Draft version April 30, 2019
Typeset using LaTeX **twocolumn** style in AASTeX62

Entrainment of low Mach number thermals in stratified domains

Evan H. Anders,[1,2] Daniel Lecoanet,[3] and Benjamin P. Brown[1,2]

[1] *Dept. Astrophysical & Planetary Sciences, University of Colorado – Boulder, Boulder, CO 80309, USA*
[2] *Laboratory for Atmospheric and Space Physics, Boulder, CO 80303, USA*
[3] *Stuff*

(Received April 30, 2019; Revised April 30, 2019; Accepted ??)

Submitted to ApJ

ABSTRACT

Keywords: hydrodynamics — turbulence — entrainment

1. INTRODUCTION

(Para 1) Brief intro to solar convective conundrum.

(Para 2) Entropy rain history and expansion by Axel

(Para 3) Boussinesq thermals and their filling factor vs. depth.

(Para 4) What we're doing here, how it fits into this context.

(Para 5) Standard last paragraph of an intro?

2. THEORY

The evolution of thermals as buoyant vortex rings has been well described in the unstratified, Boussinesq limit for decades (as early as e.g., CITE, and see Lecoanet & Jeevanjee (2018) for other sources). Buoyant motions in the atmospheres of stars and planets are generally large enough to feel the atmospheric stratification, and therefore a more thorough tratment of the evolution of thermals in stratified domains is required to understand the nature of thermal entrainment in nature.

In this work, we focus on the non-dissipative, low Mach number regime, in which the ideal anelastic equations are a decent approximation to the fully compressible equations. In this regime, the buoyancy is directly proportional to the specific entropy. In the absence of diffusion, or in the limit where diffusivities are sufficiently small, the volume-integrated total entropy is constant,

$$B \equiv \int_{\mathcal{V}} \rho\, s\, dV = \text{const}, \tag{1}$$

Corresponding author: Evan H. Anders
evan.anders@colorado.edu

where $s = c_V \ln T - R \ln \rho$ is the specific entropy, with c_V the specific heat at constant volume and R the ideal gas constant.

In a stratified domain, the hydrodynamic impulse is defined Shivamoggi (2010),

$$\boldsymbol{I} = \frac{1}{2}\int_{\mathcal{V}} \boldsymbol{x} \times (\nabla \times (\rho \boldsymbol{u}))dV, \tag{2}$$

where $\boldsymbol{x}$ is the position vector, $\boldsymbol{u}$ is vector velocity, ρ is density, and $\mathcal{V}$ is the volume being integrated over. Furthermore, changes in the impulse can be expressed

$$\frac{\partial \boldsymbol{I}}{\partial t} = \int_{\mathcal{V}} \frac{\partial}{\partial t}(\rho \boldsymbol{u})dV = B\hat{z} + S, \tag{3}$$

where S is a combination of surface terms that disappears upon appropriate boundary conditions on $\mathcal{V}$ (Shivamoggi 2010). As we have assumed that the buoyancy, B, is constant, we can straightforwardly integrate

$$I_z = Bt + I_0, \tag{4}$$

for some constant I_0.

(need to work through this section carefully). Assuming an adiabatically stratified atmosphere in hydrostatic equilibrium, the vertical momentum equation is

$$= -\partial_z \varpi + g\frac{S_1}{c_P}, \tag{5}$$

where w is vertical velocity, S_1 is specific entropy, ϖ is the reduced pressure, $D/Dt = \partial_t + (\boldsymbol{u}\cdot\nabla)$ is the (eulerian? lagrangian?) derivative, g is gravity, and c_P is the specific heat at constant pressure. In the anelastic approximation, the density stratification is constant in time. OH GOD I DON'T KNOW THIS MATH WELL

MAGIC MAGIC MAGIC. By doing magic, we arrive at

$$P_z = \beta B t + P_0. \tag{6}$$

Under the assumption that the thermal develops into a thin-core propagating vortex ring whose vortex core is radius r away from the axis of symmetry, the impulse can be approximated as $I_z \approx \pi \rho r^2 \Gamma$, where Γ is the circulation of the thermal vortex, which we assume to be constant. Rearranging, we find our first result,

$$r = \sqrt{\frac{Bt + I_0}{\pi \rho \Gamma}}. \tag{7}$$

In the limit where $\rho \rightarrow$ constant, as in the Boussinesq regime, we retrieve the $r \propto \sqrt{t}$ scaling found in the Boussinesq regime by Lecoanet & Jeevanjee (2018). We find that the inclusion of stratification adds the additional complexity of $r \propto \rho^{-1/2}$, such that downward-propogating vortex rings (as studied here) will not entrain to the same degree as boussinesq thermals, and upward-propogating rings will entrain more.

We further assume that the volume-integrated momentum, $P_z \approx \rho V w_{th}$, where w_{th} is the vertical velocity of the thermal as a whole, and the volume of the fluid region propogating with the thermal is a spheroid, $V = V_0 r^3$, for some constant V_0. Plugging Eqn. 7 into this approximation, we find that

$$\rho^{-1/2} w = \left(\frac{(\pi\Gamma)^{3/2}}{V_0}\right) \frac{\beta B t + M_0}{(Bt + I_0)^{3/2}}. \tag{8}$$

Substituting $w = dz/dt$ and $\tau \equiv (Bt+I_0)/\Gamma$, We retrieve

$$\frac{dz}{\sqrt{\rho(z)}} = \left(\frac{\pi^{3/2}\Gamma}{V_0}\right)\left(\beta\Gamma\tau^{-1/2} + [M_0 - \beta I_0]\tau^{-3/2}\right) d\tau. \tag{9}$$

If the atmospheric stratification in which the thermal is falling is known, this result can be straightforwardly integrated with $\rho(z)$ plugged in in order to find the position of the thermal versus time. While we will leave this result general for now, we will plug in our polytropic stratification at the end of section 3, and show that the resulting expression does a good job of explaining the evolution of thermals in these atmospheres in section 4.

3. EXPERIMENT

blah blah blah polytrope blah blah blah (do it with a general length scale).

blah blah blah 2D anelastic blah blah blah, nondimm'd for a direct comparison with boussinesq.

blah blah blah 3D Fully compressible blah blah blah

blah blah blah thermal initial conditions, pressure equilibrium blah blah blah.

blah blah blah comparison of 2D and 3D blah blah blah. z v t, r v z, w v z. Also side-by-side pictoral comparison with diff.

blah blah blah T(t) and derivative blah blah blah.

4. RESULTS

Picture of a big old grid of z v t, r v z, w v z.

Pretty picture showing thermal evolution (comparing low and high stratification).

Verification of theory by simulations

Table of found parameters for the fits

Two regimes: stalling and falling.

5. DISCUSSION

Wild speculation about extensions to the solar regime. Do things on the sun shrink to the point where they viscously dissipate?

Talk about what would happen if we were to study up-thermals.

Extensions, and the fact that we trust these results will likely hold in the solar regime.

This work was supported by NASA Headquarters under the NASA Earth and Space Science Fellowship Program – Grant 80NSSC18K1199. This work was additionally supported by NASA LWS grant number NNX16AC92G. Computations were conducted with support by the NASA High End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center on Pleiades with allocation GID s1647.

APPENDIX

A. THERMAL TRACKING

B. TABLE OF SIMULATIONS

REFERENCES

Lecoanet, D., & Jeevanjee, N. 2018, arXiv e-prints, arXiv:1804.09326

Shivamoggi, B. K. 2010, Physics Letters A, 374, 4736