Convective Penetration Paper Outline

May 8, 2021

1 Intro & Background

- The outline should be comprised of a few paragraphs, which basically say these things:
  1. "Convection is important in stars."
  2. "Understanding convective boundaries is important (because of all of these observational and simulation and theoretical data points that show we kinda don't).'
  3. People have thought about this problem theoretically in these ways.
  4. We think the theoretical arguments of Zahn (1991), who suggested that this is what convective penetration is, are really promising.
  5. Here's what we do in this work.
- Observational & Theoretical context: "convective overshoot" and the problems it relates to. Include sources on e.g.,
  - stellar Lithium abundances (fig 4 of Pinsonneault, 1997; Dumont et al., 2021),
  - solar modeling problem (Basu & Antia, 2004; Bahcall et al., 2005; Zhang & Li, 2012; Vinyoles et al., 2017; Asplund et al., 2021),
  - helioseismology (Christensen-Dalsgaard et al., 2011),
  - black hole mass gap (**?**)
  - stellar evolution & remnants (Higgins & Vink, 2020),
  - stellar core convection overshoot (Claret & Torres, 2018; Jermyn et al., 2018; Viani & Basu, 2020; Martinet et al., 2021),
  - parameterizations (Shaviv & Salpeter, 1973; Maeder, 1975; Herwig, 2000)
  - "convective boundaries" in MESA papers (Paxton et al., 2011, 2013, 2018, 2019).
  - Other things?
- Terminology of overshoot vs penetration.

- Note that the state of the field has been reviewed many times by many authors (Marcus et al., 1983; Zahn, 1991; Browning et al., 2004; Rogers et al., 2006; Viallet et al., 2015; Korre et al., 2019). We will do a brief review but these are additional resources.

- Mention miscellaneous theoretical or semianalytical arguments or models (Roxburgh, 1965, 1978, 1989, 1992, 1998; Marcus et al., 1983; Zahn, 1991; Hurlburt et al., 1994; Rempel, 2004; Canuto, 2011; Viallet et al., 2015; Rieutord, 2019; Korre et al., 2019).

- Quick mention of penetration in the geophysical context (including neat lab experiments like Deardorff et al., 1969).

- Briefly review simulations:

  - "Early" simulations of e.g., plumes (Schmitt et al., 1984).
  - Dynamical Cartesian simulations in both fully compressible and Boussinesq equation sets (Musman, 1968; Moore & Weiss, 1973; Hurlburt et al., 1986, 1994; Singh et al., 1995; Saikia et al., 2000; Brummell et al., 2002; Rogers & Glatzmaier, 2005; Käpylä et al., 2007; Tian et al., 2009; Kitiashvili et al., 2016; Lecoanet et al., 2016; Käpylä et al., 2017; Couston et al., 2017; Toppaladoddi & Wettlaufer, 2018; Käpylä, 2019; Cai, 2020).
  - Dynamical spherical geometry 2D or 3D simulations (fully compressible, anelastic, boussinesq, some solar-like, some not) (Browning et al., 2004; Rogers et al., 2006; Brun et al., 2017; Pratt et al., 2017; Dietrich & Wicht, 2018; Higl et al., 2021)

- Zahn's energetics arguments and the various times it has cropped up in the literature (Zahn, 1991; Hurlburt et al., 1994; Rempel, 2004; Rogers et al., 2006).

- Simulations that have noticed that overshoot and/or penetration depends in some way on the flux (Singh et al., 1998; Hotta, 2017; Käpylä, 2019).

- State the conclusions of this work very clearly. *The penetration depth depends on how steeply the radiative flux changes with depth near the radiative-convective boundary.* Thus, the penetration depth depends on the slope of the radiative conductivity / the opacity near the boundaries of the CZ We have designed two simple numerical setups which demonstrate this point.

2 Theory

- The dimensional modified Boussinesq equations (note that Zahn did the fully compressible one, we're simplifying for easier reading and math and alignment with sims).

- Walk through $F_{\text{conv}} = F_0 W^3$ argument in both the CZ and an adiabatic PZ in general terms. Importantly, we need to leave this part general as a ratio of integrals of the flux around the point where $\boldsymbol{\nabla}_{\text{ad}} = \boldsymbol{\nabla}_{\text{rad}}$.

- If this theory is correct, it should work regardless of the shape of k. So we will do two experiments with different predictions to test the overall idea of this theory. One where k is discontinuous, and one where $\partial_z k$ is discontinuous.

- Solve out the theory to get function forms of $\delta_P/L_{\rm cz}$ for the two simulations we're doing.

- Comment briefly on the fact that this work will only be able to get at the *shape of conductivity* portion of this theoretical prediction. Future work which includes density stratification would be required to understand the geometrical effects of the plumes, but that's beyond the scope of this work.

3 Simulation Details

- Nondimensionalization details & nondimensional eqns.

- Parameters ($\mathcal{P}$, $\mathcal{R}$, Pr, $\mathcal{S}$, Q, ζ). We used fixed values of Pr, Q, ζ in this work and vary the others.

- Specification of conductivity profiles (erf and linear), $\boldsymbol{\nabla}_{\rm ad}$, internal heating.

- The magic sauce of internal heating that allows splitting $\mathcal{S}$ and $\mathcal{P}$, and a brief discussion of some prior work where those were implicitly tied.

- Mention Dedalus (Burns et al., 2020) and timestepping details.

- Define a set of "standard parameters:" e.g., R = 400, Pr = 0.5, $Q = 1$, $\zeta = 10^{-3}$, $\mathcal{S} = 10^3$, $\mathcal{P} = 4$. All of our cuts through our multi-D parameter space cut through these parameters, varying one parameter ($\mathcal{S}$, $\mathcal{P}$, $\mathcal{R}$) and holding the rest constant. Note the different standard parameters for the erf (subscript E?) and linear (subscript L?) simulations.

- Forward ref an appendix on our initial conditions / accelerated evolution (appendix A) and also an appendix of tables of simulation details (appendix B).

4 Results

4.1 Qualitative Description of Simulations

- Description of the *evolved state* of a turbulent simulation. This is the "put in some pretty convective dynamics picture" section to orient people into what our simulations look like. And to show them that we get sims where the standard answer (edge of CZ is where $\boldsymbol{\nabla}_{\rm rad} = \boldsymbol{\nabla}_{\rm ad}$) does a really bad job of describing the final state. Dynamics picture will show: pretty CZ dynamics, a developed PZ with the sign of w and T reversed from the CZ.

- Here we'll also show one of our $|\boldsymbol{\nabla} T|$ plots to show that convection flattens $\boldsymbol{\nabla} \to \boldsymbol{\nabla}_{\rm ad}$ far above top of nominal CZ. This is a good time to re-explain the definitions of penetration vs overshoot in words again with a picture. This is also a good time to show pictorally what our measure of the height of the PZ is (currently it's where $\boldsymbol{\nabla}$ is 50% between $\boldsymbol{\nabla}_{\rm rad}$ and $\boldsymbol{\nabla}_{\rm ad}$). Could even label "CZ", "PZ", "OZ", "RZ" (convection, penetration, overshoot, radiative zones) on the profile picture.

4.2 Measured penetration zone scalings

- Time-averaged 1D profile plots showing verification of theory. Does W go from W_0 to $\mathcal{O}(0)$ at the top of the PZ? Is it an e-folding? Also how does the rms Pe or Re scale with height? Does Zahn's "thermal adjustment layer" with Pe ~ 1 hold in the OZ?
- Show that the predicted $\mathcal{P}$ scalings do a good job (think about what the relevant error bars are on our measures). This will include some parameter space plots (probably all in one figure) that show:
  - δ_P depends on $\mathcal{P}$.
  - δ_P does not really depend strongly on $\mathcal{S}$ or $\mathcal{R}$.
- An additional figure (like above) for the linear sims, where we only vary $\mathcal{P}$.
- Mention that 2D simulations produce more pronounced PZs (larger by a factor of ~ 2) but similar scalings with theory; we choose to omit 2D results and focus on more realistic 3D results.
- our theory has $\delta_P = A\mathcal{P}$ (for erf sims) and $\delta_P = B\mathcal{P}^{1/2}$; we should report good-fits for A and B. These can be plugged directly into MESA as first guesses as the penetration depth there.

5 A modified solar model

- Here we briefly describe a new MESA algorithm where we have a parameterization like $\delta_P/\ell = \xi B\mathcal{P}^{1/2}$ put into MESA, where ξ is a knob the user can turn and ℓ is the mixing length or some such. We have two options here: state that $\mathcal{P} = 1$ is always true, or have MESA do a simple linear interpolation of $\boldsymbol{\nabla}_{\text{rad}}$ over some depth like $\ell/2$ above and below the CZ boundary to get a value for $\mathcal{P}$.
- We probably can use the lame / naive assumption that below δ_P, MESA latches immediately onto $\boldsymbol{\nabla}_{\text{rad}}$ (this means there is no OZ; stars are stiff, after all). Assume everything is mixed within the full CZ-PZ.
- We calculate a few solar models with $\xi = [0.5, 1, 2]$ and show how these models change the structure at the base of the solar CZ.

6 Discussion

- Say what we said again. Penetration depth depends on how quickly F_{rad} changes with height.
- Discuss the subtleties associated with filling factors and how Boussinesq / incompressible can't really appropriately capture that. (And the confusion that Massaguer et al. (1984) have a model that suggests *more* penetration for downflows than upflows in stratified domains).
- Discuss open questions (rotation, stratification [above], magnetism, geometry).

- Round it off by calling back to how awesome it is that $\mathcal{P}$ totally determines δ_P in our work and that $\mathcal{S}$ and $\mathcal{R}$ don't, which suggests we can actually use simulations to crack this long-standing problem in stellar physics.

7 Appendix A: Accelerated Evolution

- Description of time evolution of simulation starting from "schwarzschild" ICs and how brutal the time evolution is.
- Description of newton iter-like procedure.
- Figure that shows things converging to the same answer to within a few %.

8 Appendix B: Table-o-sims

- Housekeeping; show resolutions, etc.

References

Asplund, M., Amarsi, A. M., & Grevesse, N. 2021, arXiv e-prints, arXiv:2105.01661. https://arxiv.org/abs/2105.01661

Bahcall, J. N., Serenelli, A. M., & Basu, S. 2005, ApJL, 621, L85, doi: 10.1086/428929

Basu, S., & Antia, H. M. 2004, ApJL, 606, L85, doi: 10.1086/421110

Browning, M. K., Brun, A. S., & Toomre, J. 2004, ApJ, 601, 512, doi: 10.1086/380198

Brummell, N. H., Clune, T. L., & Toomre, J. 2002, ApJ, 570, 825, doi: 10.1086/339626

Brun, A. S., Strugarek, A., Varela, J., et al. 2017, ApJ, 836, 192, doi: 10.3847/1538-4357/aa5c40

Burns, K. J., Vasil, G. M., Oishi, J. S., Lecoanet, D., & Brown, B. P. 2020, Physical Review Research, 2, 023068, doi: 10.1103/PhysRevResearch.2.023068

Cai, T. 2020, ApJ, 891, 49, doi: 10.3847/1538-4357/ab711c

Canuto, V. M. 2011, A&A, 528, A80, doi: 10.1051/0004-6361/201014450

Christensen-Dalsgaard, J., Monteiro, M. J. P. F. G., Rempel, M., & Thompson, M. J. 2011, MNRAS, 414, 1158, doi: 10.1111/j.1365-2966.2011.18460.x

Claret, A., & Torres, G. 2018, ApJ, 859, 100, doi: 10.3847/1538-4357/aabd35

Couston, L. A., Lecoanet, D., Favier, B., & Le Bars, M. 2017, Physical Review Fluids, 2, 094804, doi: 10.1103/PhysRevFluids.2.094804

Deardorff, J. W., Willis, G. E., & Lilly, D. K. 1969, Journal of Fluid Mechanics, 35, 7, doi: 10.1017/S0022112069000942

Dietrich, W., & Wicht, J. 2018, Frontiers in Earth Science, 6, 189, doi: 10.3389/feart.2018.00189

Dumont, T., Palacios, A., Charbonnel, C., et al. 2021, A&A, 646, A48, doi: 10.1051/0004-6361/202039515

Herwig, F. 2000, A&A, 360, 952. https://arxiv.org/abs/astro-ph/0007139

Higgins, E. R., & Vink, J. S. 2020, A&A, 635, A175, doi: 10.1051/0004-6361/201937374

Higl, J., Müller, E., & Weiss, A. 2021, A&A, 646, A133, doi: 10.1051/0004-6361/202039532
Hotta, H. 2017, ApJ, 843, 52, doi: 10.3847/1538-4357/aa784b
Hurlburt, N. E., Toomre, J., & Massaguer, J. M. 1986, ApJ, 311, 563, doi: 10.1086/164796
Hurlburt, N. E., Toomre, J., Massaguer, J. M., & Zahn, J.-P. 1994, ApJ, 421, 245, doi: 10.1086/173642
Jermyn, A. S., Tout, C. A., & Chitre, S. M. 2018, MNRAS, 480, 5427, doi: 10.1093/mnras/sty1831
Käpylä, P. J. 2019, A&A, 631, A122, doi: 10.1051/0004-6361/201834921
Käpylä, P. J., Korpi, M. J., Stix, M., & Tuominen, I. 2007, in Convection in Astrophysics, ed. F. Kupka, I. Roxburgh, & K. L. Chan, Vol. 239, 437–442, doi: 10.1017/S1743921307000865
Käpylä, P. J., Rheinhardt, M., Brandenburg, A., et al. 2017, ApJL, 845, L23, doi: 10.3847/2041-8213/aa83ab
Kitiashvili, I. N., Kosovichev, A. G., Mansour, N. N., & Wray, A. A. 2016, ApJL, 821, L17, doi: 10.3847/2041-8205/821/1/L17
Korre, L., Garaud, P., & Brummell, N. H. 2019, MNRAS, 484, 1220, doi: 10.1093/mnras/stz047
Lecoanet, D., Schwab, J., Quataert, E., et al. 2016, ApJ, 832, 71, doi: 10.3847/0004-637X/832/1/71
Maeder, A. 1975, A&A, 40, 303
Marcus, P. S., Press, W. H., & Teukolsky, S. A. 1983, ApJ, 267, 795, doi: 10.1086/160915
Martinet, S., Meynet, G., Ekström, S., et al. 2021, A&A, 648, A126, doi: 10.1051/0004-6361/202039426
Massaguer, J. M., Latour, J., Toomre, J., & Zahn, J. P. 1984, A&A, 140, 1
Moore, D. R., & Weiss, N. O. 1973, Journal of Fluid Mechanics, 61, 553, doi: 10.1017/S0022112073000868
Musman, S. 1968, Journal of Fluid Mechanics, 31, 343, doi: 10.1017/S0022112068000194
Paxton, B., Bildsten, L., Dotter, A., et al. 2011, ApJS, 192, 3, doi: 10.1088/0067-0049/192/1/3
Paxton, B., Cantiello, M., Arras, P., et al. 2013, ApJS, 208, 4, doi: 10.1088/0067-0049/208/1/4
Paxton, B., Schwab, J., Bauer, E. B., et al. 2018, ApJS, 234, 34, doi: 10.3847/1538-4365/aaa5a8
Paxton, B., Smolec, R., Schwab, J., et al. 2019, ApJS, 243, 10, doi: 10.3847/1538-4365/ab2241
Pinsonneault, M. 1997, Annual Review of Astronomy and Astrophysics, 35, 557, doi: 10.1146/annurev.astro.35.1.557
Pratt, J., Baraffe, I., Goffrey, T., et al. 2017, A&A, 604, A125, doi: 10.1051/0004-6361/201630362
Rempel, M. 2004, ApJ, 607, 1046, doi: 10.1086/383605
Rieutord, M. 2019, in EAS Publications Series, Vol. 82, EAS Publications Series, 153–165, doi: 10.1051/eas/1982015
Rogers, T. M., & Glatzmaier, G. A. 2005, ApJ, 620, 432, doi: 10.1086/423415
Rogers, T. M., Glatzmaier, G. A., & Jones, C. A. 2006, ApJ, 653, 765, doi: 10.1086/508482
Roxburgh, I. W. 1965, MNRAS, 130, 223, doi: 10.1093/mnras/130.3.223
—. 1978, A&A, 65, 281
—. 1989, A&A, 211, 361
—. 1992, A&A, 266, 291
—. 1998, Astrophysics and Space Science, 261, 43, doi: 10.1023/A:1002029105802
Saikia, E., Singh, H. P., Chan, K. L., Roxburgh, I. W., & Srivastava, M. P. 2000, ApJ, 529, 402, doi: 10.1086/308249
Schmitt, J. H. M. M., Rosner, R., & Bohn, H. U. 1984, ApJ, 282, 316, doi: 10.1086/162205
Shaviv, G., & Salpeter, E. E. 1973, ApJ, 184, 191, doi: 10.1086/152318
Singh, H. P., Roxburgh, I. W., & Chan, K. L. 1995, A&A, 295, 703
—. 1998, A&A, 340, 178

Tian, C.-L., Deng, L.-C., & Chan, K.-L. 2009, MNRAS, 398, 1011, doi: 10.1111/j.1365-2966.2009.15178.x

Toppaladoddi, S., & Wettlaufer, J. S. 2018, Physical Review Fluids, 3, 043501, doi: 10.1103/PhysRevFluids.3.043501

Viallet, M., Meakin, C., Prat, V., & Arnett, D. 2015, A&A, 580, A61, doi: 10.1051/0004-6361/201526294

Viani, L. S., & Basu, S. 2020, ApJ, 904, 22, doi: 10.3847/1538-4357/abba17

Vinyoles, N., Serenelli, A. M., Villante, F. L., et al. 2017, ApJ, 835, 202, doi: 10.3847/1538-4357/835/2/202

Zahn, J. P. 1991, A&A, 252, 179

Zhang, Q. S., & Li, Y. 2012, ApJ, 746, 50, doi: 10.1088/0004-637X/746/1/50