Internally heated, stratified, compressible convection

Evan H. Anders and Benjamin P. Brown

Dept. Astrophysical & Planetary Sciences, University of Colorado – Boulder, Boulder, CO 80309, USA and
Laboratory for Atmospheric and Space Physics, Boulder, CO 80303, USA

An abstract will go here eventually

I. INTRODUCTION

1. Importance of internal heating in natural convective systems

2. Shape of internal heating in something like Sun

3. Studies in Rayleigh-Benard

4. Goals of this paper:

   (a) Show how to set up internally heated polytropes, with stable layers (or without)

   (b) Show how the resultant stratification depends on combo of lower flux + internal heating profile choices

   (c) Show how stable layer affects surface power spectrum

II. EXPERIMENT

We study direct numerical simulations of an ideal gas whose equation of state is $P = \rho T$ and whose adiabatic index is $\gamma = 5/3$ by evolving the fully compressible Navier-Stokes equations,

$$\frac{\partial \ln \rho}{\partial t} + \nabla \cdot \boldsymbol{u} = -\boldsymbol{u} \cdot \nabla \ln \rho, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla T - T\nabla \ln \rho + \boldsymbol{g} - \nabla \cdot \bar{\bar{\boldsymbol{\Pi}}}, \tag{2}$$

$$\frac{\partial T}{\partial t} + \boldsymbol{u} \cdot \nabla T + (\gamma - 1)T\nabla \cdot \boldsymbol{u} + \frac{1}{\rho c_V}\nabla \cdot (-\kappa \nabla T) = -(\bar{\bar{\boldsymbol{\Pi}}} \cdot \nabla) \cdot \boldsymbol{u} + \kappa H, \tag{3}$$

with the viscous stress tensor given by

$$\Pi_{ij} \equiv -\mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla \cdot \boldsymbol{u} \right), \tag{4}$$

where δ_{ij} is the Kronecker delta function. In Eq. 3, H specifies the magnitude of internal heating, and in this study we study H which is constant in space and time. We assume here that κ and μ, the thermal conductivity and dynamic viscosity, are constant in space and time.

The initial atmosphere is constructed under the assumptions of hydrostatic equilibrium and thermal equilibrium in the presence of constant gravity, $g = (\gamma - 1)^{-1} + 1 - \epsilon$, where ϵ is a control parameter which sets the superadiabaticity and is similar to the superadiabatic excess in polytropic atmospheres [1, 2]. An atmosphere which satisfies these initial conditions takes the form

$$T_0(z) = -\frac{H}{2}z^2 + (HL_z - 1)z + \left(1 - \frac{H}{2}L_z^2 + L_z\right), \qquad P_0(z) = \left(\frac{\xi + \nabla T_0}{\xi - \nabla T_0} \cdot \frac{\xi + 1}{\xi - 1}\right)^{g/\xi}, \tag{5}$$

where L_z is the depth of the atmosphere, $\xi \equiv \sqrt{1 + 2H}$, $\nabla T_0 = \partial_z T_0(z) = -Hz + (H_L z - 1)$, and the density profile is $\rho_0(z) = P_0(z)/T_0(z)$.

Stratified systems evolve towards a characteristic adiabatic profile. An adiabatically stratified atmosphere composed of an ideal gas in hydrostatic equilibrium has a temperature gradient specified by the gravity, $\nabla T_{ad} = -\boldsymbol{g}/c_P$, where $c_P = \gamma/(\gamma - 1)$. In these internally heated systems,

$$\nabla T_0 - \nabla T_{ad} = H(L_z - z) - \frac{\epsilon}{c_P}, \tag{6}$$

and there is a special point in the initial atmosphere, $z_{\text{cross}} \equiv L_z - \epsilon/Hc_P$, at which the temperature gradient is exactly the adiabatic temperature gradient. Above that point, the temperature gradient is superadiabatic and unstable to convection. Below that point, the temperature gradient is subadiabatic. Thus, the depth of the region that is convectively unstable is $d_{\text{conv}} = L_z - z_{\text{cross}} = \epsilon/Hc_P$. From this, we retrieve the magnitude of the internal heating term,

$$H \equiv \frac{\epsilon}{d_{\text{conv}}c_P}. \tag{7}$$

If $L_z <= d_{\text{conv}}$, the whole atmosphere is unstable or marginally stable. If $L_z > d_{\text{conv}}$, there is a stable radiative zone beneath the convective zone. We specify the depth of this radiative zone through a new parameter, $r \equiv L_z/d_{\text{conv}} - 1$. We specify the depth of the convective zone by specifying the number of density scale heights, n_ρ, it spans. To achieve this we use an iterative, root finding algorithm find when $f(L_z) = \rho_0(z_{\text{cross}})/\rho_0(L_z) - e^{n_\rho}$ is zero.

Diffusivities in the system are specified by choosing a value of the Rayleigh Number and Prandtl number. The thermal diffusivity, $\chi = \kappa/\rho$ and viscous diffusivity, $\nu = \mu/\rho$ are constrained by

$$\text{Ra}(z) = \frac{gd_{\text{conv}}^4 \left|\nabla s/c_P\right|}{\nu\chi} = \frac{gd_{\text{conv}}^4}{\kappa\mu} \left|\frac{\nabla s}{c_P}(z)\right| \rho_0^2(z), \qquad \text{Pr} = \frac{\nu}{\chi}, \tag{8}$$

where

$$\frac{\nabla s}{c_P} = \frac{1}{\gamma}\nabla \ln T - \frac{\gamma - 1}{\gamma}\nabla \ln \rho. \tag{9}$$

We specify the value of Ra at the first moment of the $\nabla T - \nabla T_{ad} = T\nabla s/c_P$,

$$L_{sm1} = \frac{\int_{z_{\text{cross}}}^{L_z} zT\nabla s\, dz}{\int_{z_{\text{cross}}}^{L_z} T\nabla s\, dz}. \tag{10}$$

In the limit of classic, polytropic atmospheres, this reduces to the midplane of the atmosphere, which is a commonly chosen location to specify the value of Ra [3]. We choose this location as it minimizes the variation of the critical value of Ra as other parameters (n_ρ, r, ϵ) change.

III. RESULTS & DISCUSSION

---

[1] E. H. Anders and B. P. Brown, "Convective heat transport in stratified atmospheres at low and high Mach number," Physical Review Fluids **2**, 083501 (2017), arXiv:1611.06580 [physics.flu-dyn].

[2] E. Graham, "Numerical simulation of two-dimensional compressible convection," J. Fluid Mech. **70**, 689–703 (1975).

[3] N. E. Hurlburt, J. Toomre, and J. M. Massaguer, "Two-dimensional compressible convection extending over multiple scale heights," Astrophys. J. **282**, 557–573 (1984).