Sonification of Gravity Waves in Stars

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1 Data sonification and creation of sound clips for sonic analogy

We create ten supplementary sound files to illustrate the transfer function. These files are:

1. A 20-second audio clip of *Jupiter*, by Gustav Holst. Created using Musescore, an open source music notation software (`https://github.com/musescore/MuseScore`).
2. (3 files) 20-second clips of how *Jupiter* would sound if it were filtered through the stellar transfer functions.
3. (3 files) Short ($\sim$ 1-second) clips of sonified excitation sampled in the radiative zone close to the convection zone.
4. (3 files) 15-second clips of sonified waveforms corresponding to the predicted photometric varaibility presented in Fig. 2 of the main manuscript.

We describe each of these clips in some detail below.

1.1 The pure sound of a song

We present an audio clip of *Jupiter* from The Planets, an orchestral suite by Gustav Holst (see supplemental sound `Jupiter_Holst_Clip.wav`). This clip is made using Musescore, an open source music notation software, which supports MIDI output for a variety of instruments. It is arranged for a string quartet of violin, viola, cello, and bass. The melody line is played by the cello in the first half of the clip and then transitions to the violin in the second half of the clip. The sheet music for this arrangement is available online in the `https://github.com/evanhanders/gmode_variability_paper` repository, in the `sound/Jupiter_Sheet_Music.pdf` file.

The different sizes of these instruments determine their different resonant modes, with the smaller instruments (violin, viola) having higher frequency resonances than the larger instruments (cello, bass). Throughout this text, we will refer to both "resonant modes" and "standing modes". "Resonant modes" are those which resonate within the instrument cavity and are thus affected by the instrument's size, whereas "standing modes" resonate within the star and are thus determined by the stellar mass.

In figure 1, we plot both the waveform amplitude and power spectrum of the *Jupiter* audio clip.

1.2 The song as heard after propagating through the star

To understand how a star affects waves, we will appeal to a sonic analogy. We imagine that the core convection is arranged in precisely the right way so that it generates a wave luminosity with the same waveform as *Jupiter*. We note that in this analogy, the wave luminosity does not depend on stellar mass. We convolve the wave signal with the stellar transfer function to determine how the frequency spectrum of the song changes as it propagates through each of the stars that we study. The output of this operation is three audio clips which answer the question, "if we could hear gravity waves, how would propagation through different stellar envelopes change the sound of gravity waves?"

Gravity waves in massive stars have low frequencies ($\lesssim 10^{-4}$ Hz) which are outside the range of human hearing. In order to produce an audio aid, we uniformly increase the frequencies of all waves, and thus

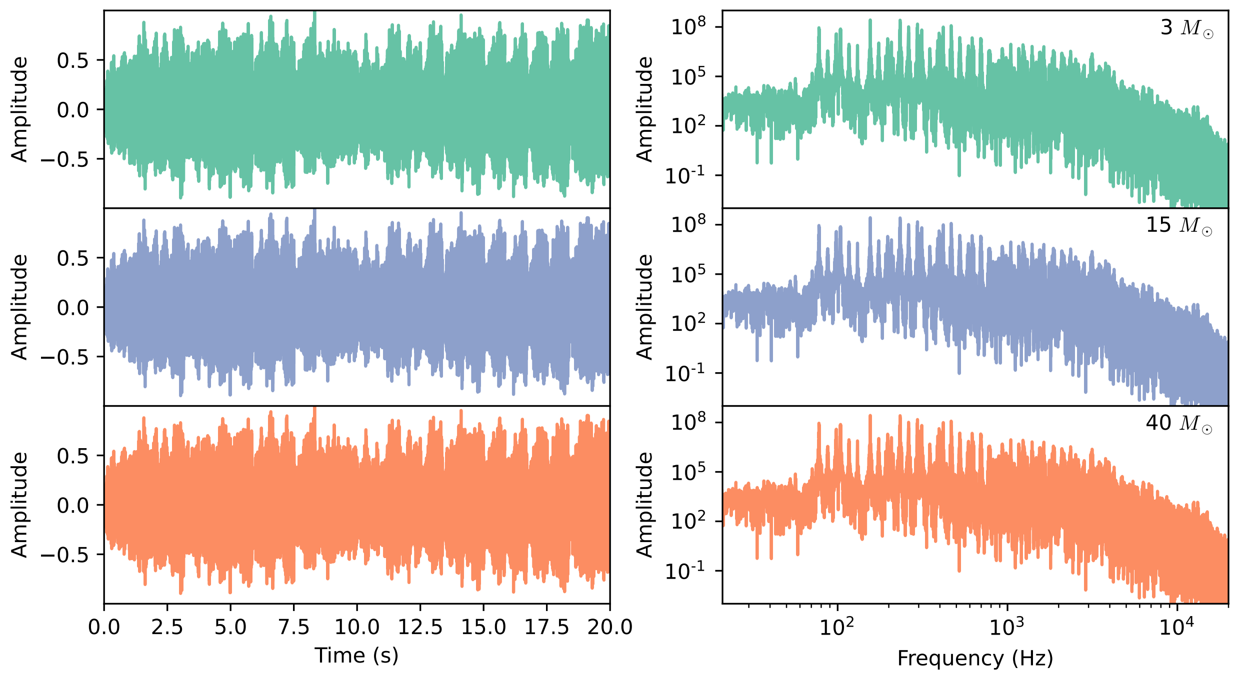

Figure 1: Left panels: Normalized amplitude of the *Jupiter* audio clip waveform. We assume that this waveform is analogous to the wave luminosity of a 3 $M_\odot$ (top, green), 15 $M_\odot$ (middle, purple), or 40 $M_\odot$ (bottom, orange) star. Right panels: Amplitude of the wave power in each frequency for the 3 $M_\odot$ (top, green), 15 $M_\odot$ (middle, purple), and 40 $M_\odot$ (bottom, orange) stars. Each of the rows displays the same data; we choose to present all three rows to facilitate direct comparison with the later Fig. 3.

the transfer function, so that its domain coincides with that of human hearing. To accomplish this, we multiply all gravity mode frequencies by 3.4×10^7, so that the waves occupy the 20-20,000 Hz range of human hearing. In order to precisely determine the effect of the transfer function on *Jupiter*, we should multiply the amplitude spectrum of the song by the transfer function (or equivalently, we could multiply the power by the square root of the transfer function). However, the standing mode peaks in the transfer function vary by many orders of magnitude, and these sharp peaks can dominate other signals, making it extremely difficult to auditorily interpret any differences between the stars of different masses. We therefore take the cube root of the transfer function before convolving it with the power spectrum of the song.

The resulting cube root transfer function with the shifted frequencies are shown in figure 2. Each transfer function has a high frequency cutoff around the characteristic Brunt-Väisälä frequency of the star. This cutoff moves to lower frequencies as the stellar mass increases. Each transfer function is characterized by a number of standing mode peaks and a low frequency "bump" of power, whose characteristic frequency shifts to lower frequency with increasing stellar mass.

We interpolate the shifted, cube root transfer function into the frequency bins that *Jupiter* is sampled in. The interpolated transfer function is convolved with the spectrum of the song, then transformed back into the time domain. The resulting time series is normalized to the absolute maximum amplitude and the audio files `J_in_3_msol.wav`, `J_in_15_msol.wav`, and `J_in_40_msol.wav` are created for the 3 $M_\odot$, 15 $M_\odot$, and 40 $M_\odot$ stars respectively. The timeseries amplitude of the waveform is shown in the left hand panels of figure 3 for each mass of star studied. The right hand panels show the power spectrum of the left hand timeseries. We see these power spectra follow the same pattern as the transfer functions, with a high frequency cutoff that lowers as stellar mass increases. In comparing to the original waveforms in figure 1, we see that the normalized waveform amplitude exhibits a time-varying envelope. Additionally, we see peaks in the power spectrum at the surface from both resonant modes in the instruments (as in Fig. 1) as well as from gravity wave standing modes; these standing modes can be heard in the `J_in_3_msol.wav` and `J_in_15_msol.wav` files.

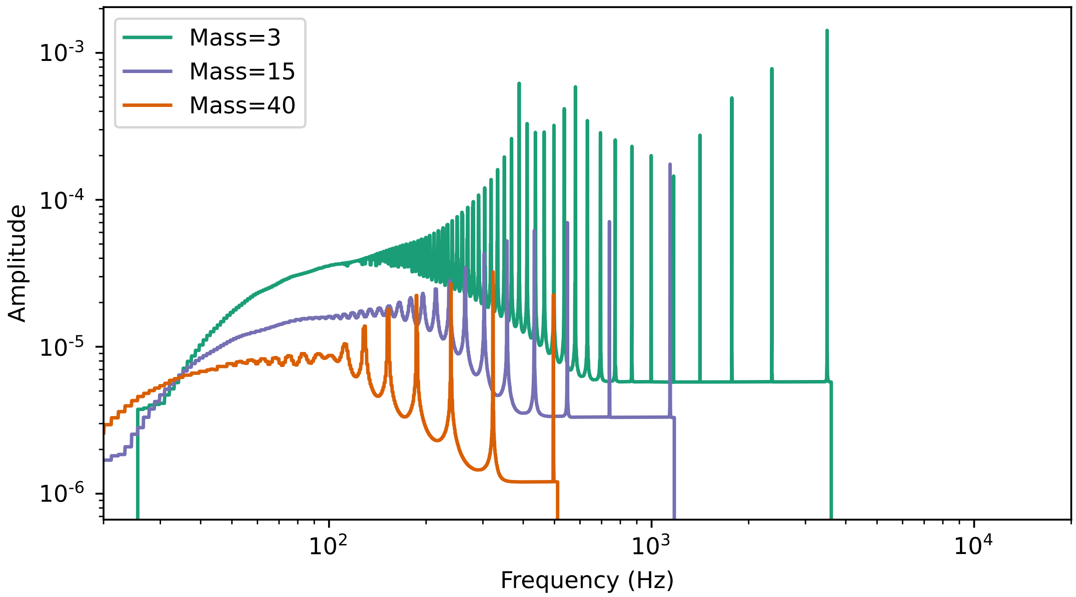

Figure 2: The amplitude of the cube root of the transfer function for the audible range of frequencies for a 3 $M_\odot$ (green), 15 $M_\odot$ (purple), and 40 $M_\odot$ (orange) star.

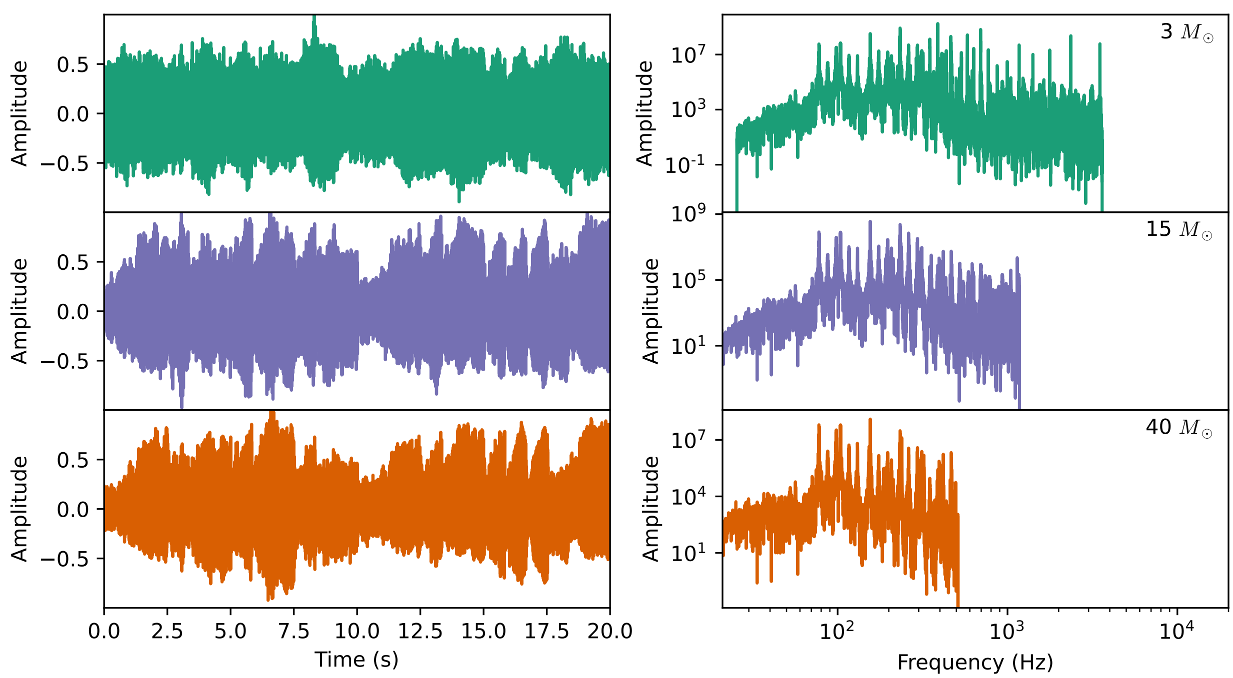

Figure 3: Same as figure 1, but for sound waves near the surface of each star showing the effect of the stellar transfer function on the *Jupiter* audio clip.

When listening to the resulting wav files, the effects of wave propagation are clear. The `J_in_3_msol.wav` clip has high frequency standing modes which dominate the clip. The instruments are weakly audible in the background.

The `J_in_15_msol.wav` clip has medium frequency standing modes in the background. All of the instruments are audible with resonant modes in the bass which are noticeably stronger than that of the violin.

The `J_in_40_msol.wav` clip has no strong standing modes in the background. Notably, the violin part is not audible at all, and the viola part is extremely faint, indicating that the high frequencies are heavily damped as supported by the power spectrum. Additionally, we hear much stronger resonant modes in the bass than the other instruments.

1.3 Wave excitation inside a star

The previous analogy demonstrates how the stellar transfer function affects music, but the wave luminosity generated by convection does sound like music. We next aim to understand how the wave luminosity sounds.

We measure the wave luminosity in our simulations close to the convection zone then shift the frequency of that spectrum by a factor of 8.1×10^7 into the audible frequency range. We then set the luminosity spectrum to zero outside the human audible range. The measured luminosity has no inherent phase information, so we assign a random phase to each frequency bin in the luminosity spectrum. We transform the luminosity into the time domain, which produces a very short clip, so we concatenate the soundwaves 20 times in a row. We plot the resulting timeseries in the left panel of figure 4. As stellar mass increases, the wave period appears to increase overall leading us to expect lower frequency waves overall. In the power spectrum (right panel), we see each star follows roughly the same shape, with a differing location of the low frequency cutoff and more high frequency peaks as the mass increases. We note that the change in characteristic frequency we see here is likely due to differences in the diffusivities of our simulations, rather than due to the excitation frequency of the waves by convection.

When listening to the sonification (see supplemental audios `E_in_3_msol.wav`, `E_in_15_msol.wav`, and `E_in_40_msol.wav`) we hear that the apparent frequency of the noise generated does not simply decrease as stellar mass increases. Instead, the frequency appears higher for 15 $M_\odot$ than for 3 $M_\odot$, with the 40 $M_\odot$ star having the lowest frequency. While not immediately obvious, this discrepancy comes from the shape

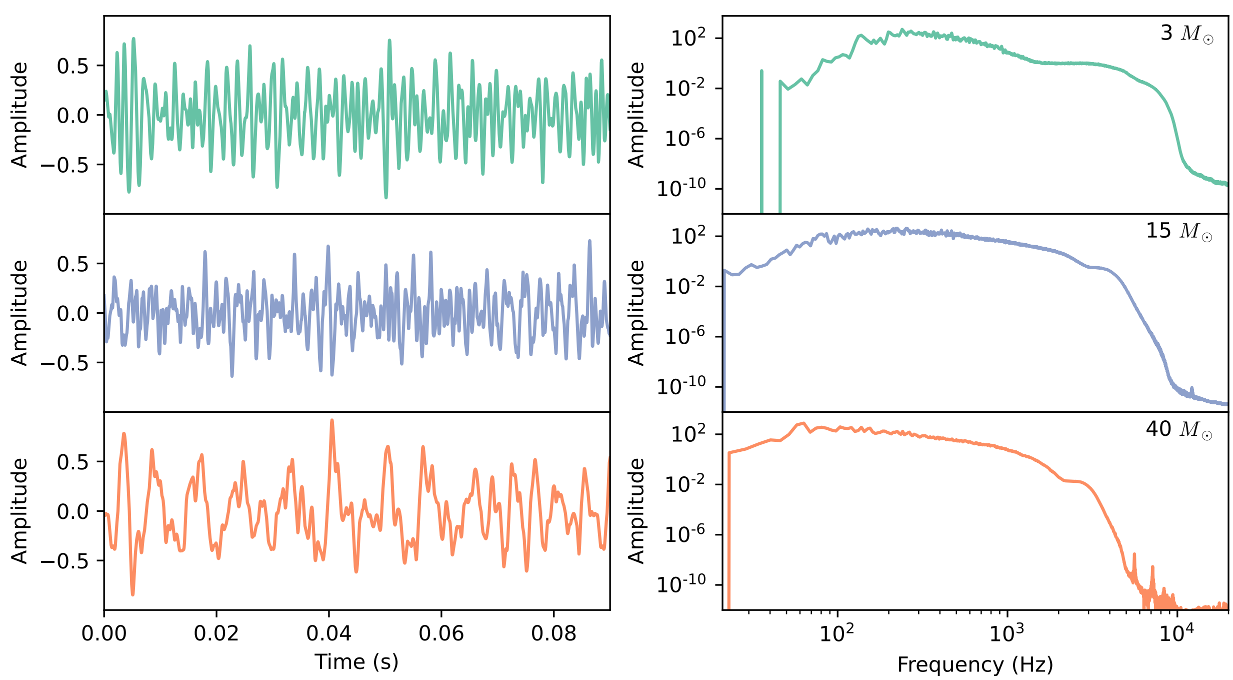

Figure 4: Same as figure 1 but showing the wave luminosity near the core of waves excited by convection.

of the power spectrum. We see that at frequencies of 1500 Hz, the power has fallen off more steeply in the 3 $M_{\odot}$ case than the 15 $M_{\odot}$ case. This more gradual decline in the power spectra leads to the the 15 $M_{\odot}$ sonification sounding like it is at higher frequencies than the 3 $M_{\odot}$ sonification. This result is difficult to see from the power spectrum alone, and sonification allowed us to hear this subtlety.

1.4 What do we hear at the surface?

As waves travel through the star the sound will be altered by the stellar transfer function, just as *Jupiter* was altered in the above analogy.

The waves excited at the surface are once again in frequency ranges lower than the audible range of human hearing. We shift the frequencies by the same factor as above (8.1×10^7), and assign a random phase to the fluctuations. As in the main manuscript, we assume that wave excitation is characterized as a power-law; we expect this to be the case for realistic stellar diffusivities within the audible range we consider, and this prevents the low-frequency cutoff which arose from unphysically large simulation diffusivities in the previous section from imprinting onto this signal. We transform the signal into the time domain and normalize to the absolute maximum amplitude. We note that this creates a very short sound clip (0.4 seconds), so we stack the waveform many times in order to create a longer final sound clip. The waveforms are shown in the left panels of figure 5 with the period of oscillation getting longer as stellar mass increases. Additionally the right hand panel of the power spectra shows that the high frequency cutoff shifts lower as mass increases. All indicating that the overall frequencies in our sonifcation should become lower as mass increases.

The sonifications are in supplemental wav files `sonified_3msol.wav`, `sonified_15msol.wav`, and `sonified_40msol.wav` respectively. The sound waves near the surface can be described by two parts: the high pitched tones and a low pitch (red) noise.

The high pitched tones follow the same pitch pattern as the excitation does, with the 3 $M_{\odot}$ star having the highest pitch followed by the 15 $M_{\odot}$ and the 40 $M_{\odot}$ stars. This indicates that the higher pitched tones are set by the internal excitation, i.e. the ‘song’ that is played by the star.

The lower pitched tones, however, show a decrease in frequency as the stellar mass increases. These are analogous to the standing modes seen in the *Jupiter* clip at the stellar surface. The pitch of these is once again set by the size of the stellar cavity and thus will only depend on the stellar mass.

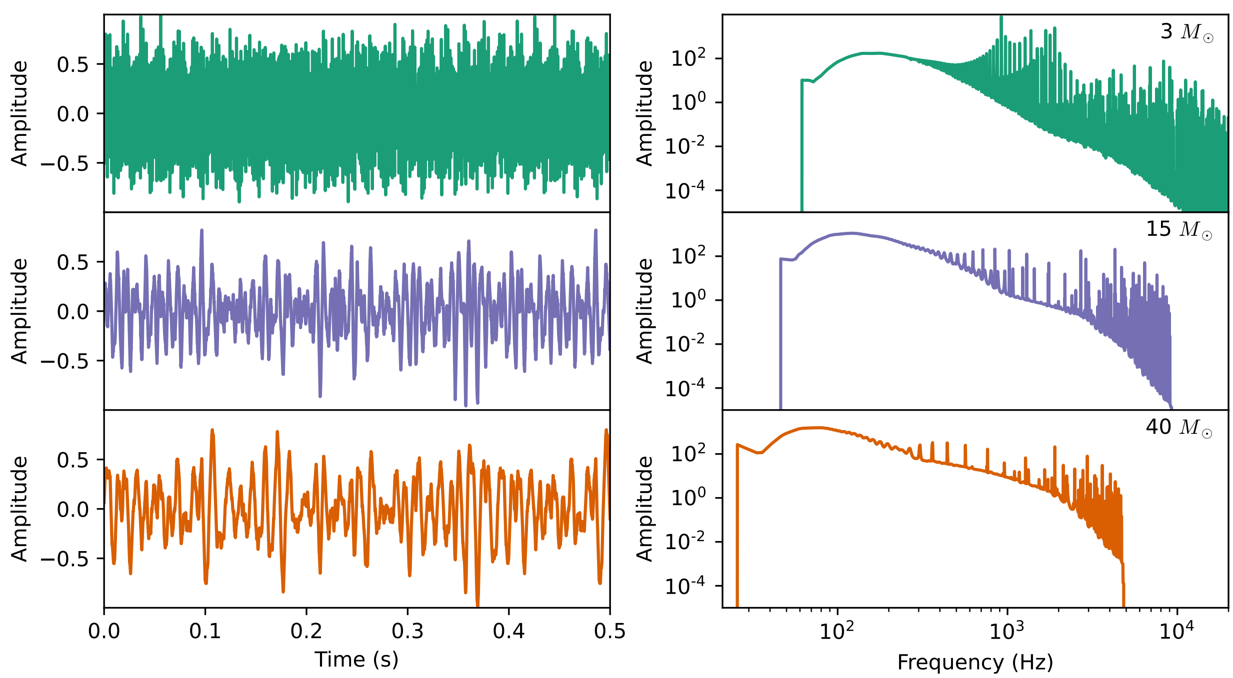

Figure 5: Same as figure 1, but for the sound waves near the stellar surface, showing the effect of the stellar transfer function on the convective waves.