Homework 3 - Coordinate Descent

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1. Method

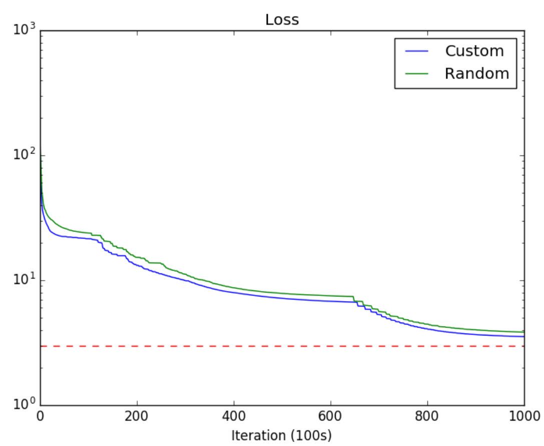
Coordinate descent is method for solving an optimization problem by optimizing over one coordinate at a time. The objective in this homework was to minimize $L(w)$ where $w \in \mathbb{R}^p$. Once a coordinate j is selected, $w(t+1) = \text{argmin}\{L(w-\overline{\alpha}), L(w), L(w+\overline{\alpha})\}$ where $\overline{\alpha}$ is a one-hot vector with $\overline{\alpha}_j = \alpha$ and α is an exponentially decaying step size. The coordinate with the largest previous decrease in loss is chosen at each iteration, with previous decreases initialized as large values, and a reset of these values occurs every q iterations. This algorithm does not require knowledge or continuity of the gradient of the loss.

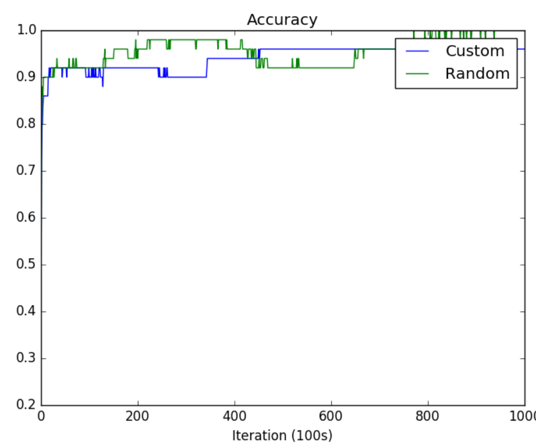
2. Convergence

This algorithm will converge to the optimal loss if the loss is a convex function and if the step size starts out large enough and asymptotically approaches zero.

3. Experimental Results

The experimental results show that the custom algorithm converges faster than random coordinate selection. Both converge to the same accuracy, but the custom algorithm converges with less noise.





4. Critical Evaluation

This coordinate descent algorithm could be improved in a few ways. If the gradient of the loss function is known, then the step size at each iteration can be calculated as a function of the gradient at the current point, to more accurately achieve a minimum with respect to a given coordinate. The prioritization of previous improvements with constant resets is quite heuristic, perhaps approximating how much the gradient has changed in each direction after each update would result in a smarter sequence of updates.