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The Processes and Structure of Convective Boundary Mixing

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ABSTRACT

Convective motions extend beyond the nominal boundaries of a convection zone. The mechanisms which drive these motions are collectively called "convective boundary mixing" (CBM). In this note, we discuss three distinct fluid dynamical processes: convective overshoot, entrainment, and penetrative convection. We describe the structure of a convective boundary when all of these processes are active. To resolve discrepancies between models and observations, the stellar astrophysics community must distinguish between these processes and parameterize each of them separately in 1D evolutionary models.

Keywords: Stellar convection zones (301), Stellar physics (1621); Stellar evolutionary models (2046)

1. CBM PROCESSES

Observational inferences make it clear that 1D stellar models fail to produce realistic convective boundary mixing (CBM, Pinsonneault 1997; Claret & Torres 2018; Pedersen et al. 2021). In this section, we will briefly describe three distinct CBM processes: convective overshoot, entrainment, and penetrative convection. In the following discussion of each CBM process, "convective boundary" refers to the location coinciding with the sign change of either the Schwarzschild or Ledoux discriminant.

1.1. *Convective Overshoot*

Convective overshoot occurs because the convective boundary is not the location where convective velocities are zero, but rather the location where the *buoyant acceleration* of the fluid is zero. The process of convective overshoot is shown in the upper left panel of Fig. 1, where motions from the white convection zone (CZ) overshoot into the purple stable radiative zone (RZ). Flows buoyantly decelerate beyond the convective boundary, so there is an extended overshoot zone (OZ) with nonzero convective velocities.

A simple $\Delta x = u\Delta t$ argument provides an estimate for how far convective motions overshoot. Here Δx is the overshoot distance, u is the convective velocity, and $\Delta t \approx N^{-1}$ where N is the Brunt-Väisälä frequency in the stable region. In stellar environments, this estimate generally retrieves $\Delta x \ll H_P$, where H_P is the pressure scale height. We note that there is disagreement regarding precisely how to calculate Δx, but this estimate provides the proper flavor.

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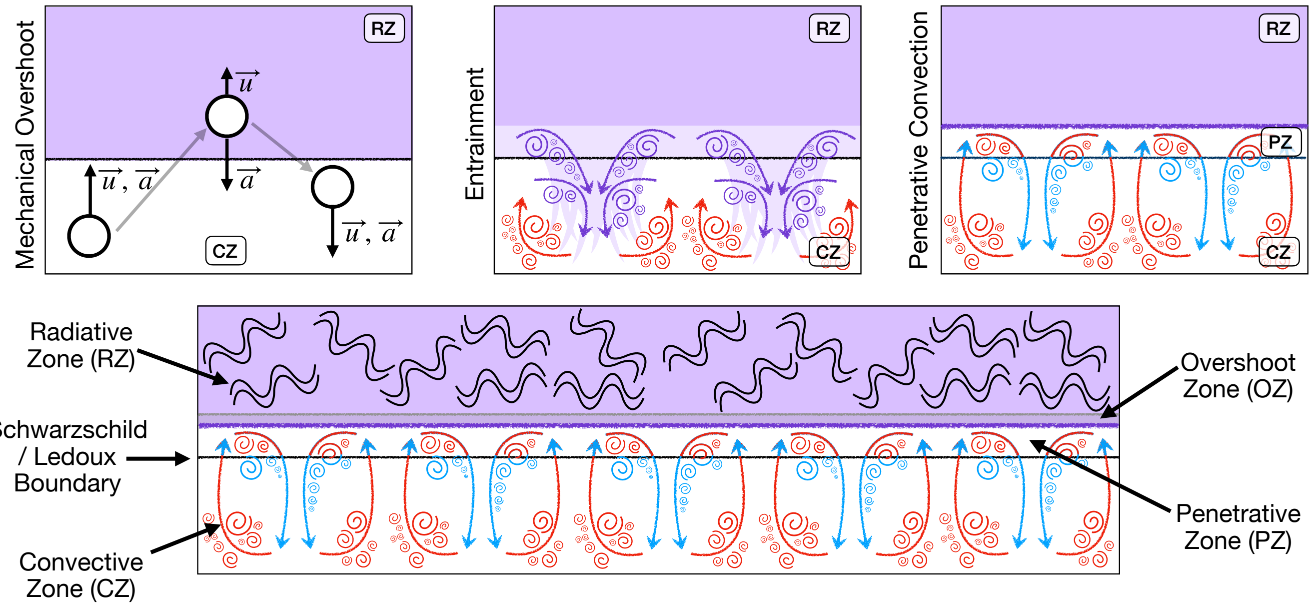

Figure 1. The three processes discussed in Sec. 1 are schematically demonstrated in the top row. White fluid has the properties of the well-mixed CZ, while purple fluid is the stable RZ. (Left) Convective overshoot occurs when a fluid parcel from the CZ crosses into the RZ; due to a strong positive entropy gradient in the RZ, the parcel is accelerated back into the CZ. (Middle) Motions generated by overshooting fluid parcels drag fluid from the RZ into the CZ in a process called entrainment. (Right) If a region of fluid beyond the convective boundary becomes well-mixed, it is a PZ; a divergence of the radiative flux acts as an internal cooling term, changing the buoyant signature of both upflows and downflows. In the bottom panel, we show the structure of a statistically-stationary convective boundary. The CZ sits below a well-mixed PZ. Above the PZ, the fluid is stable but there is a small OZ where convective overshoot occurs. Above this region is the stable RZ where there are internal gravity waves excited by the convection.

The exponential overshoot parameterization (per e.g., Herwig 2000) which is frequently implemented in 1D models describes this process fairly well, but 1D models generally use a much larger $\Delta x/H_P \sim \mathcal{O}(0.1)$ than hydrodynamical simulations suggest should happen when low Mach number flows encounter very stable interfaces. Such simulations have been discussed and contextualized in e.g., Korre et al. (2019).

1.2. *Entrainment*

The process of entrainment is shown in the upper middle panel of Fig. 1. Return flows from overshooting convection carry fluid with the chemical and thermodynamic signature of the RZ. This material then rapidly turbulently mixes in the convection zone. As a result, convective motions which overshoot and entrain materials can gradually move convective boundaries. Since entrainment is linked to convective overshooting, the overshoot distance Δx directly relates to the *rate* of entrainment (which can be inferred from frequently-plotted entrainment rate laws; Meakin & Arnett 2007).

Entrainment has been modeled in 1D stellar evolution software instruments by Staritsin (2013) and Scott et al. (2021), but their implementations differ from one another and entrainment is not standard in any instrument. Hydrodynamical simulations of entrainment have been discussed and contextualized in e.g., Fuentes & Cumming (2020).

1.3. *Penetrative Convection*

The process of penetrative convection is shown in the upper right panel of Fig. 1. Through continual overshoot and entrainment, convection creates well-mixed regions beyond the convective boundary. These well-mixed extensions of the CZ are known as penetration zones (PZs). Since PZs grow gradually over many dynamical times, they can extend beyond the overshoot distance Δx.

Penetrative convection most closely resembles "step overshoot" employed in 1D models, but penetrative convection mixes both entropy and composition. Hydrodynamical simulations of penetrative convection have been discussed and contextualized in e.g., Anders et al. (2021).

2. CONVECTIVE BOUNDARY STRUCTURE

The structure of a convective boundary in steady state is shown in the bottom panel of Fig. 1. The convective boundary as determined by the Schwarzschild or Ledoux criterion is denoted by a black horizontal line, and the CZ is below that line. In the CZ, upflows are hot and downflows are cold. A well-mixed PZ is above the CZ. In the PZ, the radiative luminosity exceeds the system luminosity, so the convective luminosity is negative resulting in cold upflows and warm downflows. Both entropy and composition are mixed throughout the CZ and PZ. The upper boundary of the PZ is marked by a purple line. Above the PZ, there is a stably-stratified OZ where composition is mixed and a strong positive entropy gradient rapidly decelerates convective motions. A thin grey line denotes the top of the OZ. Above the OZ is an RZ filled with internal gravity waves. There is no entrainment zone, but entrainment can change the structure of the other zones by bringing material from the OZ into the PZ.

3. CONCLUSION

In conclusion, convective boundary mixing (CBM) is a conglomeration of a few distinct dynamical processes. These processes include convective overshoot, entrainment, and penetrative convection. A thorough understanding and parameterization of each of these processes can reduce discrepancies between models and observations.

We have not made any distinction between radiative zones which are *thermally* stable and those which are *compositionally* stable. Composition gradients are subject to the processes discussed in this work in the same way as thermal gradients. The only distinction is that the radiative flux, and divergences in it, act to restore the thermal gradients to their radiative values, while composition gradients have no restoring flux.

Modeling of convective boundaries has plagued stellar structure modelers for many years (Paxton et al. 2011, 2018, 2019). Unfortunately, throughout the stellar structure literature, "convective overshoot", "convective penetration", and "convective boundary mixing" are often used interchangeably, which increases confusion regarding this tricky topic. Coming to an agreement as a community about the terminology of convective boundary mixing and the processes that terminology refers to will help us pinpoint areas where models behave poorly and design experiments to improve those models.

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