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Waves!

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ABSTRACT

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1. INTRODUCTION

Here's what Evan needs to do:

1. Get all of the damping layer simulations run.
2. Get a really long simulation run without a damping layer.
3. Do the transfer function derivation again.
4. Make figures and write out the story!

2. GRAIVITY WAVES IN THE TERMINOLOGY OF SOUND WAVES

3. SIMULATION DETAILS

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We solve a version of the LBR Anelastic equations based on Eqns. 118-120 of Vasil et al. (2013),

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho_0 = 0, \tag{1}$$

$$\partial_t\boldsymbol{u} + \boldsymbol{\nabla}\varpi + s_1\boldsymbol{\nabla}T_0 + \boldsymbol{u}\mathcal{D} = -\boldsymbol{u}\cdot\boldsymbol{\nabla}\boldsymbol{u} + \boldsymbol{\nabla}\cdot\Pi, \tag{2}$$

$$\begin{aligned}&\partial_t s_1 + \boldsymbol{u}\cdot\boldsymbol{\nabla}s_0 = \\ &-\boldsymbol{u}\cdot\boldsymbol{\nabla}s_1 + \frac{1}{\rho_0 T_0}\left(\boldsymbol{\nabla}\cdot[\rho_0 T_0\chi_R\boldsymbol{\nabla}s_1] + \mathcal{H} + \mathrm{VH}\right),\end{aligned} \tag{3}$$

where $\boldsymbol{u}$ is the velocity, s is the specific entropy, and $\varpi = h + \phi - T_0 s_1$ is the dynamic pressure with h the enthalpy and ϕ the gravitational potential. The background density ρ_0, temperature T_0, and specific entropy s_0 stratification are loaded from a MESA stellar model. We define the viscous stress tensor

$$\Pi_{ij} = 2\rho_0\nu\left(e_{ij} - \frac{1}{3}\delta_{ij}\boldsymbol{\nabla}\cdot\boldsymbol{u}\right), \tag{4}$$

where $e = 0.5(\boldsymbol{\nabla}\boldsymbol{u} + [\boldsymbol{\nabla}\boldsymbol{u}]^T)$ is the rate of strain tensor and ν is the kinematic viscosity. The viscous heating term is then $\mathrm{VH} = 2\rho\nu(\mathrm{Tr}(e\cdot e) - (1/3)(\boldsymbol{\nabla}\cdot\boldsymbol{u})^2)$ where Tr is the trace operation. The efective internal heating accounts for nuclear energy generation and flux carried radiative $\mathcal{H} = \rho\epsilon - \boldsymbol{\nabla}\cdot(F_{\mathrm{rad}})$ with ϵ the nuclear energy generation rate per unit mass. Finally, χ_R is the radiative conductivity; we assume this is a constant in the interior of the star, but use the real value in the outer portions of the stellar envelope where diffusivity is large.

The momentum equation includes a damping term ($\boldsymbol{u}\mathcal{D}$). In simulations which include damping, $\mathcal{D} = f_{\text{char}} H(r = r_{\text{transition}})$ where f_{char} is a characteristic frequency and H is the Heaviside step function, which is zero below $r = r_{\text{transition}}$. In simulations without damping, we set $\mathcal{D} = 0$.

For full details about how we implement these equations, we refer the reader to appendix A.

4. RESULTS OF SIMULATIONS WITH DAMPING LAYERS

This is the "recording studio" section – it's useful to report the amplitude here because the theory doesn't give that.

We should run simulations as turbulent as possible here. Hopefully more turbulnce doesn't change the result. We will ignore the low-resolution laminar runs.

We report that we see powerlaw wave fluxes at all turbulence values, and no scaling with turbulence (hopefully).

5. SURFACE MANIFESTATION OF GRAVITY WAVES

This is the "Band plays in the bar" section – we can measure sound waves and resonances here.

We should run simulations for a long time here, at a resolution like 256^3.

6. TRANSFER FUNCTION: LINEAR THEORY DESCRIBES SURFACE MANIFESTATION

This is the section where we open Garage Band and put our bar filter on the recordings we took earlier. We see if the filter actually reproduces the resonances of the bar, etc.

Need to check through the transfer function derivation and theory here carefully. It may be useful to include Kyle here?

7. TRANSFER FUNCTION EXTRAPOLATION TO REAL STARS

This is where we say, based on the last section, the bar filter does a pretty good job of turning studio into bar. So, let's now see what happens if we use a stadium filter instead of a bar filter (whole star instead of sim). We obviously can't actually simulate a whole star, but our hope is that this gives us an idea of what gravity waves would look like on stars in gyre.

8. CONCLUSIONS & DISCUSSION

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APPENDIX

A. NUMERICAL METHODS

We nondimensionalize Eqns. 1-3. Our nondimensional length scale L_{nd} is the radial coordinate of the outer boundary of the core convection zone. We nondimensionalize the temperature as the temperature at the stellar core $T_{\text{nd}} = T(r = 0)$ and we use the central density to nondimensionalize the mass $m_{\text{nd}} = \rho(r = 0)L_{\text{nd}}^3$. We use the central value of the nuclear generation rate $H_0 = (\rho\epsilon)(r = 0)$ to define the nondimensional timescale $\tau_{\text{nd}} = (m_{\text{nd}}/[H_0 L_{\text{nd}}])^{1/3}$. The important remaining control parameters are then the nondimensional viscosity $\nu_{\text{nd}} = \nu\tau_{\text{nd}}/L_{\text{nd}}^2$ and thermal diffusivity $\chi_{\text{nd}} = \chi_R \tau_{\text{nd}}/L_{\text{nd}}^2$.

Under this nondimensionalization, the equations of evolution become

$$\boldsymbol{\nabla}\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\boldsymbol{\nabla}\ln\rho_0 = 0, \tag{A1}$$

$$\begin{aligned}\partial_t \boldsymbol{u} &+ \boldsymbol{\nabla}\varpi + s_1(\boldsymbol{\nabla} T_0) + \boldsymbol{u}\mathcal{D} \\ &- \nu_{\text{nd}}\left(\boldsymbol{\nabla}\cdot\bar{\boldsymbol{\sigma}} + \bar{\boldsymbol{\sigma}}\cdot\boldsymbol{\nabla}\ln\rho_0\right) \\ &= \boldsymbol{u}\times(\boldsymbol{\nabla}\times\boldsymbol{u}),\end{aligned} \tag{A2}$$

$$\begin{aligned}\partial_t s_1 &+ \boldsymbol{u}\cdot(\boldsymbol{\nabla} s_0) - \boldsymbol{\nabla} s_1\cdot\boldsymbol{\nabla}\chi_{\text{nd}} \\ &- \chi_{\text{nd}}(\boldsymbol{\nabla}^2 s_1 + [\boldsymbol{\nabla} s_1]\cdot[\boldsymbol{\nabla}\ln\rho_0 + \boldsymbol{\nabla}\ln T_0]) \\ &= -\boldsymbol{u}\cdot\boldsymbol{\nabla} s_1 + \frac{\nu_{\text{nd}}}{T_0}\mathcal{V} + \tilde{\mathcal{H}},\end{aligned} \tag{A3}$$

where we define $\tilde{\mathcal{H}} = \mathcal{H}/(\rho_0 T_0)$ before nondimensionalizing. The state variables are $\boldsymbol{u}$ the velocity, s_1 the fluctuations of specific entropy, and ϖ the dynamic reduced pressure. We define the stress tensor $\overline{\boldsymbol{\sigma}} = \Pi/(\rho_0\nu)$. The background stellar structure is that of a $15M_\odot$ MESA model (see appendices B & D). This provides radial profiles of $\boldsymbol{\nabla}\ln\rho_0$, $\boldsymbol{\nabla}\ln T_0$, $\boldsymbol{\nabla}T_0$, T_0, $\boldsymbol{\nabla}s_0$, χ_R, and the effective heating $\mathcal{H}$. We refer the reader to appendix B for information on how these are calculated and what differences exist between MESA and Dedalus.

$\mathcal{D}$ is a radial profile of a damping coefficient. For the runs with damping presented in sec. 4, we set $\mathcal{D} = 0$ in the interior and $\mathcal{D} = f_b$ where f_b is the frequency associated with the peak of N^2 in our simulation domain.

TODO: be more precise about $\mathcal{D}$

TODO: Update this to be true for this paper.

We time-evolve equations A1-A3 using the Dedalus pseudospectral solver (Burns et al. 2020, git commit 1339061) using timestepper SBDF2 (Wang & Ruuth 2008) and safety factor 0.3. All variables are spectral expansions of Chebyshev coefficients in the vertical (z) direction ($n_z = 512$ between $z = [0, 2.25]$ plus $n_z = 64$ between $z = [2.25, 3]$) and as $(n_x,\ n_y) = (192,\ 192)$ Fourier coefficients in the horizontally periodic (x, y) directions. Our domain spans $x \in [0, L_x]$, $y \in [0, L_y]$, and $z \in [0, L_z]$ with $L_x = L_y = 4$ and $L_z = 3$. To avoid aliasing errors, we use the 3/2-dealiasing rule in all directions. To start our simulations, we add random noise temperature perturbations with a magnitude of 10^{-6} to the initial temperature profile.

Spectral methods with finite coefficient expansions cannot capture true discontinuities. To approximate discontinuous functions such as Eqns. **??** & **??**, we define a smooth Heaviside step function centered at $z = z_0$,

$$H(z; z_0, d_w) = \frac{1}{2}\left(1 + \mathrm{erf}\left[\frac{z - z_0}{d_w}\right]\right). \tag{A4}$$

where erf is the error function and we set $d_w = 0.05$. The simulation in this work uses $\mathcal{P} = 3.2 \times 10^3$, $\mathrm{R}_0^{-1} = 10$, $\mathrm{Pr} = \tau = 0.5$, $\tau_0 = 1.5 \times 10^{-3}$, and $\kappa_{T,0} = \mathcal{P}^{-1}[(\partial T/\partial z)_{\mathrm{rad}}|_{z=0}]/(\partial T/\partial z)_{\mathrm{rad}}$

We produce figures **??** and **??** using matplotlib (Hunter 2007; Caswell et al. 2021). We produce figure **??** using plotly (Inc. 2015) and matplotlib. All of the Python scripts used to run the simulations in this paper and to create the figures in this paper are publicly available in a git repository (https://github.com/evanhanders/d3_stars) and in a Zenodo repository (**?**).

B. FROM MESA TO DEDALUS

We read in a 40 $M_\odot$ ZAMS MESA stellar model (`LOGS/6.data` at github.com/matteocantiello/MESA_Models_Dedalus_Full_Sphere). From this model, we load the radial profiless of the mass m, radial coordinate r, density ρ, pressure P, temperature T, energy generation rate per unit mass ϵ, opacity κ, logarithmic temperature gradient $\nabla = d\ln T/d\ln P$, adiabatic temperature gradient ∇_{ad}, $\chi_\rho = (d\ln P/d\ln\rho)|_T$, $\chi_T = (d\ln P/d\ln T)|_\rho$, Brunt-Väisälä frequency N^2, specific heat at constant volume c_V and pressure c_P, convective luminosity L_{conv}, and square sound speed c_s^2. From these quantities, we calculate the radiative diffusivity,

$$\chi_{\mathrm{rad}} = \frac{16\sigma_{\mathrm{SB}}T^3}{3\rho^2 c_P\kappa}, \tag{B5}$$

the gravitational acceleration $g = Gm/r^2$, the logarithmic pressure derivative $d\ln P/dr = -\rho g/P$, the logarithmic density gradient $(d\ln\rho/dr = d\ln P/dr)(\chi_T/\chi_\rho)(\nabla_{\mathrm{ad}} - \nabla) - g/c_s^2$, and the logarithmic temperature gradient $d\ln T/dr = (d\ln P/dr)\nabla$. The heating profile is found from the convective velocity $H = \hat{e}_r \cdot \boldsymbol{\nabla}L_{\mathrm{conv}}/(4\pi r^2)$. We nondimensionalize the length L on the radius of the core convection zone; the density, temperature, heating, and specific heat at constant pressure are nondimensionalized at their core values $\rho_c = \rho(r=0)$ and $T_c = T(r=0)$, $H_c = H(r=0)$, $c_{P,c} = c_P(r=0)$; the timescale is then set to be the core heating timescale $\tau_H = (H_c/L^2/\rho_c)^{-1/3}$ and the nondimensiona velocity is $u_H = L/\tau_H$.

Given this nondimensionalization and these MESA profiles, we compute the following nonconstant coefficients for our Dedalus model:

1. $\mathrm{Pe}^{-1} = \chi_{\mathrm{rad}}/(\mathrm{u_H L})$. When $\mathrm{Pe}^{-1} < \mathrm{Re}^{-1}$, a constant, we set $\mathrm{Pe}^{-1} = \mathrm{Re}^{-1}$.
2. $\nabla\mathrm{Pe}^{-1}$ is computed as the gradient of Pe^{-1}.
3. $\ln\rho$ is set to be $\ln(\rho/\rho_c)$.
4. $\ln T$ is set to be $\ln(T/T_c)$.
5. T is set to be T/T_c.
6. $\boldsymbol{\nabla}T$ is set to be $(T/T_c)\,L\,d\ln T/dr$.
7. $\mathcal{H} = (H/H_c)\,\rho_c T_C/(\rho T)$.
8. $\nabla s_0 = c_P N^2/g(L/s_c)$
   TODO: write down what s_c is

Some fields with particularly sharp transitions (e.g., $\boldsymbol{\nabla}s_0$, $\mathcal{H}$) require additional smoothing. In Fig.
TODO: put fig in
, we show comparisons between the stratification felt by the Dedalus simulation and the original MESA model.

C. TRANSFORMS AND SPECTRA

All wave flux and power spectra in this work are calculated from long time series of data of our state variables projected onto a 2D sphere. Given a 3D data cube of a variable $a(t,\phi,\theta)$, we transform the data into frequency space as

$$a(t,\phi,\theta) \xrightarrow[\text{SHT}]{} \hat{a}_{\ell,m}(t) \xrightarrow[\text{FT}]{} \hat{A}_{\ell,m}(f). \tag{C6}$$

We perform the spherical harmonic transform (SHT) using Dedalus. Dedalus returns two coefficient amplitudes, b_1 and b_2, corresponding to the $\cos(m\phi)$ and $-\sin(m\phi)$ components. From these, we construct the spherical harmonic amplitude $\hat{a}_{\ell,m}(t) = b_1 + ib_2$.

We calculate the Discrete Fourier Transform using NumPy and define its normalization such that

$$\hat{A}_{\ell,m,f} = \frac{1}{N}\sqrt{\frac{8}{3}}\sum_{j=0}^{N-1} H_N(j)\hat{a}_{\ell,m}(t_j)\text{exp}\left\{-2\pi i\frac{jf}{N}\right\} \tag{C7}$$

In this definition, $H_N(j)$ is the jth point of the Hanning window defined over N total data points, and the factor of $\sqrt{8/3}$ accounts for this window.

To calculate a power-like quantity (power spectrum, wave flux spectrum), we multiply one transformed field with the complex conjugate of another, e.g., $P_A(f) = \hat{A}^*\hat{A}$. To properly account for power in negative frequencies, we define $P_A(f) = P_A(f) + P_A(-f)$ for all $f \geq 0$.

To calculate the wave flux, we output $p = \varpi - 0.5|\boldsymbol{u}|^2$ and $u_r = \boldsymbol{u}\cdot\hat{e}_r$. We take transforms per the above recipe and calculate the wave flux $W(f) = R^2\rho(R)\hat{P}^*\hat{U}_r$, where R is the radial coordinate at which the shell is sampled.

TODO: call it wave luminosity and add 4π

D. MESA

The MESA EOS is a blend of the OPAL (Rogers & Nayfonov 2002), SCVH (Saumon et al. 1995), FreeEOS (Irwin 2004), HELM (Timmes & Swesty 2000), and PC (Potekhin & Chabrier 2010) EOSes.

Radiative opacities are primarily from OPAL (Iglesias & Rogers 1993, 1996), with low-temperature data from Ferguson et al. (2005) and the high-temperature, Compton-scattering dominated regime by Buchler & Yueh (1976). Electron conduction opacities are from Cassisi et al. (2007).

Nuclear reaction rates are from JINA REACLIB (Cyburt et al. 2010) plus additional tabulated weak reaction rates Fuller et al. (1985); Oda et al. (1994); Langanke & Martínez-Pinedo (2000). Screening is included via the prescription of Chugunov et al. (2007). Thermal neutrino loss rates are from Itoh et al. (1996).

E. DATA AVAILABILITY

The inlists and plotting scripts used in this work may be found at **Cite Github and Zenodo**.

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