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INTERNALLY HEATED, STRATIFIED, COMPRESSIBLE CONVECTION

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ABSTRACT

An abstract will go here eventually

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1. INTRODUCTION

People who study the Sun don't understand what convective velocities are doing. They're either way smaller than simulations and mixing length theory predict (Hanasoge et al. 2012), or they are roughly aligned with what we would expect (Greer et al. 2015). Power spectra at the solar surface show clear signals at granular and supergranular scales, but other predicted length scales, most notably giant cells, are missing from the surface power spectrum (Hathaway et al. 2015). Understanding how convection is driven in stellar interiors is important in constructing proper models of stellar evolution and structure. Thus, this convective conundrum must be figured out.

Recently, Brandenburg (2016) extended stellar mixing length theory to include an additional flux term which does not depend on the local entropy gradient but rather parameterizes the nonlocal flux carrying of convective downdrafts. This Deardorff flux, if sufficiently strong, could be the bulk carrier of convective flux through an adiabatic lower convection zone, and then large "giant cells" won't be driven there Lord et al. (2014). Recently, Käpylä et al. (2017) studied penetrative convection in simulations with realistic opacities. In these simulations, they reported "Deardorff zones," or portions of the convective domain in which enthalpy flux points upwards but the entropy gradient is positively and is nominally stable to convection. They show that, for a specific simulation, classical penetrative convection, such as that studied by Hurlburt et al. (1986), does not physically capture the same mechanisms. They conclude that a realistic, Kramers-like opacity is required to study Deardorff zones.

Studies of internally heated boussinesq convection show that for the proper boundary conditions, stable layers can be achieved (Goluskin & van der Poel 2016). Here we show that, using simple principles studies in well-understood Boussinesq convection, stable layers are simple to achieve in internally heated atmospheres with a simple (constant) opacity profile. In these atmospheres, we see a reduction of power at the surface of the atmosphere due to the stably stratified Deardorff zone in the lower convective domain. Deardorff zones naturally arise in these systems, and the extent of the Deardorff zone can be understood from the initial conditions of the atmosphere.

2. EXPERIMENT

We study direct numerical simulations of an ideal gas whose equation of state is $P = \rho T$ and whose adiabatic index is $\gamma = 5/3$ by evolving the fully compressible

Navier-Stokes equations,

$$\frac{\partial \ln\rho}{\partial t} + \nabla\cdot\boldsymbol{u} = -\boldsymbol{u}\cdot\nabla\ln\rho, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} = -\nabla T - T\nabla\ln\rho + \boldsymbol{g} - \nabla\cdot\bar{\bar{\boldsymbol{\Pi}}}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u}\cdot\nabla T + (\gamma-1)T\nabla\cdot\boldsymbol{u} + \frac{1}{\rho c_V}\nabla\cdot(-\kappa\nabla T) = -(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla)\cdot\boldsymbol{u} + \kappa H, \tag{3}$$

with the viscous stress tensor given by

$$\Pi_{ij} \equiv -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right), \tag{4}$$

where δ_{ij} is the Kronecker delta function. In Eq. 3, H specifies the magnitude of internal heating, and in this study we study H which is constant in space and time. We assume here that κ and μ, the thermal conductivity and dynamic viscosity, are constant in space and time.

The initial atmosphere is constructed under the assumptions of hydrostatic equlibrium and thermal equilibrium in the presence of constant gravity, $g = (\gamma-1)^{-1} + 1 - \epsilon$, where ϵ is a control parameter which sets the superadiabaticity and is similar to the superadiabatic excess in polytropic atmospheres Anders & Brown (2017); Graham (1975). An atmosphere which satisfies these initial conditions takes the form

$$T_0(z) = -\frac{H}{2}z^2 + (HL_z - 1)z + \left(1 - \frac{H}{2}L_z^2 + L_z\right),$$
$$P_0(z) = \left(\frac{\xi + \nabla T_0}{\xi - \nabla T_0}\cdot\frac{\xi+1}{\xi-1}\right)^{g/\xi}, \tag{5}$$

where L_z is the depth of the atmosphere, $\xi \equiv \sqrt{1+2H}$, $\nabla T_0 = \partial_z T_0(z) = -Hz + (H_L z - 1)$, and the density profile is $\rho_0(z) = P_0(z)/T_0(z)$.

Stratified systems evolve towards a characteristic adiabatic profile. An adiabatically stratified atmosphere composed of an ideal gas in hydrostatic equilibrium has a temperature gradient specified by the gravity, $\nabla T_{ad} = -\boldsymbol{g}/c_P$, where $c_P = \gamma/(\gamma-1)$. In these internally heated systems,

$$\nabla T_0 - \nabla T_{ad} = H(L_z - z) - \frac{\epsilon}{c_P}, \tag{6}$$

and there is a special point in the initial atmosphere, $z_{\text{cross}} \equiv L_z - \epsilon/Hc_P$, at which the temperature gradient is exactly the adiabatic temperature gradient. Above that point, the temperature gradient is superadiabatic and unstable to convection. Below that point, the temperature gradient is subadiabatic. Thus, the depth of the region that is convectively unstable is $d_{\text{conv}} = L_z - z_{\text{cross}} = \epsilon/Hc_P$. From this, we retrieve the magnitude of the internal heating term,

$$H \equiv \frac{\epsilon}{d_{\text{conv}}c_P}. \tag{7}$$

If $L_z <= d_{\text{conv}}$, the whole atmosphere is unstable or marginally stable. If $L_z > d_{\text{conv}}$, there is a stable radiative zone beneath the convective zone. We specify the depth of this radiative zone through a new parameter, $r \equiv L_z/d_{\text{conv}} - 1$. We specify the depth of the convective zone by specifying the number of density scale heights, n_ρ, it spans. To achieve this we use an iterative, root finding algorithm find when $f(L_z) = \rho_0(z_{\text{cross}})/\rho_0(L_z) - e^{n_\rho}$ is zero.

Diffusivities in the system are specified by choosing a value of the Rayleigh Number and Prandtl number. The thermal diffusivity, $\chi = \kappa/\rho$ and viscous diffusivity, $\nu = \mu/\rho$ are constrained by

$$\text{Ra}(z) = \frac{g d_{\text{conv}}^4 \left|\nabla s/c_P\right|}{\nu\chi} = \frac{g d_{\text{conv}}^4}{\kappa\mu}\left|\frac{\nabla s}{c_P}(z)\right|\rho_0^2(z),$$
$$\text{Pr} = \frac{\nu}{\chi}, \tag{8}$$

where

$$\frac{\nabla s}{c_P} = \frac{1}{\gamma}\nabla\ln T - \frac{\gamma-1}{\gamma}\nabla\ln\rho. \tag{9}$$

We specify the value of Ra at the first moment of the $\nabla T - \nabla T_{ad} = T\nabla s/c_P$,

$$L_{sm1} = \frac{\int_{z_{\text{cross}}}^{L_z} zT\nabla s dz}{\int_{z_{\text{cross}}}^{L_z} T\nabla s dz}. \tag{10}$$

In the limit of classic, polytropic atmospheres, this reduces to the midplane of the atmosphere, which is a commonly chosen location to specify the value of Ra (Hurlburt et al. 1984). We choose this location as it minimizes the variation of the critical value of Ra as other parameters (n_ρ, r, ϵ) change.

2.1. *Stability*

We decompose thermodynamic variables such that $\ln\rho = (\ln\rho)_0 + (\ln\rho)_1$ and $T = T_0 + T_1$. We assume that the background terms are constant with respect to time, and this allows us to subtract out the background thermal equilibrium and hydrostatic equilibrium. The *linearized* equations of motion are then

$$\frac{\partial(\ln\rho)_1}{\partial t} + \nabla\cdot\boldsymbol{u} + \boldsymbol{u}\cdot\nabla(\ln\rho)_0 = 0$$
$$\frac{\partial\boldsymbol{u}}{\partial t} + \nabla T_1 + T_1\nabla(\ln\rho)_0 + T_0\nabla(\ln\rho)_1 + \nabla\cdot\bar{\bar{\boldsymbol{\Pi}}} = 0$$
$$\frac{\partial T_1}{\partial t} + \boldsymbol{u}\cdot\nabla T_0 + (\gamma-1)T_0\nabla\cdot\boldsymbol{u} - \kappa e^{-(\ln\rho)_0}\nabla^2 T_1 = 0 \tag{11}$$

We assume that all fluctuations $\{T_1, (\ln\rho)_1, \boldsymbol{u}\} = f(z)g(x,y)e^{i\omega t}$, and we use Dedalus to solve eigenvalue

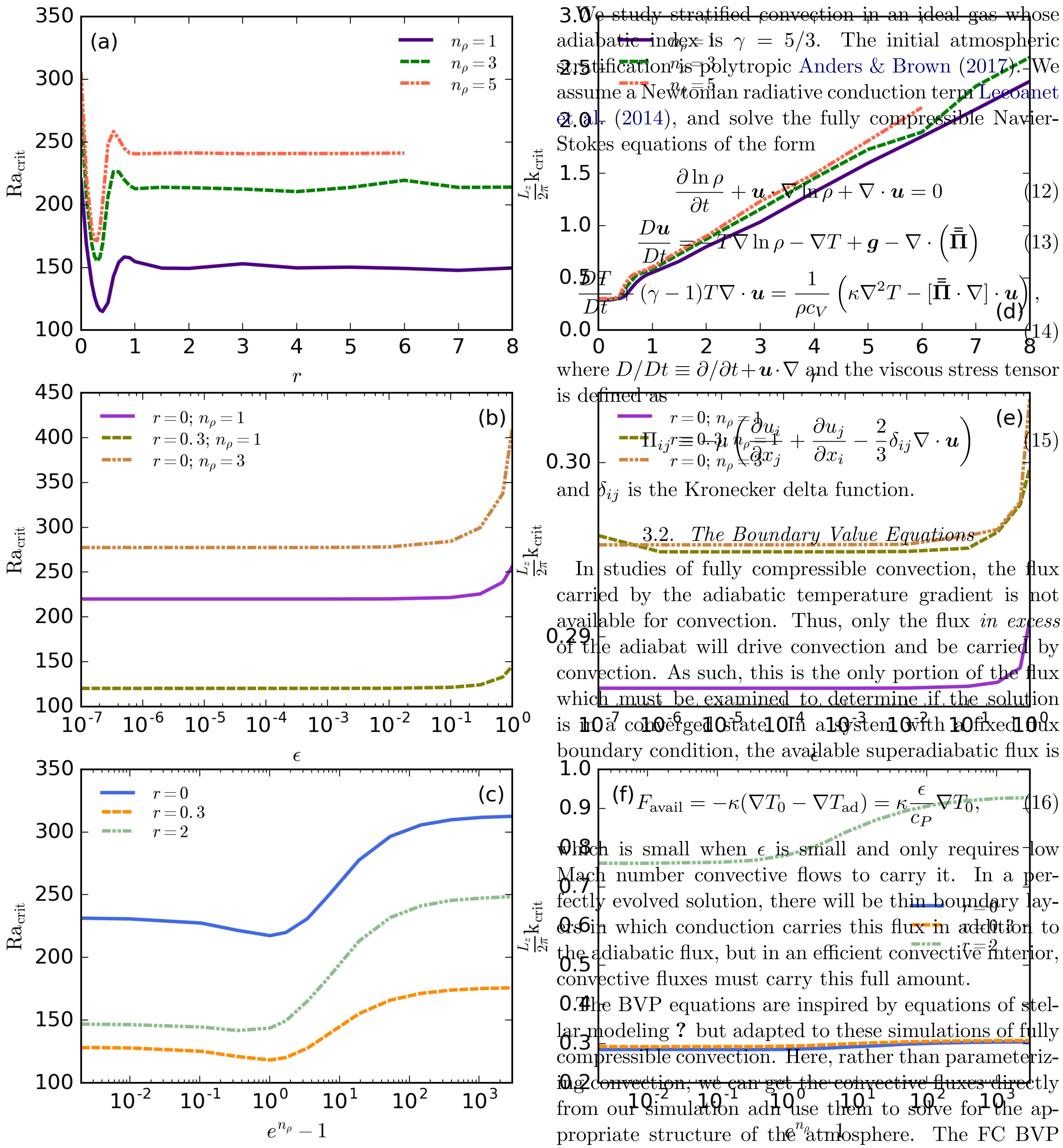

Figure 1. The value of the critical Rayleigh number and normalized critical wavenumber as the control parameters of the problem are varied.

problems to determine when $\omega = 0$. From this, we find Fig. 1.

3. FULLY COMPRESSIBLE CONVECTION

3.1. *Governing Equations*

We study stratified convection in an ideal gas whose adiabatic index is $\gamma = 5/3$. The initial atmospheric stratification is polytropic Anders & Brown (2017). We assume a Newtonian radiative conduction term Lecoanet et al. (2014), and solve the fully compressible Navier-Stokes equations of the form

$$\frac{\partial \ln \rho}{\partial t} + \boldsymbol{u}\cdot\nabla \ln \rho + \nabla\cdot\boldsymbol{u} = 0 \tag{12}$$

$$\frac{D\boldsymbol{u}}{Dt} = -T\nabla \ln \rho - \nabla T + \boldsymbol{g} - \nabla\cdot\left(\bar{\bar{\boldsymbol{\Pi}}}\right) \tag{13}$$

$$\frac{DT}{Dt} + (\gamma - 1)T\nabla\cdot\boldsymbol{u} = \frac{1}{\rho c_V}\left(\kappa\nabla^2 T - [\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla]\cdot\boldsymbol{u}\right), \tag{14}$$

where $D/Dt \equiv \partial/\partial t + \boldsymbol{u}\cdot\nabla$ and the viscous stress tensor is defined as

$$\Pi_{ij} \equiv -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right) \tag{15}$$

and δ_{ij} is the Kronecker delta function.

3.2. *The Boundary Value Equations*

In studies of fully compressible convection, the flux carried by the adiabatic temperature gradient is not available for convection. Thus, only the flux *in excess* of the adiabat will drive convection and be carried by convection. As such, this is the only portion of the flux which must be examined to determine if the solution is in a converged state. In a system with a fixed flux boundary condition, the available superadiabatic flux is

$$F_{\mathrm{avail}} = -\kappa(\nabla T_0 - \nabla T_{\mathrm{ad}}) = \kappa\frac{\epsilon}{c_P}\nabla T_0, \tag{16}$$

which is small when ϵ is small and only requires low Mach number convective flows to carry it. In a perfectly evolved solution, there will be thin boundary layers in which conduction carries this flux in addition to the adiabatic flux, but in an efficient convective interior, convective fluxes must carry this full amount.

The BVP equations are inspired by equations of stellar modeling **?** but adapted to these simulations of fully compressible convection. Here, rather than parameterizing convection, we can get the convective fluxes directly from our simulation and use them to solve for the appropriate structure of the atmosphere. The FC BVP equations are

$$\frac{dM_1}{dz} = \rho_1 \tag{17}$$

$$T_0\nabla\rho_1 + T_1\nabla\rho_0 + \rho_1 g = -T_0\nabla\rho_0 - T_1\nabla\rho_1 - \rho_0 g \tag{18}$$

$$\kappa\frac{d^2 T_1}{dz^2} = -\frac{d}{dz}F_{\mathrm{conv,\ z}}, \tag{19}$$

which ensure mass conservation, thermal equilibrium, and that the atmosphere is, on average, in hydrostatic equilibrium. We couple these equations with four boundary conditions (mixed flux / temperature boundary conditions, as well as setting $M_1 = 0$ at the top and bottom of the atmosphere).

3.3. *Results*

4. RESULTS & DISCUSSION

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