

The Processes and Structure of Convective Boundary Mixing

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ABSTRACT

Convective motions extend beyond the nominal boundaries of a convection zone. The mechanisms which drive these motions are collectively called “convective boundary mixing” (CBM). In this note, we discuss the three fluid dynamical processes of convective overshoot, entrainment, and convective penetration. We describe the structure of a convective boundary which accounts for all of these processes. To resolve discrepancies between models and observations, the stellar astrophysics community must distinguish between these processes and parameterize each of them into 1D evolutionary models.

Keywords: Stellar convection zones (301), Stellar physics (1621); Stellar evolutionary models (2046)

1. INTRODUCTION

Observations tell us that we do not understand convective boundary mixing (CBM).

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In order to resolve this problem, we need to build a community understanding of CBM processes. In this note, we will briefly describe three fluid dynamical processes: convective overshoot, entrainment, and convective penetration. Each of these processes likely occurs at convective boundaries in stars, and each should be separately parameterized and employed in 1D stellar evolution models.

2. CBM PROCESSES

We will now describe each CBM process in turn. In the following discussion, when we say “convective boundary,” we are referring to the location of the convective boundary determined by the sign change of either the Schwarzschild or Ledoux discriminant.

2.1. Mechanical Overshoot

The process of mechanical overshoot (or convective overshoot) is shown in the upper left panel of Fig. 1. Mechanical overshoot occurs because the convective boundary is not the location where convective velocities are zero, but rather the location where the *buoyant acceleration* of the fluid is zero. Flows buoyantly decelerate beyond the convective boundary, so there is an extended region with nonzero convective velocities.

A simple $\Delta x = u \Delta t$ argument provides an estimate for how deeply convective motions overshoot. Here Δx is the overshoot depth, u is the convective velocity, and $\Delta t \approx N^{-1}$ where N is the Brunt-Väisälä frequency in the stable region. In stellar environments, this estimate generally retrieves $\Delta x \ll H_P$, where H_P is the pressure scale height.

The exponential overshoot parameterization (per e.g., Herwig 2000) which is implemented in many 1D stellar evolution models describes this process fairly well,

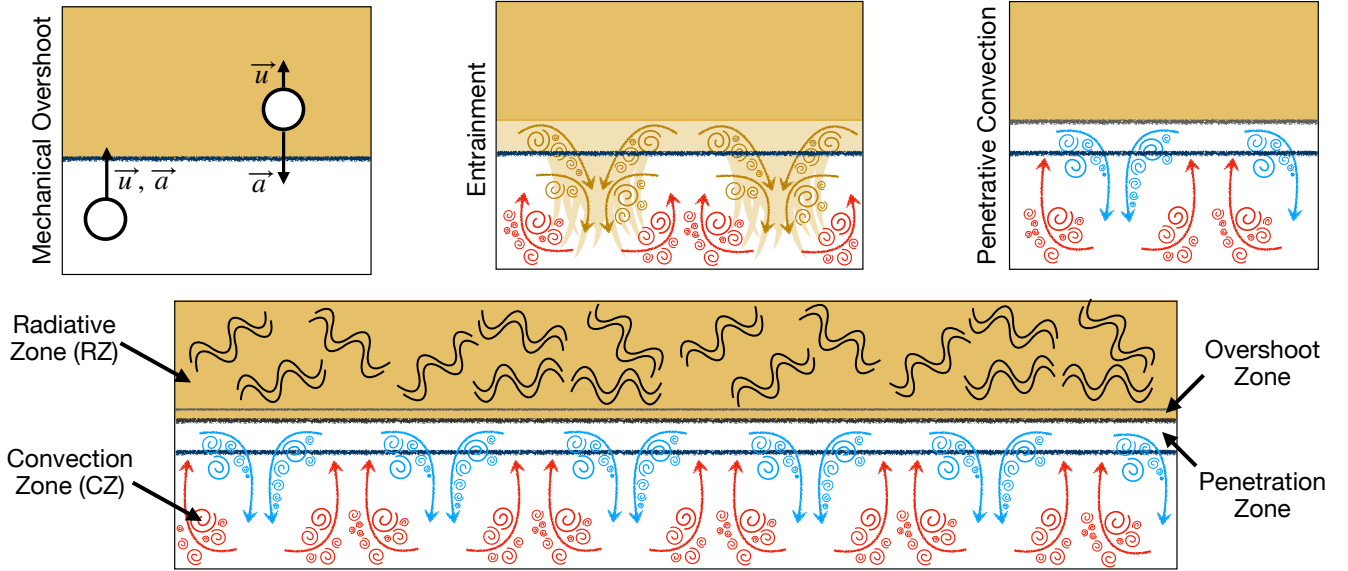


Figure 1.

but 1D modelers generally use $\Delta x/H_P \sim \mathcal{O}(0.1)$, much larger than our earlier estimate. Hydrodynamical simulations of overshoot have been discussed and contextualized in e.g., [Korre et al. \(2019\)](#).

2.2. Entrainment

The process of entrainment is shown in the upper middle panel of Fig. 1. Return flows from overshooting convection carry fluid with the chemical and thermodynamic signature of the RZ. This material then rapidly turbulently mixes in the convection zone. As a result, convective motions which overshoot and entrain materials can cause convective boundaries to gradually advance. As entrainment is linked to the overshooting process described above, the overshoot length scale Δx is directly related to how rapidly a convective boundary can advance by entrainment.

Entrainment has been modeled in 1D stellar evolution codes by [Staritsin \(2013\)](#) and [Scott et al. \(2021\)](#), but their implementations differ from one another and entrainment is not standard in any code. Hydrodynamical simulations of entrainment have been discussed and contextualized in e.g., [Meakin & Arnett \(2007\)](#) & [Fuentes & Cumming \(2020\)](#).

2.3. Penetrative Convection

The process of penetrative convection is shown in the upper right panel of Fig. 1. Through continual overshoot and entrainment, well-mixed convective regions can extend beyond the convective boundary. In these adiabatic extensions to the convection zone, weak buoy-

ancy forces decelerate convective flows over appreciable length scales. Over time, this process creates an extended, nearly-adiabatic “penetrative zone” beyond the convective boundary. Since penetrative zones are established over many dynamical times, they can have length scales much larger than the previously mentioned overshoot length scale Δx .

Penetrative convection most closely resembles “step overshoot” employed in 1D stellar evolution models, but penetrative convection mixes both entropy and composition. Hydrodynamical simulations of penetrative convection have been discussed and contextualized in e.g., [Anders et al. \(2021\)](#), following on the theory laid down in [Roxburgh \(1978, 1989\)](#) and [Zahn \(1991\)](#).

3. CONVECTIVE BOUNDARY STRUCTURE

The structure of a convective boundary in its stationary state is shown in the bottom panel of Fig. 1. The convective boundary as determined by the Schwarzschild or Ledoux criterion is denoted by a black horizontal line. Below that sits the convection zone, which is well-mixed as denoted by its white background, and where upflows are hot and downflows are cold. Above the convection zone lies a well-mixed penetrative zone, where upflows are cold and downflows are hot (showing that flows are buoyantly decelerating but still have appreciable velocity). Above the penetrative zone sits a thin, stably-stratified overshoot zone where convective motions are rapidly buoyantly restored and decelerated by a strong positive entropy gradient. Above the overshoot zone lies a radiative zone, where waves have been excited by the

117 convection. Note that there is no “entrainment zone.”
 118 The entrainment which establishes the penetrative zone
 119 is a transient process, and while there may be some en-
 120 trainment of material from the overshoot zone into the
 121 penetrative zone, that process does not have a separate
 122 region.

123 4. CONCLUSION

124 In conclusion, convective boundary mixing (CBM) is
 125 a conglomeration of a few distinct dynamical processes.
 126 These processes include mechanical overshoot, entrain-
 127 ment, and penetrative convection. A thorough under-
 128 standing and parameterization of each of these processes
 129 can reduce discrepancies between models and observa-
 130 tions.

131 Modeling of convective boundaries has plagued stellar
 132 structure modelers for many years
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 134 Unfortunately, throughout the stellar structure litera-
 135 ture, “convective overshoot,” “convective penetration,”
 136 and “convective boundary mixing” are often used in-
 137 terchangeably, which increases confusion regarding this
 138 tricky topic. Coming to an agreement as a community
 139 about the terminology of convective boundary mixing
 140 and the processes that terminology refers to will help us
 141 pinpoint areas where models behave poorly and design
 142 experiments to improve those models.

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