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Waves!

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ABSTRACT

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1. INTRODUCTION

Massive stars are the cornerstone of many fields of astrophysics. The ionizing radiation emitted at the surfaces of these stars serves as an important feedback mechanism which regulates star formation and the ISM structure (Lancaster et al. 2021), and this radiation was important in the reionization of the early universe (Bromm & Larson 2004). Massive stars end their lives in dynamic explosions which produce supernovae and observable high-energy transients (Heger et al. 2003). These explosions produce compact remnants such as black holes and neutron stars and ground-based gravitational wave interferometers like LIGO have recently begun to probe the mass distribution of these objects (Abbott et al. 2018). The complex evolutionary history of massive stars determines what type of explosion these stars end their lives in and what type of remnant they leave behind (Farmer et al. 2016). A detailed understanding of the internal structure and evolution of massive stars is therefore desired.

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1. Our best tool for probing the insides of massive stars is Asteroseismology. This is going really well for SPB and γ Dor stars.

2. Recently, stochastic low-frequency variability has been seen on even hotter, more massive stars, and there is a current debate about whether that signal is gravity waves driven by core convection or something else.

3. In order to resolve this debate, and to compare simulations of waves in massive stars to observations of massive stars, a framework is required for computing the spectra that would be observed at the surface of theoretical "stars" given the flow fields in simulations.

4. In this paper, we build upon the framework laid out in **??**. We demonstrate that the surface signal of linear waves can be predicted given the linear eigenvalues and a power-law description of the wave luminosity produced by the convection.

2. GRAIVITY WAVES IN THE TERMINOLOGY OF SOUND WAVES

1. Sound waves and gravity waves aren't *that* different.

2. Humans are much more familiar with music (sound waves) than internal gravity waves, so we will appeal to that intuition to intuitively describe our approach.

3. In a star, flows in the core convection zone excite gravity waves in the radiative envelope just like a band playing musical instrument excites sound waves in the room.

4. If we want to know exactly what those musicians sound like, we need a room with idealized acoustics – a recording studio. Recording studios have padded walls, so sound waves emanate from the musicians, and then are damped at the walls so they do not reflect and interfere.

5. We do not observe the pure signal of waves excited by the convection, but rather waves which propagate through the stellar envelope and reflect off of the surface and later the convection zone, establishing standing waves.

6. This is analogous to seeing your favorite band live in a venue with "bad" acoustics. Rooms have specific length scales and dimensions and therefore certain sound waves resonate in those spaces. Power is placed in these resonant modes over time and bam you get a very different sound.

7. If you want to simulate how a band will sound in a given venue, you need two pieces: (1) the pure sound of the band and (2) enough information about the venue to determine which wave modes are resonant. You can test out a theory from this information by measuring the band in the studio, measuring the band in the venue, then comparing the two tracks.

8. Similarly, if you want to simulate how gravity waves will affect the luminosity at the surface of a star, you need (1) the pure signal of the convection and (2) information about the eigenfunctions and eigenfrequencies of gravity waves in the stellar envelope. Then you can compare simulations to asteroseismic observations to see if the model matches or if pieces are missing.

9. Our goal in this paper is to understand the pure signal of convection and also to develop and test a model of gravity waves in the radiative envelope of a massive star.

3. SIMULATION DETAILS

We solve an entropy-log density version of the fully compressible Navier-Stokes equations (cite landau)

The initial background state is constructed from a 15 $M_\odot$ near-ZAMS MESA (cite) model with a low metallicity of (todo). We choose this model because all opacity-driven envelope convection zones may be thermally stabilized in the outer layers of this star (cite), so the surface manifestations of core gravity waves should be unambiguously observable in these stars without pollution from surface-driven turbulence.

4. RESULTS OF SIMULATIONS WITH DAMPING LAYERS

1. Show or refer to a picture of a simulation where the waves are obviously damped in the outer layer.

2. Discuss that these simulations are in the traveling wave regime so we can get properties of the waves as they are generated and propagate outwards with little reflection.

3. Show that the wave luminosity is constant with radius in the star, and demonstrate that this is expected from the linear eigenfunctions.

4. Show that the wave luminosity is fairly constant with increasing Reynolds number, so our answer is a good approximation for actual stars.

5. Show that the wave luminosity can be well-described by a power law. Report that power law in dimensional units. Compare to literature values.

5. SURFACE MANIFESTATION OF GRAVITY WAVES

1. Show or refer to a picture of a simulation dominated by standing waves.

2. Show the power spectrum at a few ells, and then the "observational" spectrum over one hemisphere summed over all the ells.

3. Show that the transfer function (refer to an appendix? derive?) does a good job of describing waves at different frequencies.

4. Mention the places where transfer function doesn't do great: high frequency peaks and troughs (due to timestepping? cite appendix) and the damping side of things (TODO fix this do it better.)

5. Show that the summed transfer function does a good job of describing the summed luminosity.

6. Mention that, surprisingly, we really don't have to run the simulations that long...

7. Talk about the quality factor of the peaks.

6. TRANSFER FUNCTION: LINEAR THEORY DESCRIBES SURFACE MANIFESTATION

This is the section where we open Garage Band and put our bar filter on the recordings we took earlier. We see if the filter actually reproduces the resonances of the bar, etc. It's unclear if this should be its own section; perhaps it should be wrapped up in the previous section or as a subsection there?

7. TRANSFER FUNCTION EXTRAPOLATION TO REAL STARS

This is where we say, based on the last section, the bar filter does a pretty good job of turning studio into bar. So, let's now see what happens if we use a stadium filter instead of a bar filter (whole star instead of sim). We obviously can't actually simulate a whole star, but our hope is that this gives us an idea of what gravity waves would look like on stars in gyre.

TODO find an actual star to compare to by including Matteo.

8. CONCLUSIONS & DISCUSSION

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APPENDIX

A. NUMERICAL METHODS

We nondimensionalize Eqns. **??**-**??**. Our nondimensional length scale $L_{\rm nd}$ is the radial coordinate of the outer boundary of the core convection zone. We nondimensionalize the temperature as the temperature at the stellar core $T_{\rm nd} = T(r = 0)$ and we use the central density to nondimensionalize the mass $m_{\rm nd} = \rho(r = 0)L_{\rm nd}^3$. We use the central value of the nuclear generation rate $H_0 = (\rho\epsilon)(r = 0)$ to define the nondimensional timescale $\tau_{\rm nd} = (m_{\rm nd}/[H_0 L_{\rm nd}])^{1/3}$. The important remaining control parameters are then the nondimensional viscosity $\nu_{\rm nd} = \nu\tau_{\rm nd}/L_{\rm nd}^2$ and thermal diffusivity $\chi_{\rm nd} = \chi_R\tau_{\rm nd}/L_{\rm nd}^2$.

Under this nondimensionalization, the equations of evolution become

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho_0 = 0, \tag{A1}$$

$$\begin{aligned}&\partial_t\boldsymbol{u} + \boldsymbol{\nabla}\varpi + s_1(\boldsymbol{\nabla}T_0) + \boldsymbol{u}\mathcal{D}\\ &\quad - \nu_{\rm nd}\left(\boldsymbol{\nabla}\cdot\overline{\boldsymbol{\sigma}} + \overline{\boldsymbol{\sigma}}\cdot\boldsymbol{\nabla}\ln\rho_0\right)\\ &\quad = \boldsymbol{u}\times(\boldsymbol{\nabla}\times\boldsymbol{u}),\end{aligned} \tag{A2}$$

$$\begin{aligned}&\partial_t s_1 + \boldsymbol{u}\cdot(\boldsymbol{\nabla}s_0) - \boldsymbol{\nabla}s_1\cdot\boldsymbol{\nabla}\chi_{\rm nd}\\ &\quad - \chi_{\rm nd}(\boldsymbol{\nabla}^2 s_1 + [\boldsymbol{\nabla}s_1]\cdot[\boldsymbol{\nabla}\ln\rho_0 + \boldsymbol{\nabla}\ln T_0])\\ &\quad = -\boldsymbol{u}\cdot\boldsymbol{\nabla}s_1 + \frac{\nu_{\rm nd}}{T_0}\mathcal{V} + \tilde{\mathcal{H}},\end{aligned} \tag{A3}$$

where we define $\tilde{\mathcal{H}} = \mathcal{H}/(\rho_0 T_0)$ before nondimensionalizing. The state variables are $\boldsymbol{u}$ the velocity, s_1 the fluctuations of specific entropy, and ϖ the dynamic reduced pressure. We define the stress tensor $\overline{\boldsymbol{\sigma}} = \Pi/(\rho_0\nu)$. The background stellar structure is that of a $15M_\odot$ MESA model (see appendices B & D). This provides radial profiles of $\boldsymbol{\nabla}\ln\rho_0$, $\boldsymbol{\nabla}\ln T_0$, $\boldsymbol{\nabla}T_0$, T_0, $\boldsymbol{\nabla}s_0$, χ_R, and the effective heating $\mathcal{H}$. We refer the reader to appendix B for information on how these are calculated and what differences exist between MESA and Dedalus.

$\mathcal{D}$ is a radial profile of a damping coefficient. For the runs with damping presented in sec. 4, we set $\mathcal{D} = 0$ in the interior and $\mathcal{D} = f_b$ where f_b is the frequency associated with the peak of N^2 in our simulation domain.

TODO: be more precise about $\mathcal{D}$

TODO: Update this to be true for this paper.

We time-evolve equations A1-A3 using the Dedalus pseudospectral solver (Burns et al. 2020, git commit 1339061) using timestepper SBDF2 (Wang & Ruuth 2008) and safety factor 0.3. All variables are spectral expansions of Chebyshev coefficients in the vertical (z) direction ($n_z = 512$ between $z = [0, 2.25]$ plus $n_z = 64$ between $z = [2.25, 3]$) and as $(n_x,\ n_y) = (192,\ 192)$ Fourier coefficients in the horizontally periodic $(x,\ y)$

directions. Our domain spans $x \in [0, L_x]$, $y \in [0, L_y]$, and $z \in [0, L_z]$ with $L_x = L_y = 4$ and $L_z = 3$. To avoid aliasing errors, we use the 3/2-dealiasing rule in all directions. To start our simulations, we add random noise temperature perturbations with a magnitude of 10^{-6} to the initial temperature profile.

Spectral methods with finite coefficient expansions cannot capture true discontinuities. To approximate discontinuous functions such as Eqns. **??** & **??**, we define a smooth Heaviside step function centered at $z = z_0$,

$$H(z; z_0, d_w) = \frac{1}{2}\left(1 + \mathrm{erf}\left[\frac{z - z_0}{d_w}\right]\right). \tag{A4}$$

where erf is the error function and we set $d_w = 0.05$. The simulation in this work uses $\mathcal{P} = 3.2 \times 10^3$, $\mathrm{R}_0^{-1} = 10$, $\mathrm{Pr} = \tau = 0.5$, $\tau_0 = 1.5 \times 10^{-3}$, and $\kappa_{T,0} = \mathcal{P}^{-1}[(\partial T/\partial z)_{\mathrm{rad}}|_{z=0}]/(\partial T/\partial z)_{\mathrm{rad}}$

We produce figures **??** and **??** using matplotlib (Hunter 2007; Caswell et al. 2021). We produce figure **??** using plotly (Inc. 2015) and matplotlib. All of the Python scripts used to run the simulations in this paper and to create the figures in this paper are publicly available in a git repository (https://github.com/evanhanders/d3_stars) and in a Zenodo repository (**?**).

B. FROM MESA TO DEDALUS

We use a 15 $M_\odot$ ZAMS MESA stellar model to construct our background stratification. From this model, we load the radial profiless of the mass m, radial coordinate r, density ρ, pressure P, temperature T, energy generation rate per unit mass ϵ, opacity κ, logarithmic temperature gradlent $\nabla = d\ln T/d\ln P$, adiabatic temperature gradient ∇_{ad}, $\chi_\rho = (d\ln P/d\ln\rho)|_T$, $\chi_T = (d\ln P/d\ln T)|_\rho$, Brunt-Väisälä frequency N^2, specific heat at constant pressure c_P, luminosity L_*, convective luminosity fraction $f_L = L_{\mathrm{conv}}/L_*$, and sound speed c_s. We secondarily calculate

$$\frac{d\ln P}{dr} = -\frac{\rho g}{P}, \tag{B5}$$

$$\frac{d\ln\rho}{dr} = \frac{d\ln P}{dr}\frac{\chi_T}{\chi_\rho}\left(\nabla_{\mathrm{ad}} - \nabla\right) - \frac{g}{c_s^2}, \tag{B6}$$

$$\frac{d\ln T}{dr} = \frac{d\ln P}{dr}\nabla, \tag{B7}$$

and we define $L_{\mathrm{conv}} = L_* f_L$ and $dT/dr = T d\ln T/dr$ and gravitational acceleration $g = Gm/r^2$ and specific entropy gradient $ds/dr = c_P N^2/g$. We calculate the radiative conductivity,

$$k_{\mathrm{rad}} \equiv \frac{16\sigma_{\mathrm{SB}} T^3}{3\rho\kappa} = \frac{L_* - L_{\mathrm{conv}}}{4\pi r^2 dT/dr}, \tag{B8}$$

where we use the second definition because it produces a smoother profile. We define the heating function

$$\begin{aligned}\mathcal{H} &= \rho\epsilon - \frac{1}{4\pi r^2}\frac{d}{dr}\left(-4\pi r^2 k_{\mathrm{rad}}\frac{dT}{dr}\right)\\ &= \frac{1}{4\pi r^2}\left[\frac{dL_*}{dr} - \frac{d}{dr}\left(-4\pi r^2 k_{\mathrm{rad}}\frac{dT}{dr}\right)\right]\end{aligned} \tag{B9}$$

We find that the second equality produces a smoother profile than the first quantity, but use the first quantity (with $\rho\epsilon$) for the innermost radial points where the numerical derivative of L_* is less well-defined. We nondimensionalize using $L_{\mathrm{nd}} = 8.21 \times 10^{10}$ cm, $T_{\mathrm{nd}} = 3.63 \times 10^7$ K, $m_{\mathrm{nd}} = 4.30 \times 10^{33}$ g, and $\tau_{\mathrm{nd}} = 4.66 \times 10^5$ s.

Given this nondimensionalization and these MESA profiles, we control the turbulence and resolution requirement of a simulation by specifying a nondimensional viscosity ν_{nd} We then compute the following nondimensional radial profiles:

1. $\chi_{\mathrm{nd}} = (k_{\mathrm{rad}}/[\rho c_P])(\tau_{\mathrm{nd}}/L_{\mathrm{nd}}^2)$. When $\chi_{\mathrm{nd}} < \nu_{\mathrm{nd}}$, we set $\chi_{\mathrm{nd}} = \nu_{\mathrm{nd}}$.
2. $\ln\rho$ is set to be $\ln[\rho(L_{\mathrm{nd}}^3/m_{\mathrm{nd}})]$.
3. $\ln T$ is set to be $\ln(T/T_{\mathrm{nd}})$.
4. T is set to be T/T_{nd}.
5. $\boldsymbol{\nabla} T$ is set to be $(d\ln T/dr)(L_{\mathrm{nd}}/T_{\mathrm{nd}})$.
6. $\tilde{\mathcal{H}} = (H/[\rho T])(T_{\mathrm{nd}}\tau_{\mathrm{nd}}^3/L_{\mathrm{nd}}^2)$.
7. $\nabla s_0 = (ds/dr)(T_{\mathrm{nd}}\tau_{\mathrm{nd}}^2/L_{\mathrm{nd}})$

Some fields with particularly sharp transitions (e.g., $\boldsymbol{\nabla} s_0$, $\mathcal{H}$) require additional smoothing. In Fig.
TODO: put fig in
, we show comparisons between the stratification felt by the Dedalus simulation and the original MESA model.

C. TRANSFORMS AND SPECTRA

All wave flux and power spectra in this work are calculated from long time series of data of our state variables projected onto a 2D sphere. Given a 3D data cube of a variable $a(t, \phi, \theta)$, we transform the data into frequency space as

$$a(t, \phi, \theta) \underset{\mathrm{SHT}}{\longrightarrow} \hat{a}_{\ell,m}(t) \underset{\mathrm{FT}}{\longrightarrow} \hat{A}_{\ell,m}(f). \tag{C10}$$

We perform the spherical harmonic transform (SHT) using Dedalus. Dedalus returns two coefficient amplitudes, b_1 and b_2, corresponding to the $\cos(m\phi)$ and $-\sin(m\phi)$ components. From these, we construct the spherical harmonic amplitude $\hat{a}_{\ell,m}(t) = b_1 + ib_2$.

We calculate the Discrete Fourier Transform using NumPy and define its normalization such that

$$\hat{A}_{\ell,m,f} = \frac{1}{N}\sqrt{\frac{8}{3}}\sum_{j=0}^{N-1} H_N(j)\hat{a}_{\ell,m}(t_j)\exp\left\{-2\pi i\frac{jf}{N}\right\} \tag{C11}$$

In this definition, $H_N(j)$ is the jth point of the Hanning window defined over N total data points, and the factor of $\sqrt{8/3}$ accounts for this window.

To calculate a power-like quantity (power spectrum, wave flux spectrum), we multiply one transformed field with the complex conjugate of another, e.g., $P_A(f) = \hat{A}^*\hat{A}$. To properly account for power in negative frequencies, we define $P_A(f) = P_A(f) + P_A(-f)$ for all $f \geq 0$.

To calculate the wave flux, we output $p = \varpi - 0.5|\boldsymbol{u}|^2$ and $u_r = \boldsymbol{u}\cdot\hat{e}_r$. We take transforms per the above recipe and calculate the wave flux $W(f) = R^2\rho(R)\hat{P}^*\hat{U}_r$, where R is the radial coordinate at which the shell is sampled.

TODO: call it wave luminosity and add 4π

D. MESA

The MESA EOS is a blend of the OPAL (Rogers & Nayfonov 2002), SCVH (Saumon et al. 1995), FreeEOS (Irwin 2004), HELM (Timmes & Swesty 2000), and PC (Potekhin & Chabrier 2010) EOSes.

Radiative opacities are primarily from OPAL (Iglesias & Rogers 1993, 1996), with low-temperature data from Ferguson et al. (2005) and the high-temperature, Compton-scattering dominated regime by Buchler & Yueh (1976). Electron conduction opacities are from Cassisi et al. (2007).

Nuclear reaction rates are from JINA REACLIB (Cyburt et al. 2010) plus additional tabulated weak reaction rates Fuller et al. (1985); Oda et al. (1994); Langanke & Martínez-Pinedo (2000). Screening is included via the prescription of Chugunov et al. (2007). Thermal neutrino loss rates are from Itoh et al. (1996).

E. DATA AVAILABILITY

The inlists and plotting scripts used in this work may be found at **Cite Github and Zenodo**.

REFERENCES

Abbott, B. P., Abbott, R., Abbott, T. D., et al. 2018, LRR, 21, 3, doi: 10.1007/s41114-018-0012-9

Bromm, V., & Larson, R. B. 2004, Annual Review of Astronomy and Astrophysics, 42, 79, doi: 10.1146/annurev.astro.42.053102.134034

Buchler, J. R., & Yueh, W. R. 1976, ApJ, 210, 440, doi: 10.1086/154847

Burns, K. J., Vasil, G. M., Oishi, J. S., Lecoanet, D., & Brown, B. P. 2020, Physical Review Research, 2, 023068, doi: 10.1103/PhysRevResearch.2.023068

Cassisi, S., Potekhin, A. Y., Pietrinferni, A., Catelan, M., & Salaris, M. 2007, ApJ, 661, 1094, doi: 10.1086/516819

Caswell, T. A., Droettboom, M., Lee, A., et al. 2021, matplotlib/matplotlib: REL: v3.3.4, v3.3.4, Zenodo, doi: 10.5281/zenodo.4475376

Chugunov, A. I., Dewitt, H. E., & Yakovlev, D. G. 2007, PhRvD, 76, 025028, doi: 10.1103/PhysRevD.76.025028

Cyburt, R. H., Amthor, A. M., Ferguson, R., et al. 2010, ApJS, 189, 240, doi: 10.1088/0067-0049/189/1/240

Farmer, R., Fields, C. E., Petermann, I., et al. 2016, ApJS, 227, 22, doi: 10.3847/1538-4365/227/2/22

Ferguson, J. W., Alexander, D. R., Allard, F., et al. 2005, ApJ, 623, 585, doi: 10.1086/428642

Fuller, G. M., Fowler, W. A., & Newman, M. J. 1985, ApJ, 293, 1, doi: 10.1086/163208

Heger, A., Fryer, C. L., Woosley, S. E., Langer, N., & Hartmann, D. H. 2003, ApJ, 591, 288, doi: 10.1086/375341

Hunter, J. D. 2007, Computing in Science and Engineering, 9, 90, doi: 10.1109/MCSE.2007.55

Iglesias, C. A., & Rogers, F. J. 1993, ApJ, 412, 752, doi: 10.1086/172958

—. 1996, ApJ, 464, 943, doi: 10.1086/177381

Inc., P. T. 2015, Collaborative data science, Montreal, QC: Plotly Technologies Inc. https://plot.ly

Irwin, A. W. 2004, The FreeEOS Code for Calculating the Equation of State for Stellar Interiors. http://freeeos.sourceforge.net/

Itoh, N., Hayashi, H., Nishikawa, A., & Kohyama, Y. 1996, ApJS, 102, 411, doi: 10.1086/192264

Lancaster, L., Ostriker, E. C., Kim, J.-G., & Kim, C.-G. 2021, ApJ, 914, 89, doi: 10.3847/1538-4357/abf8ab

Langanke, K., & Martínez-Pinedo, G. 2000, Nuclear Physics A, 673, 481, doi: 10.1016/S0375-9474(00)00131-7

Oda, T., Hino, M., Muto, K., Takahara, M., & Sato, K. 1994, Atomic Data and Nuclear Data Tables, 56, 231, doi: 10.1006/adnd.1994.1007

Potekhin, A. Y., & Chabrier, G. 2010, Contributions to Plasma Physics, 50, 82, doi: 10.1002/ctpp.201010017

Rogers, F. J., & Nayfonov, A. 2002, ApJ, 576, 1064, doi: 10.1086/341894

Saumon, D., Chabrier, G., & van Horn, H. M. 1995, ApJS, 99, 713, doi: 10.1086/192204

Timmes, F. X., & Swesty, F. D. 2000, ApJS, 126, 501, doi: 10.1086/313304

Wang, D., & Ruuth, S. J. 2008, Journal of Computational Mathematics, 26, 838. http://www.jstor.org/stable/43693484