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Entrainment of low Mach number thermals in stratified atmospheres

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ABSTRACT

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1. INTRODUCTION

2. EXPERIMENT

We perform direct numerical simulations of the evolution of dry thermals in an ideal gas whose pressure (P), temperature (T), and density (ρ) follow a nondimensional ideal gas law of $P = \rho T$. We evolving the temperature, log density ($\ln\rho$), and velocity vector ($\boldsymbol{u} = u\hat{x} + v\hat{y} + z\hat{z}$) according to the fully compressible Navier-Stokes equations,

$$\frac{D\ln\rho}{Dt} = -\nabla\cdot(\boldsymbol{u}) \tag{1}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla T - T\nabla\ln\rho + \boldsymbol{g} + \nabla\cdot\left(\bar{\bar{\boldsymbol{\Pi}}}\right), \tag{2}$$

$$\frac{DT}{Dt} + (\gamma-1)T\nabla\cdot(\boldsymbol{u}) + \frac{1}{\rho c_V}\nabla\cdot(-\rho\chi\nabla T) = \frac{1}{\rho c_V}(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla)\cdot\boldsymbol{u}, \tag{3}$$

where χ is the thermal diffusivity and with the viscous stress tensor given by

$$\Pi_{ij} \equiv \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right), \tag{4}$$

where δ_{ij} is the Kronecker delta and with $\mu = \rho\nu$ is the dynamic viscosity and ν is the kinematic viscosity. We split our thermodynamics into a time-stationary adiabatic background component and a time-evolving fluctuation away from that background as $T = T_0(z) + T_1(x,y,z,t)$ and $\ln\rho = \ln\rho_0(z) + \ln\rho_1(x,y,z,t)$. We specify the background atmosphere as an adiabatic polytrope, as in **?**, so that

$$T_0(z) = (1 + L_z - z), \qquad \ln\rho_0(z) = m_{ad}\ln(T_0), \tag{5}$$

where L_z is the domain depth, and $m_{ad} = 1/(\gamma-1) = 1.5$ is the adiabatic polytropic index. In the construction of these atmospheres, we assume that gravity $\boldsymbol{g} = -g\hat{z}$ is everywhere constant and that the background atmosphere is in hydrostatic equilibrium such that $\partial_z T_0 + T_0\partial_z\ln\rho_0 - g = 0$.

We assume a Pradtl number of unity, $\mathrm{Pr} = \nu/\chi = 1$, everywhere in the domain. We furthermore assume that ν and χ are uniform everywhere in the domain, and that a negligibly small diffusivity, $\chi_B = 0$ acts on the pure background terms in the thermal flux such that

$$\frac{1}{\rho_0 c_V}\nabla\cdot(-\rho\chi_B\nabla T) = 0.$$

Under all of these assumptions, the equations we solve become

$$\frac{D\ln\rho_1}{Dt} = -\nabla\cdot(\boldsymbol{u}) - w\partial_z\ln\rho_0 \tag{6}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla T_1 - T_0\nabla\ln\rho_1 - T_1\partial_z\rho_0\hat{z} - T_1\nabla\ln\rho_1 + \nabla\cdot\left(\bar{\bar{\boldsymbol{\Pi}}}\right), \tag{7}$$

$$\frac{DT_1}{Dt} + w\partial_z T_0 + (\gamma-1)(T_0+T_1)\nabla\cdot(\boldsymbol{u}) - \frac{1}{c_V}\left\{\chi\left(\nabla^2 T_1 + \partial_z\ln\rho_0\partial_z T_1 + \partial_z\ln\rho_1\partial_z T_0 + \nabla\ln\rho_1\cdot\nabla T_1\right)\right\} = \frac{1}{\rho c_V}(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla)\cdot\boldsymbol{u}, \tag{8}$$

3. RESULTS

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4. DISCUSSION

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APPENDIX

A. TABLE OF SIMULATIONS