $\delta[n]$ ⇒ h[n] + h[n-1] = $\delta[n-1]$. h[0] + h[-1] = $\delta[-1] = 0$ since the system at initially at rest, h[n] = for $n < 0$. h[1] = 1, h[2] = -1, h[3] = 1.Thus; h[n] = $(-1)^{n-1}$ for $n > 0$. h[0] = 0.

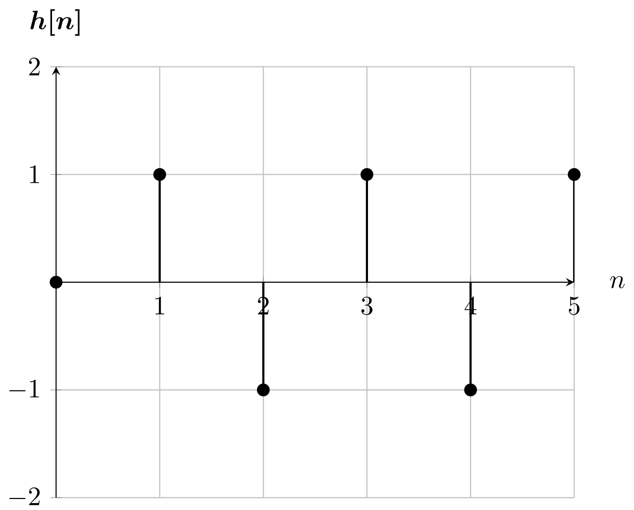

Figure 1: n vs. $h[n]$.

   (c) y[n] = $\delta[n-1]$, x[n] = $\delta[n] + \delta[n-1]$. y[n+1] + y[n] = x[n]. Thus; y[n] + y[n-1] = x[n-1].

   (d)

3. (a) y[n] = $(\delta[n-3] + 2\delta[n+1]) * (\delta[n-1] + 3\delta[n+2])$. With distributive property of convolution ⇒ y[n] = $\delta[n-4] + 3\delta[n-1] + 2\delta[n] + 6\delta[n+3]$

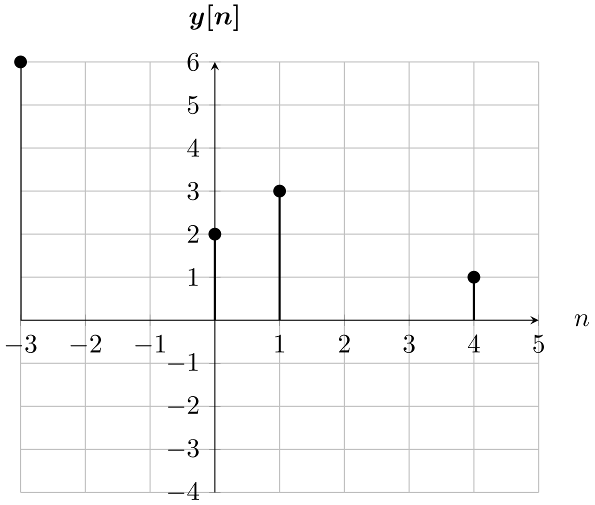

Figure 2: n vs. $y[n]$.

(b) y[n] = $\sum_{k=-\infty}^{\infty}$ x[k]h[n-k] = $\sum_{k=-\infty}^{\infty}$ (u[k+3]-u[k])(u[n-k-1]-u[n-k-3]) = (u[n+2]-u[n]) + (u[n+1]-u[n-1]) + (u[n]-u[n-2]). (for k=-3, k=-2 and k=-1 respectively) $\Rightarrow$ y[n] = u[n+2] + u[n+1] - u[n-1] - u[n-2].

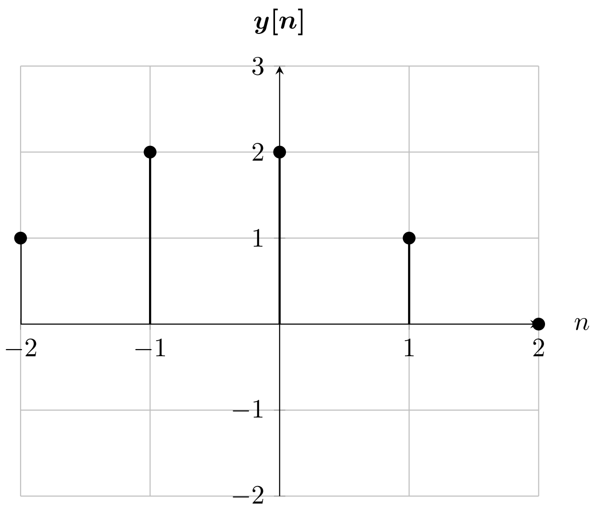

Figure 3: n vs. $y[n]$.

4. (a) y(t) = $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_0^t e^{-2\tau}e^{-3(t-\tau)}d\tau = e^{-3t}\int_0^t e^{\tau}d\tau \Rightarrow$ y(t) = $e^{-3t}(e^t-1)u(t) = (e^{-2t}-e^{-3t})u(t)$
   (b) y(t) = $\int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_0^{\infty} e^{2\tau}(u(t-\tau)-u(t-\tau-2))d\tau$. $u(t-\tau)-u(t-\tau-2)$ is nonzero only in the range $(t-2)<\tau<t$. Thus; for $t \leq 0$ integral evaluates zero. For $0<t\leq 2$, y(t) = $\int_0^t e^{2\tau}d\tau = \dfrac{e^{2t}-1}{2}$
   For $t>2$, y(t) = $\int_{t-2}^t e^{2\tau}d\tau = \dfrac{e^{2t}-e^{2t-4}}{2}$. Thus; y(t) = 0 for $-\infty<t\leq 0$, $\dfrac{e^{2t}-1}{2}$ for $0<t\leq 2$, $\dfrac{e^{2t}-e^{2t-4}}{2}$ for $t>2$.

5. (a) h[n] = s[n] - s[n-1] = nu[n] - (n-1)u[n-1] = u[n-1]
   (b) $h[n]*h^{-1}[n]=\delta[n]$. We also know that $\delta[n]=u[n]-u[n-1]$. Thus; $u[n-1]*h^{-1}[n] \Rightarrow h^{-1}[n]=\delta[n+1]-\delta[n]$
   $h^{-1}[n]*y[n]=x[n] \Rightarrow (\delta[n+1]-\delta[n])*(\delta[n]-\delta[n-1]) = \delta[n+1]-2\delta[n]+\delta[n-1]$ = x[n]
   (c) y[n] = $\delta[n]-\delta[n-1]$, y[n+1] = $\delta[n+1]-\delta[n]$. Thus; y[n+1] - y[n] = x[n] $\Rightarrow$ y[n] - y[n-1] = x[n-1].

6. h(t) = $\dfrac{ds(t)}{dt}$ = tu(t). y(t) = $\int_{-\infty}^{\infty} x(\tau)h(t-\tau)d\tau = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau = \int_0^t e^{-(t-\tau)}\tau d\tau = e^{-t}\int_0^t e^{\tau}\tau d\tau$ $(u=\tau, dv = e^{\tau}d\tau)$ = $(e^{-t}+t-1)$u(t)

7. (a) h(t) = $u(t)*(\delta(t-3)-\delta(t-5))$ = u(t-3) - u(t-5)

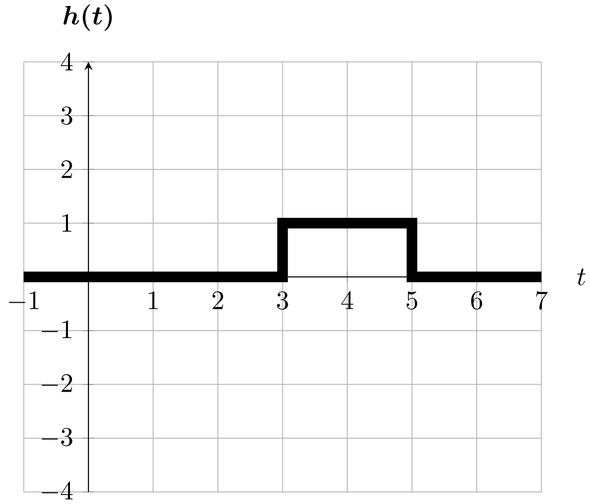

Figure 4: t vs. $h(t)$.

(b) y(t) = x(t) * h(t) $\Rightarrow$ y(t) = $\int_{-\infty}^{\infty} h(\tau)x(t-\tau)d\tau = \int_{0}^{\infty} e^{-3\tau}(u(t-\tau-3)-u(t-\tau-5))d\tau$. $u(t-\tau-3)-u(t-\tau-5)$ is nonzero only in the range $(t-5) < \tau < (t-3)$. Thus; for $t \leq 3$ integral evaluates zero. For $3 < t \leq 5$, y(t) = $\int_{0}^{t-3} e^{-3\tau}d\tau = \dfrac{1-e^{-3(t-3)}}{3}$. For $t > 5$, y(t) = $\int_{t-5}^{t-3} e^{-3\tau}d\tau = \dfrac{(1-e^{-6})(e^{-3(t-5)})}{3}$. Thus; y(t) = 0 for $-\infty < t \leq 3$, $\dfrac{1-e^{-3(t-3)}}{3}$ for $3 < t \leq 5$, $\dfrac{(1-e^{-6})(e^{-3(t-5)})}{3}$ for $t > 5$.

(c) $\dfrac{dh(t)}{dt} = \delta(t-3) - \delta(t-5)$. g(t) = $(\delta(t-3) - \delta(t-5) * x(t)$ = x(t-3) - x(t-5)