I. WORK ON UNDERSTANDING KRAMERS OPACITY ATMOSPHERES (E.G., [1])

In this work, we have a two-layer atmosphere: an unstable layer lying above a stable layer. Let's basicallly try to make the exact same experiment as Kapyla did, so let's assume that the atmosphere has polytropic stratification, or

$$T = \begin{cases} T_b + \nabla T_{RZ} z, & z <= L_{RZ} \\ T_m + \nabla T_{CZ}(z - L_{RZ}), & z > L_{RZ} \end{cases}, \qquad \rho = \begin{cases} \rho_b \left(1 + \frac{\nabla T_{RZ}}{T_b} z\right)^{m_{RZ}}, & z <= L_{RZ} \\ \rho_m \left(1 + \frac{\nabla T_{CZ}}{T_m}(z - L_{RZ})\right)^{m_{CZ}}, & z > L_{RZ} \end{cases}. \tag{1}$$

There's a lot of different variables in there, so let's break them down a bit. T_b and T_m are the temperature values at the bottom (b) and match (m) points in the atmosphere. In other words, they're the temperatures at the bottom of the RZ and the bottom of the CZ, respectively. ρ_b and ρ_m are the density analogues of those two. L_{RZ} is the geometric depth of the radiative zone. ∇T_{RZ} and ∇T_{CZ} are the temperature gradients of the two polytropes. m_{RZ} and m_{CZ} are the polytropic indices of the two layers. Thanks to polytropes having some really easy-to-understand properties, here's some definitions on some of the variables we have kicking around:

$$\begin{aligned}
T_m &= T_b e^{-n_{\rho,RZ}/m_{RZ}} \\
\rho_m &= \rho_b e^{-n_{\rho,RZ}} \\
L_{RZ} &= \frac{T_b}{|\nabla T_{RZ}|}\left(e^{-n_{\rho,RZ}/m_{RZ}} - 1\right) \\
L_{CZ} &= \frac{T_b}{|\nabla T_{CZ}|} e^{-n_{\rho,RZ}/m_{RZ}}\left(e^{-n_{\rho,CZ}/m_{CZ}} - 1\right) \\
\nabla T_{CZ} &= \frac{\boldsymbol{g}}{R(1+m_{CZ})} \\
\nabla T_{RZ} &= \frac{\boldsymbol{g}}{R(1+m_{RZ})} \\
\nabla T_{ad} &= \boldsymbol{g}/c_P.
\end{aligned} \tag{2}$$

OK, so that takes care of a lot of things, and if we specify the number of density scale heights in both parts of the atmosphere, the polytropic indicies, the temperature and density at the bottom of the atmosphere, and the profile of gravity as a function of height, then we end up with a fully specified stitched polytrope.

The interesting new thing that makes setting up this problem a mess is the radiative conductivity profile,

$$\kappa(z) = \kappa_0 \rho^{-(1+a)} T^{3-b}, \tag{3}$$

where $a = 1$ and $b = -7/2$ for free-free interactions. Note that if we have a constant gravity $\boldsymbol{g} = -\text{constant} \cdot \hat{z}$, then setting $m = (3-b)/(1+a)$ will produce a stratification that has a constant conductivity.

[1] Petri J. Käpylä, Matthias Rheinhardt, Axel Brandenburg, Rainer Arlt, Maarit J. Käpylä, Andreas Lagg, Nigul Olspert, and Jörn Warnecke, "Extended Subadiabatic Layer in Simulations of Overshooting Convection," Astrophys. J. **845** (2017), 10.3847/2041-8213/aa83ab.