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1. STATE OF THE PROBLEM

In these thermal simulations, as far as I can tell, we have *eight* variables

1. B, the integrated total entropy leading to buoyancy
2. V, the thermal volume
3. R, the radius of the thermal
4. h, the thermal's vertical extent ('radius' in z-direction)
5. w, the thermal bulk vertical velocity
6. z, the height of the thermal
7. Γ, the circulation of the vortex ring
8. ρ, the local density at the height of the thermal.
9. t, time.

From the simulations we've done so far, when viscous heating is neglected, we have the following understanding of thermals:

1. $B = \text{const} \propto \rho s_1$
2. $h = AR$, where A is a constant.
3. $V = (4/3)\pi h R^2 = (4/3)\pi A R^3 = m R^3$.
4. $w = \frac{\partial z}{\partial t}$
5. $\rho = (1 + L_z - z)^{1.5}$ (for our polytropes)
6. $\rho V w \sim Bt$ (momentum linearly increases)
7. $\rho \pi R^2 \Gamma \sim Bt$ (e.g., eqn 36 of Shivamoggi 2010).

That's 9 unknowns, and 7 equations. There's two pieces of the puzzle we're missing. Ideally, $\Gamma = \text{const}$, and then we're left with one more equation, but we can't even be sure that's the case.

2. PUTTING THE PIECES TOGETHER IN THE BOUSSINESQ-Y LIMIT

In the boussinesq limit, we can make two more assumptions:

1. $\Gamma = \text{const}$
2. $\rho = \text{const}$.

By these assumptions, eqn 7 above gives us $R \propto t^{1/2}$. Plugging that in to eqn 6 gives $w \propto t^{-1/2}$, and thus $z(t) \propto t^{1/2}$.

3. WHAT DO WE NEED TO DO IN THE STRATIFIED LIMIT?

Come up with two more constraints. We need to understand how and why Γ changes with time, and we need to understand how w, or r, or z varies with time.

REFERENCES

Shivamoggi, B. K. 2010, Physics Letters A, 374, 4736