BVP Methods

I. BRIEF DESCRIPTION OF METHOD

In a really simple sense, this is the procedure that is being done when I do the "BVP solve" for getting a more converged atmosphere.

1. Start up IVP, and run it.
2. Once the flows hit Re $= 1$, wait some time, $t_{\text{transient}}$.
3. After $t_{\text{transient}}$, start taking horizontal and time averages of specified profiles in the atmosphere.
4. Once the profiles are converged (the change in the profiles at the next timestep change, on average, less than f, where f is a fraction. If $f = 0.01$, the profiles change no more than 1% from the previous timestep.), feed them into a 1D problem.
5. In the 1D problem, adjust the profiles from the atmosphere intelligently. Feed those profiles into a BVP that obeys the same thermal boundary conditions as the IVP, then adjust the mean thermal profile of the IVP atmosphere with the result of the BVP.
6. Keep running the IVP. If desired, wait some time, t_{equil}, and then restart from step 3.

II. MORE THOROUGH WRITE-UP, BOUSSINESQ CONVECTION

In boussinesq convection, the time-steady energy equation can be written as

$$\nabla \cdot (\boldsymbol{u}T - \kappa \nabla T) = 0$$

When we take horizontal averages and time averages, and assume constant κ, this equation reduces to

$$\partial_z \left\langle (wT) \right\rangle - \kappa \partial_z^2 \left\langle (T_0 + T_1) \right\rangle = 0,$$

where angles represent a time- and horizontal average. Further, due to symmetry, when we take a time- and horizontal average, we find that most of the terms in the Rayleigh-Benard momentum equation go away. Even if they hadn't, the main thing we want from the momentum equation is an update to the hydrostatic balance of the atmosphere, and that means that we need to update

$$\partial_z \left\langle p \right\rangle = \left\langle T_1 \right\rangle .$$

So, put simply, the BVP that we need to solve in RBC is

$$\begin{aligned} \frac{\partial T_1}{\partial z} - T_{1z} &= 0 \\ \kappa \frac{\partial T_{1z}}{\partial z} &= \frac{\partial}{\partial z} \left\langle wT \right\rangle - \kappa \frac{\partial^2 T_0}{\partial z^2} \\ \frac{\partial p}{\partial z} - T_1 &= 0. \end{aligned} \tag{1}$$

These three equations, coupled with two properl thermal boundary conditions, retrieve the full thermodynamic state of the atmosphere. I've written the equations in their dedalus-like form, for clarity.

In a real way, the profile of the enthalpy flux thus sets the profile of the thermal structure of the atmosphere.