I. THE EXPERIMENT – SINGLE LAYER ATMOSPHERE

Our initial conditions are an adiabatic polytrope,

$$T = L_z + 1 - z \qquad \rho = T^{m_{ad}},$$

where $L_z = np.exp(m_{ad}/n_r ho) - 1$ is the depth, $z = [0, L_z]$, and $m_{ad} = (\gamma - 1)^{-1}$. The opacity is a Kramer's opacity [1] such that the radiative conductivity is

$$\kappa(z) = \kappa_0 \left(\frac{\rho}{\rho_c}\right)^{-(1+a)} \left(\frac{T}{T_c}\right)^{(3-b)},$$

where $a = 1$, $b = -7/2$, and the thermal diffusivity is $\chi = \kappa/(\rho c_P)$. In general, for a polytropic atmosphere, $\rho \propto T^m$, there is a polytropic index that achieves a constant Kramer's opacity [2]

$$m_{kram} = \frac{3-b}{1+a}.$$

We hypothesize that, in general, if m_{kram} differs from m_{ad} by O(ϵ), then the initially adiabatic atmosphere will drive convective motions like the ϵ parameter we published previously [3]. In this work, we will fix $a = 1$, and study $b = (0, -3.5]$, from very small $b = -10^{-4}$, up to $b = -1$ and $b = -3.5$, the true value of b for free-free interactions, deep in the Sun. For our choice of $a = 1$,

$$m_{kram} = 1.5 - b/2 = m_{ad} - b/2. \tag{1}$$

So b will be one of our control parameters in our experiment. In general, for our polytropic initial conditions, and with a radiative flux of $F = -\kappa_0 \rho^{-(1+a)} T^{(3-b)} \nabla T$,

$$\frac{F_{top}}{F_{bot}} = \frac{-\kappa_0 \cdot 1 \cdot 1 \cdot -1}{-\kappa_0 \cdot e^{-(1+a)n_\rho}(L_z+1)^{(3-b)} \cdot -1} = e^{(1+a)n_\rho - (3-b)n_\rho/m_{ad}} \tag{2}$$

Assuming that the amount of the flux that needs to be carried at the top of the atmosphere is proportional to the size of entropy perturbations that carry the flux, and that the flux at the bottom is the fixed flux that must be transported through the atmosphere, then I assume that the characteristic entropy perturbations are of the order

$$|S| \sim \left|1 - e^{(1+a)n_\rho - (3-b)n_\rho/m_{ad}}\right| = \left|1 - e^{bn_\rho/m_{ad}}\right|,$$

and when b is small, $e^{bn_\rho/m_{ad}} = 1 + bn_\rho/m_{ad}$, and $|S|\ bn_\rho/m_{ad}$.

We will also define a Rayleigh number,

$$\mathrm{Ra} = \frac{g L_z^3 |S|/c_P}{\nu_t \chi_t}, \tag{3}$$

which will determine

$$\kappa_0 = \frac{\chi_t c_P}{e^{n_\rho(-(1+a)+(3-b)/m_{ad}}}$$

We also set T_c and ρ_c to be the values of T and ρ at the bottom of the atmosphere, $T_c = L_z + 1$, $\rho_c = e^{n_\rho}$, such that $\kappa = \kappa_0$ at the bottom of the atmosphere, and thus κ_0 directly controls the flux through the system. In order to study systems with comprehensible prandtl numbers, we set

$$\mu = \mathrm{Pr} \cdot \kappa(\rho_0, T_0)/c_P,$$

where $\mathrm{Pr} \equiv \nu/\chi$. This keeps a constant Pr with depth in the intial state, although with μ fixed, Pr is allowed to evolve.

So under the specification of Pr, Ra, b, and n_ρ, our convective system is fully specified, as far as I can tell.

The characteristic convective timescale is still the buoyancy time,

$$t_b = \sqrt{\frac{L_z}{g(|S|/c_P)}}.$$

[1] Petri J. Käpylä, Matthias Rheinhardt, Axel Brandenburg, Rainer Arlt, Maarit J. Käpylä, Andreas Lagg, Nigul Olspert, and Jörn Warnecke, "Extended Subadiabatic Layer in Simulations of Overshooting Convection," Astrophys. J. **845** (2017), 10.3847/2041-8213/aa83ab.

[2] C. A. Jones, "Acoustic overstability in a polytropic atmosphere." MNRAS **176**, 145–159 (1976).

[3] Evan H. Anders and Benjamin P. Brown, "Convective heat transport in stratified atmospheres at low and high Mach number," Physical Review Fluids **2** (2017), 10.1103/PhysRevFluids.2.083501.