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Snappy and to the point.

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ABSTRACT

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1. INTRODUCTION

What is the solar convective conundrum, why is it a problem, and what are people doing to combat it?

Studies of Kramer's opacities and entropy rain are one path that people are using to figure out what's happening. Some pioneering work on that has been done recently, and they've found blah.

In this paper, we study hydrodynamic convection in the optically thick limit such that a Newton-like conductivity law with a radiative conductivity is valid. For our opacity, we use a Kramer's-like opacity law of the form

Which has been used recently by Käpylä et al. (2017, 2018). We are particularly interested in studying the effects of this fully nonlinear condcutivity and its feedback on convective flows. In this work, we fix $a = 1$ and vary $b = (0, -3.5]$, where in the limit of $b = -3.5$, this radiative conductivity takes the same form as that for free-free interactions (Weiss et al. 2004). We take this approach, as we find that b naturally controls the Mach number when the initial conditions are an adiabatic, hydrostatic polytrope. Regardless of the value of b, the radiative conductivity is highly nonlinear in both T and ρ, and so we can study the importance of its nonlinear nature when the flows are high and low Mach number. The transition from $b = 0$ to $b = -3.5$ is also interesting in that it provides a gradual transition from classic polytropic convective systems (Hurlburt et al. 1984; Brandenburg et al. 2005; Anders & Brown 2017), in which there is a large condition background conductive flux in the system, to systems in which the radiative flux becomes extremely inefficient in the interior and convection is required for nearly all energy transportation in the system.

The importance of nonlinear conductive feedback has previously been studied in the context of mantle convection in the infinite Prandtl number limit (Dubuffet et al. 2000), but there the conductivity is weakly inversely proportional to the temperature, whereas here it is strongly proportional to. Thus, we anticipate the negative feedback effects seen there to not be seen here. Blahblah.

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2. EXPERIMENT

2.1. *Model*

We study an ideal gas whose equation of state is $P = \mathcal{R}\rho T$. We assume that the gas is made up of monatomic particles such that the adiabatic index is $\gamma = 5/3$ such that the specific heats at constant pressure and volume are $c_P = 2.5\mathcal{R}$ and $c_V = 1.5\mathcal{R}$.

We study plane-parallel, polytropically stratified atmosphere (Anders & Brown 2017),

$$\rho_0(z) = \rho_t(1 + L_z - z)^m, \tag{1}$$
$$T_0(z) = T_t(1 + L_z - z), \tag{2}$$

where ρ and T are the density and temperature, respectively, ρ_t and T_t are their values at the top of the atmosphere, L_z is the depth of the atmosphere, m is the polytropic index, and z is the vertical coordinate which increases with height in the range $z = [0, L_z]$. Throughout this work we will specifically study perfectly adiabatic polytropes with $m = m_{\rm ad} = 1/(\gamma - 1) = 1.5$. The atmosphere does not vary horizontally, and our cartesian domain spans $x = [-L_x/2, L_x/2]$ and $y = [-L_y/2, L_y/2]$.

We nondimensionalize our atmosphere by choosing $\mathcal{R} = T_t = \rho_t = 1$. By this choice, the non-dimensional length scale is the inverse temperature gradient scale and the timescale is the isothermal sound crossing time, τ_I, of this unit length.

On top of this reference atmosphere, we impose a "thermal," a spherical thermal perturbation of the form [CITE DANIEL]

$$T_1(r) = \frac{A_0}{2}\left(1+\mathcal{N}\right)\left[1 - \text{erf}\left(\frac{r-r_0}{\delta}\right)\right], \tag{3}$$

$$\ln \rho_1 = \ln\left(\frac{T_0}{T_0+T_1}\right), \tag{4}$$

where $r \equiv \sqrt{(x-x_p)^2 + (y-y_p)^2 + (z-z_p)^2}$ is the distance from the center of the perturbation, located at (x_p, y_p, z_p), where

$$r_0 = \begin{cases} H_\rho(z=L_z)/2 & \text{downflow} \\ H_\rho(z=0)/10 & \text{upflow,} \end{cases}$$

is the radius of the thermal with H_ρ the local density scale height, and

$$x_p = 0, y_p = 0, z_p = \begin{cases} L_z - \frac{r_0}{2} & \text{downflow} \\ \frac{r_0}{2} & \text{upflow.} \end{cases}$$

$\delta = r_0/4$ sets the sharpness of the erf, and the erf approaches a step function as $\delta \to 0$. $\mathcal{N}$ is symmetry-breaking noise, and A_0 determines the characteristic magnitude of the perturbation. Subscripts "0" refer to background conditions, and subscripts "1" refer to fluctuations. This choice of initial conditions creates a characteristic *entropy* perturbation while being pressure-neutral, such that the initial motion of the thermal will be caused by buoyant acceleration rather than pressure equilibration. We set the amplitude of the perturbation such that

$$A_0 = \begin{cases} -(e^{\epsilon/c_P} - 1), & \text{downflow} \\ T_0(z=0)\cdot(e^{\epsilon/c_P} - 1), & \text{upflow,} \end{cases} \tag{5}$$

and by these choices the specific entropy perturbation of an upflow located near the bottom of the domain or a downflow located near the top of the domain will be ϵ, and will control the Mach number of the thermal, as in Anders & Brown (2017). $\mathcal{N}$ is a turbulent noise term whose spatial power spectrum falls in the manner of a turbulent cascade, $k^{-5/3}$, and which is normalized to have a magnitude of 0.1.

2.2. *Equations of Motion*

This fluid evolves according to the fully compressible Navier-Stokes equations of hydrodynamics:

$$\frac{D \ln \rho}{Dt} + \nabla\cdot(\boldsymbol{u}) = 0 \tag{6}$$

$$\frac{D\boldsymbol{u}}{Dt} = -\nabla T - T\nabla\ln\rho + \boldsymbol{g} + \frac{1}{\rho}\nabla\cdot\left(\bar{\bar{\boldsymbol{\Pi}}}\right), \tag{7}$$

$$\frac{DT}{Dt} + T(\gamma-1)\nabla\cdot(\boldsymbol{u}) + \frac{1}{\rho c_V}\nabla\cdot(-\kappa\nabla T) = \frac{1}{\rho c_V}(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla)\cdot\boldsymbol{u} + Q \tag{8}$$

where $D/Dt \equiv \partial/\partial t + \boldsymbol{u}\cdot\nabla$, and Q is defined in Sec. 2.4. The viscous stress tensor is defined as

$$\Pi_{ij} \equiv -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right), \tag{9}$$

We impose impenetrable, stress free, fixed temperature boundary conditions at the top and bottom of the domain, with $w = \partial_z u = T_1 = 0$ at $z = \{0, L_z\}$.

We assume that Fourier's law of conductivity (Lecoanet et al. 2014) accurately describes radiative conductivity in our system, and we assume a Kramer's like opacity such that our radiative conductivity profile is (Barekat & Brandenburg 2014; Käpylä et al. 2017, 2018)

$$\kappa(z,t) = K_0\rho^{-(1+a)}T^{3-b}. \tag{10}$$

Deep in the solar convection zone, a radiative conductivity of this form accurately describes the physics felt by the fluid there for the exponents $a = 1$ and $b = -3.5$, which describe the opacity of free-free interactions (Weiss et al. 2004). In this work, we choose $a = 1$ and study $b = (0, -3.5]$. As has been noted by previous authors (Jones 1976; Edwards 1990; Barekat & Brandenburg 2014), a polytropic stratification where

$$m = m_{\text{kram}} = \frac{3-b}{1+a}$$

provides a hydrostatic reference solution with a constant κ while the temperature gradient is constant. Our choice of $a = 1$ chooses $m_{\text{kram}} = 3/2 - b/2 = m_{\text{ad}} - b/2$, such that as the magnitude of b grows away from zero, the divergence of the conductivity profile of our adiabatic polytropic reference state grows, and in some ways b begins to resemble the classic superadiabatic excess of polytropic solutions (Graham 1975; Anders & Brown 2017).

2.3. *Numerical Methods*

We utilize the Dedalus[1] pseudospectral framework Burns et al. (2016) to time-evolve (**??**)-(**??**) using an

[1] http://dedalus-project.org/

implicit-explicit (IMEX), third-order, four-step Runge-Kutta timestepping scheme RK443 Ascher et al. (1997). Thermodynamic variables are decomposed such that $T = T_0 + T_1$ and $\ln\rho = (\ln\rho)_0 + (\ln\rho)_1$, and the velocity is $\boldsymbol{u} = w\hat{\boldsymbol{z}} + u\hat{\boldsymbol{x}} + v\hat{\boldsymbol{y}}$. In our 2D runs, $v = 0$. Subscript 0 variables, set by (**??**), have no time derivative and vary only in z. Variables are time-evolved on a dealiased Chebyshev (vertical) and Fourier (horizontal, periodic) domain in which the physical grid dimensions are 3/2 the size of the coefficient grid. By using IMEX timestepping, we implicitly step the stiff linear acoustic wave contribution and are able to efficiently study flows at high (~ 1) and low ($\sim 10^{-4}$) Ma. Our equations take the form of the FC equations in Lecoanet et al. (2014), extended to include ν and χ which vary with depth, and we follow the approach there. This IMEX approach has been successfully tested against a nonlinear benchmark of the compressible Kelvin-Helmholtz instability Lecoanet et al. (2016).

2.4. *Reference atmosphere*

Q is defined like

2.5. *Initial conditions*

We do blahblahblah at low —b—.
We do blahblahblah at high —b—.

On top of this initial stratification, T_1 is initially filled with random white noise whose magnitude is infinitesimal compared to $|b|T_0$. We filter this noise spectrum in coefficient space, such that only the lower 25% of the coefficients have power.

3. RESULTS

Here's a figure that shows how Reynolds number and Mach number scale with b / Ra:

Here's a figure of pretty dynamics at high and low Mach number. Probably a few panels: entropy deviations, kappa deviations, and vertical velocity.

Here's a figure that shows the evolved stratification (where is BZ/DZ .../oz?) at high and low Mach number (and high and low b). Shows the progression towards more nonlinearity in κ and the evolved stratification we get out. This is part of the "deviating away from polytropic convection" storyline.

Here's a figure that shows the importance of nonlinearities in the conductivity at both high and low Ma. Basically, time-averaged scalar values of $\kappa/\langle\kappa\rangle - 1$, and probably a time averaged absv profile of that. Can we say why we see the scaling we see with b?

4. CONCLUSIONS

What have we learned about opacity? Where are nonlinearities important? Where are they unimportant?

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