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Snappy and to the point.

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ABSTRACT

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1. INTRODUCTION

This section probably needs to be written after first results start happening.

We aim to answer the following questions:

1. How does the Mach number affect the importance of the nonlinearity? (low-hanging fruit)
2. How does average Plume penetration vary in 2D vs 3D? High Ma vs. low Ma?
3. Is there anything we can say about power spectrum / convective conundrum?

2. EXPERIMENT

2.1. *Fluid Description & Equations of Motion*

We study an ideal gas whose equation of state is $P = \mathcal{R}\rho T$, where P is the pressure, ρ is the density, T is the temperature, and $\mathcal{R}$ is the ideal gas constant. We assume that the gas is made up of monatomic particles such that the adiabatic index is $\gamma = 5/3$ such that the specific heats at constant pressure and volume are $c_P = 2.5\mathcal{R}$ and $c_V = 1.5\mathcal{R}$.

This gas is arranged in plane-parallel, polytropically stratified atmospheres (Anders & Brown 2017),

$$\rho_0(z) = \rho_t(1 + L_z - z)^m, \tag{1}$$

$$T_0(z) = T_t(1 + L_z - z), \tag{2}$$

where ρ_t and T_t are the values of density and temperature at the top of the atmosphere, L_z is the depth of the atmosphere, m is the polytropic index, and z is the vertical coordinate which increases with height in the range $z = [0, L_z]$. We choose here to study buoyantly stable, adiabatically stratified polytropes with $m = m_{\mathrm{ad}} = 1/(\gamma - 1) = 1.5$. The atmosphere does not vary in the horizontal direction, and our cartesian domain spans $x, y = [-L_\perp/2, L_\perp/2]$.

We nondimensionalize by setting $T_t = \rho_t = \mathcal{R} = 1$ at the top of the atmosphere. By this choice, one nondimensional unit of length is the inverse temperature gradient scale of the polytrope, and one non-dimensional time unit is the isothermal sound crossing time of that unit length.

The fluid velocity ($\boldsymbol{u} = u\hat{x} + v\hat{y} + w\hat{z}$), temperature ($T$) and log density ($\ln\rho$) evolve in time according to the fully compressible Navier-Stokes equations,

$$\frac{D\ln\rho}{Dt} + \nabla\cdot(\boldsymbol{u}) = 0 \tag{3}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla T - T\nabla\ln\rho + \boldsymbol{g} + \frac{1}{\rho}\nabla\cdot\left(\bar{\bar{\boldsymbol{\Pi}}}\right), \tag{4}$$

$$\frac{DT}{Dt} + T(\gamma-1)\nabla\cdot(\boldsymbol{u}) + \frac{1}{\rho c_V}\nabla\cdot(-\kappa\nabla T) = \frac{1}{\rho c_V}(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla)\cdot\boldsymbol{u} + Q \tag{5}$$

where $D/Dt \equiv \partial/\partial t + \boldsymbol{u}\cdot\nabla$, and Q is defined in Eqn. 10. The viscous stress tensor is defined as

$$\Pi_{ij} \equiv -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right). \tag{6}$$

We decompose thermodynamic variables into constant background atmospheric terms and evolving fluctuations about those backgrounds, $T = T_0(z) + T_1(x, y, z, t)$ and $\ln\rho = \ln\rho_0(z) + \ln\rho_1(x, y, z, t)$. The domain is horizontally periodic, and at the upper and lower boundary we impose impenetrable, stress free, fixed temperature boundary conditions, such that

$$w = \partial_z u = \partial_z v = T_1 = 0 \text{ at } z = \{0, L_z\}. \tag{7}$$

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2.2. *Diffusivities*

We study an optically thick medium in which Fourier's law of conductivity (Eqn. 5 and Lecoanet et al. (2014)) accurately describes radiative conductivity in our system, and we assume a Kramer's like opacity such that our radiative conductivity profile is (Barekat & Brandenburg 2014; Käpylä et al. 2017, 2018)

$$\kappa = K_0 \rho^{-(1+a)} T^{3-b}. \tag{8}$$

Deep in the solar convection zone, where free-free particle interactions dominate the opacity, a radiative conductivity of this form accurately describes the physics felt by the fluid there with the exponents $a = 1$ and $b = -3.5$ (Weiss et al. 2004), and we adopt these exponents in this work. As has been noted by previous authors (Jones 1976; Edwards 1990; Barekat & Brandenburg 2014), a polytropic stratification where

$$m = m_{\text{kram}} = \frac{3-b}{1+a} = 3.25$$

provides a thermally equilibrated, hydrostatically stable solution with a constant κ. Our choice of an adiabatic stratification of $m = 1.5$ results in a κ profile whose form is

$$\kappa_0 = K_0 T_0^{-b}. \tag{9}$$

The divergence of this κ profile results in a sizeable internal heating term, particularly deep in the domain, that can drastically change the stratification of the simulation over short timescales. In natural systems such as the Sun, however, any sort of atmospheric evolution from this term would take place on much longer timescales than the dynamical timescales of convective elements. As a result, we add an effective internal cooling term,

$$Q = \frac{1}{\rho c_V} \partial_z(-\kappa_0 \partial_z T_0), \tag{10}$$

which cancels out the internal heating of divergence of the conductivity. This allows us to study the motion of a plume in an adiabatic atmosphere subject to a Kramer's-like opacity, as in Eqn. 8.

For simplicity, we assume that the dynamic viscosity (μ) of the fluid is constant. To determine its value, we set the Prandtl number at the top of the atmosphere, $\text{Pr}_t = \text{Pr}(z = L_z) = \mu c_P / K_0$. As κ increases with depth, this choice results in a low Pr deep in the domain.

2.3. *Thermals*

On top of our adiabatic background state, we impose a pressure-neutral "thermal," a spherical thermal perturbation of the form [CITE DANIEL]

$$T_1 = \frac{A_0}{2}(1 + \mathcal{N})\left[1 - \text{erf}\left(\frac{r - r_0}{\delta}\right)\right], \tag{11}$$

$$\ln \rho_1 = \ln\left(\frac{T_0}{T_0 + T_1}\right), \tag{12}$$

where $r \equiv \sqrt{(x-x_p)^2 + (y-y_p)^2 + (z-z_p)^2}$ is the distance from (x_p, y_p, z_p), the center of the thermal, δ determines how geometrically steep the edges of the thermal are, A_0 determines the characteristic magnitude of the perturbation, and $\mathcal{N}$ is noise. This choice of initial conditions creates a characteristic *entropy* perturbation that will accelerate the thermal in an upwards or downwards direction depending on the sign of A_0. Values for the properties of thermals defined in Eqn. 11 are specified in Table 1.

Table 1. Assigned values for parameters in Eqn. 11 for cold (downflow) thermals and hot (upflow) thermals.

Parameter	Downflow	Upflow
δ	$r_0/4$	$r_0/4$
x_p, y_p	0	0
z_p	$L_z - r_0/2$	$r_0/2$
r_0	$H_\rho\vert_{z=L_z}/2$	$H_\rho\vert_{z=0}/10$
A_0	$-(e^{\epsilon/c_P} - 1)$	$T_0\vert_{z=0}(e^{\epsilon/c_P} - 1)$

The magnitude of A_0 is chosen so that the characteristic entropy of the thermal is ϵ, which controls the Mach number of the thermal, as in Anders & Brown (2017). $\mathcal{N}$ is a turbulent noise term whose spatial power spectrum falls in the manner of a turbulent cascade, $k^{-5/3}$, and which is normalized to have a magnitude of 0.1.

2.4. *Control Parameters*

Under these assumptions, there are a number of control parameters for our experiment: K_0, Pr_t, ϵ, and L_z, $L_\perp$. We set $L_\perp = 20 r_0$, such that the domain spans 10 thermal-widths. We desire a domain which spans four density scale neights ($n_\rho = 4$), so we set $L_z = e^{n_\rho/m} - 1 \approx 13.4$. We set $\text{Pr}_t = 1$, such that the full domain is at or below a Pr of unity. After making these choices, we are left with two free control parmeters. We define

$$\mathcal{R} = \frac{\sqrt{\epsilon}}{\mu} = \frac{\sqrt{\epsilon}}{K_0 \text{Pr}_t c_P}, \tag{13}$$

which is related to the evolved Reynolds number of the flow. In this work, we will study the dynamics of thermals at varying values of ϵ and $\mathcal{R}$, which in turn control the Mach number and turbulence of the flows.

2.5. *Numerical Methods*

We utilize the Dedalus[1] pseudospectral framework Burns et al. (2016) to time-evolve (**??**)-(**??**) using an implicit-explicit (IMEX), third-order, four-step Runge-Kutta timestepping scheme RK443 Ascher et al. (1997). We study 2D and 3D simulations, and in our 2D runs we set $v = \partial_y = 0$. Variables are time-evolved on a dealiased Chebyshev (vertical) and Fourier (horizontal, periodic) domain in which the physical grid dimensions are 3/2 the size of the coefficient grid. By using IMEX timestepping, we implicitly step the stiff linear acoustic wave contribution and are able to efficiently study flows at high (~ 1) and low ($\sim 10^{-4}$) Ma. This IMEX approach has been successfully tested against a nonlinear benchmark of the compressible Kelvin-Helmholtz instability Lecoanet et al. (2016).

3. RESULTS

Figures:

1. Pretty dynamics, what the plume looks like, etc.
2. Size of $\kappa/\langle\kappa\rangle - 1$ vs. ...Ma? Time? Both?
3. Persistence depth vs. $\mathcal{R}$, ϵ.
4. 2D vs. 3D persistence?
5. Power spectra?

4. CONCLUSIONS

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[1] http://dedalus-project.org/