Convective Penetration Paper Outline

May 6, 2021

1   Intro & Background

- Observational & Theoretical context: "convective overshoot" and the problems it relates to. Include sources on e.g.,

  - stellar Lithium abundances (Pinsonneault, 1997, fig 4),

  - solar modeling problem (Basu & Antia, 2004; Bahcall et al., 2005; Vinyoles et al., 2017; Asplund et al., 2021),

  - helioseismology (Christensen-Dalsgaard et al., 2011),

  - stellar evolution & remnants (Higgins & Vink, 2020),
    item stellar core convection overshoot (Claret & Torres, 2018),

  - parameterizations (Shaviv & Salpeter, 1973; Maeder, 1975; Herwig, 2000) and "convective boundaries" in MESA papers (Paxton et al., 2011, 2013, 2018, 2019).

  - Other things?

- Terminology of overshoot vs penetration.

- Note that the state of the field has been reviewed many times by many authors (Marcus et al., 1983; Zahn, 1991; Browning et al., 2004; Rogers et al., 2006; Viallet et al., 2015; Korre et al., 2019). We will do a brief review but these are additional resources.

- Mention miscellaneous theoretical or semianalytical arguments or models (Roxburgh, 1965, 1978, 1989, 1992, 1998; Marcus et al., 1983; Zahn, 1991; Hurlburt et al., 1994; Rempel, 2004; Canuto, 2011; Viallet et al., 2015; Korre et al., 2019).

- Briefly review simulations:

  - "Early" simulations of e.g., plumes (Schmitt et al., 1984).
  - Dynamical Cartesian simulations in both fully compressible and Boussinesq equation sets (Hurlburt et al., 1994; Brummell et al., 2002; Rogers & Glatzmaier, 2005; Käpylä et al., 2007; Kitiashvili et al., 2016; Lecoanet et al., 2016; Käpylä et al., 2017; Couston et al., 2017; Toppaladoddi & Wettlaufer, 2018).
  - Dynamical spherical geometry 2D or 3D simulations (fully compressible, anelastic, boussinesq, some solar-like, some not) (Browning et al., 2004; Rogers et al., 2006; Brun et al., 2017; Pratt et al., 2017; Dietrich & Wicht, 2018)

- Zahn's energetics arguments and the various times it has cropped up in the literature (Zahn, 1991; Hurlburt et al., 1994; Rempel, 2004; Rogers et al., 2006).

- Say very clearly that *the penetration depth depends on how steeply the radiative flux changes with depth* (and also convective structures). Say that we're going to present simulations which agree with this.

2   Theory

- The dimensional modified Boussinesq equations (note that Zahn did the fully compressible one, we're simplifying for easier reading and math and alignment with sims).

- Walk through $F_{\mathrm{conv}} = F_0 W^3$ argument in both the CZ and an adiabatic PZ in general terms. Importantly, we need to leave this part general as a ratio of integrals of the flux around the point where $\boldsymbol{\nabla}_{\mathrm{ad}} = \boldsymbol{\nabla}_{\mathrm{rad}}$.

- If this theory is correct, it should work regardless of the shape of k and thus F. So we will do two experiments with different predictions to test it. One where k is discontinuous, and one where $\partial_z k$ is discontinuous.

- Solve out the predictions for the two simulations we're going to do.

- Comment on uncertainties (rotation, magnetism, density stratification) that we won't attempt to handle in this work. Say that we won't handle them but that this is a first step.

3   Simulation Details

- Nondimensionalization details & nondimensional eqns.

- Parameters ($\mathcal{P}$, $\mathcal{R}$, Pr, $\mathcal{S}$, Q, ζ)

- Conductivity profiles (erf and linear), $\boldsymbol{\nabla}_{\mathrm{ad}}$, internal heating.

- Mention Dedalus (Burns et al., 2020) and timestepping details.

- Define a set of "standard parameters:" e.g., R = 400, Pr = 0.5, $Q = 1$, $\zeta = 10^{-3}$, $\mathcal{S} = 10^3$, $\mathcal{P} = 4$. All of our cuts through our multi-D parameter space cut through these parameters, varying one parameter ($\mathcal{S}$, $\mathcal{P}$, $\mathcal{R}$) and holding the rest constant.

- Forward ref an appendix on our initial conditions / accelerated evolution (appendix A) and also an appendix of tables of simulation details (appendix B).

4   Results

4.1   Qualitative Description of Simulations

- Description of the *evolved state* of a turbulent simulation. This is the "put in some pretty convective dynamics picture" section to orient people into what our simulations look like. And to show them that we get

sims where the "MESA answer" does a really bad job of describing the final state. Dynamics pictuer will show: pretty standard CZ dynamics; a developed PZ with the sign of w and T reversed from the CZ.

- Here we'll also show one of our $|\boldsymbol{\nabla} T|$ plots to show that convection flattens $\boldsymbol{\nabla} \to \boldsymbol{\nabla}_{\text{ad}}$ far above top of nominal CZ. This is a good time to explain the definitions of penetration vs overshoot in words again with a picture. This is also a good time to show pictorally what our measure of the height of the PZ is (currently it's where $\boldsymbol{\nabla}$ is 50% between $\boldsymbol{\nabla}_{\text{rad}}$ and $\boldsymbol{\nabla}_{\text{ad}}$).

4.2   Measured penetration zone scalings

- Profile verification of theory. Does W go from W_0 to $\mathcal{O}(0)$ at the top of the PZ? Is it an e-folding? Also how does the rms Pe or Re scale with height? Does Zahn's "thermal adjustment layer" with Pe ~ 1 hold in the PZ-RZ transition?

- Show that the predicted $\mathcal{P}$ scalings do a pretty good job. This will include some parameter space plots (probably all in one figure) that show:

    - δ_P depends on $\mathcal{P}$.
    - δ_P does not really depend strongly on $\mathcal{S}$ or $\mathcal{R}$.

- Mention that 2D simulations produce more pronounced PZs (larger by a factor of ~ 2) but similar scalings with theory; we choose to omit 2D results and focus on more realistic 3D results.

- our theory has $\delta_P = A\mathcal{P}$ (for erf sims) and $\delta_P = B\mathcal{P}^{1/2}$; we should report good-fits for A and B. These can be plugged directly into MESA as first guesses as the penetration depth there.

5   A modified solar model

- Here we briefly describe a new MESA algorithm where we have a paramterization like $\delta_P/\ell = \xi B\mathcal{P}^{1/2}$ put into MESA, where ξ is a knob

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the user can turn and ℓ is the mixing length or some such. We have two options here: state that $\mathcal{P} = 1$ is always true, or have MESA do a simple linear interpolation of $\boldsymbol{\nabla}_{\mathrm{rad}}$ over some depth like $\ell/2$ above and below the CZ boundary to get a value for $\mathcal{P}$.

- We probably can use the lame / naive assumption that below δ_P, MESA latches immediately onto $\boldsymbol{\nabla}_{\mathrm{rad}}$ (it is stiff, after all).

- We calculate a few solar models with $\xi = [0.5, 1, 2]$ and show how these models change the structure at the base of the solar CZ.

6   Discussion

- Say what we said again. Penetration depth depends on how quickly F_{rad} changes with height.

- Discuss the subtleties associated with filling factors and how boussinesq can't really appropriately capture that.

- Discuss open questions (rotation, stratification [above], magnetism, geometry).

- Round it off by calling back to how awesome it is that $\mathcal{P}$ totally determines δ_P in our work and that $\mathcal{S}$ and $\mathcal{R}]$ don't, which suggests we can actually use simulations to crack this long-standing problem in stellar physics.

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