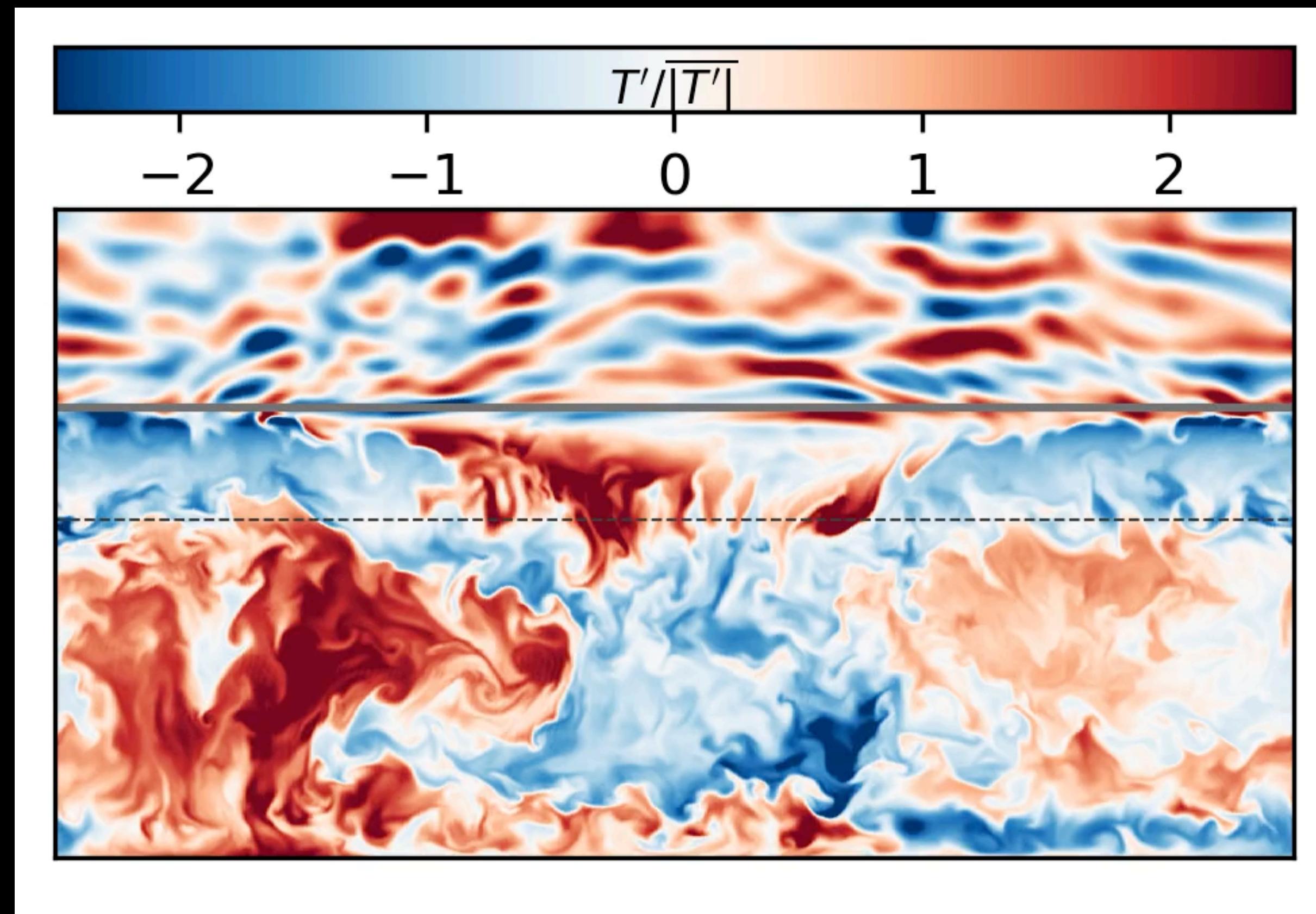


# A modern mystery in stellar convection

... &  
**Dedalus is a  
flexible tool.**

Evan Anders  
CIERA Postdoctoral Fellow  
Northwestern University

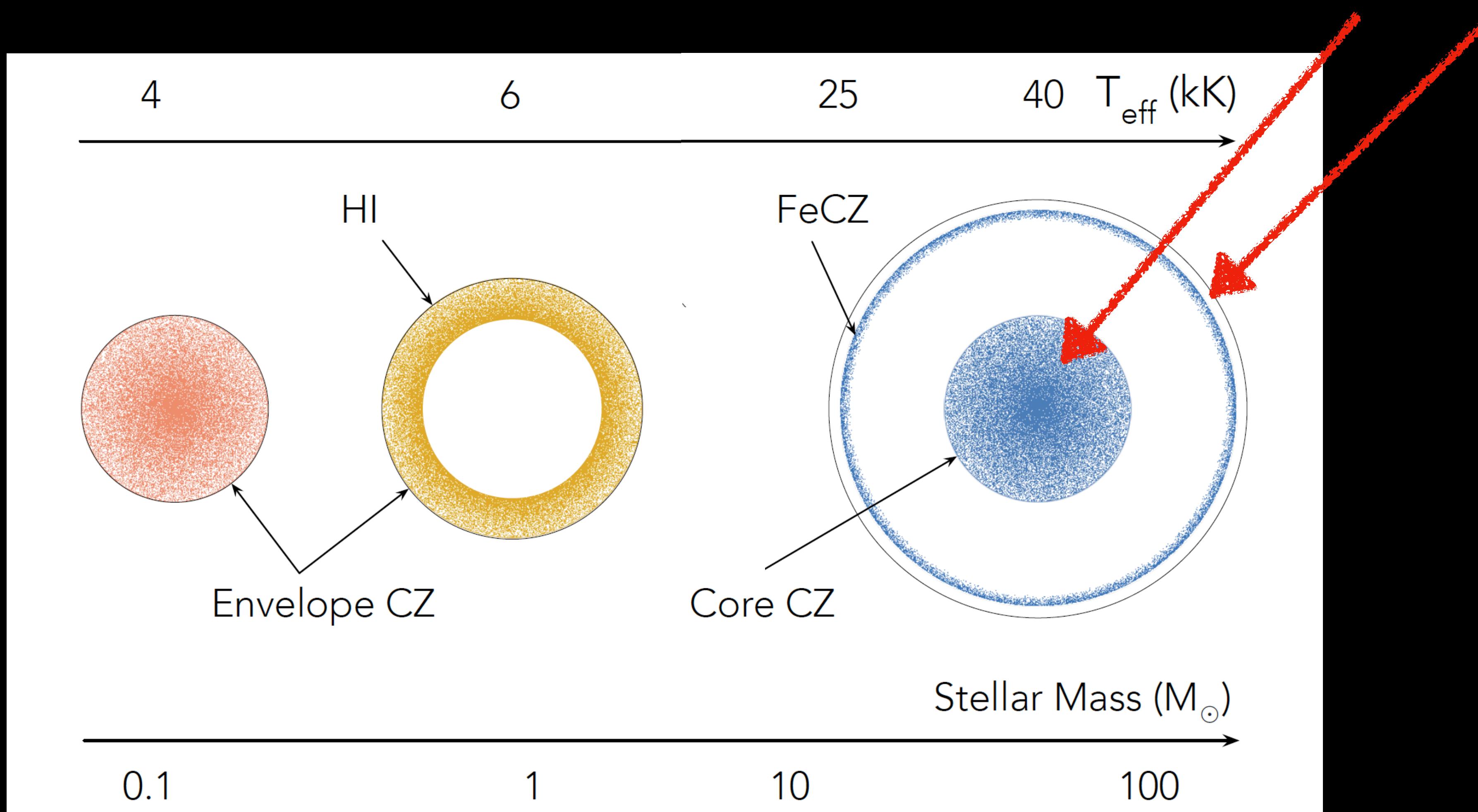


I am interested in convection.

# I am interested in convection.

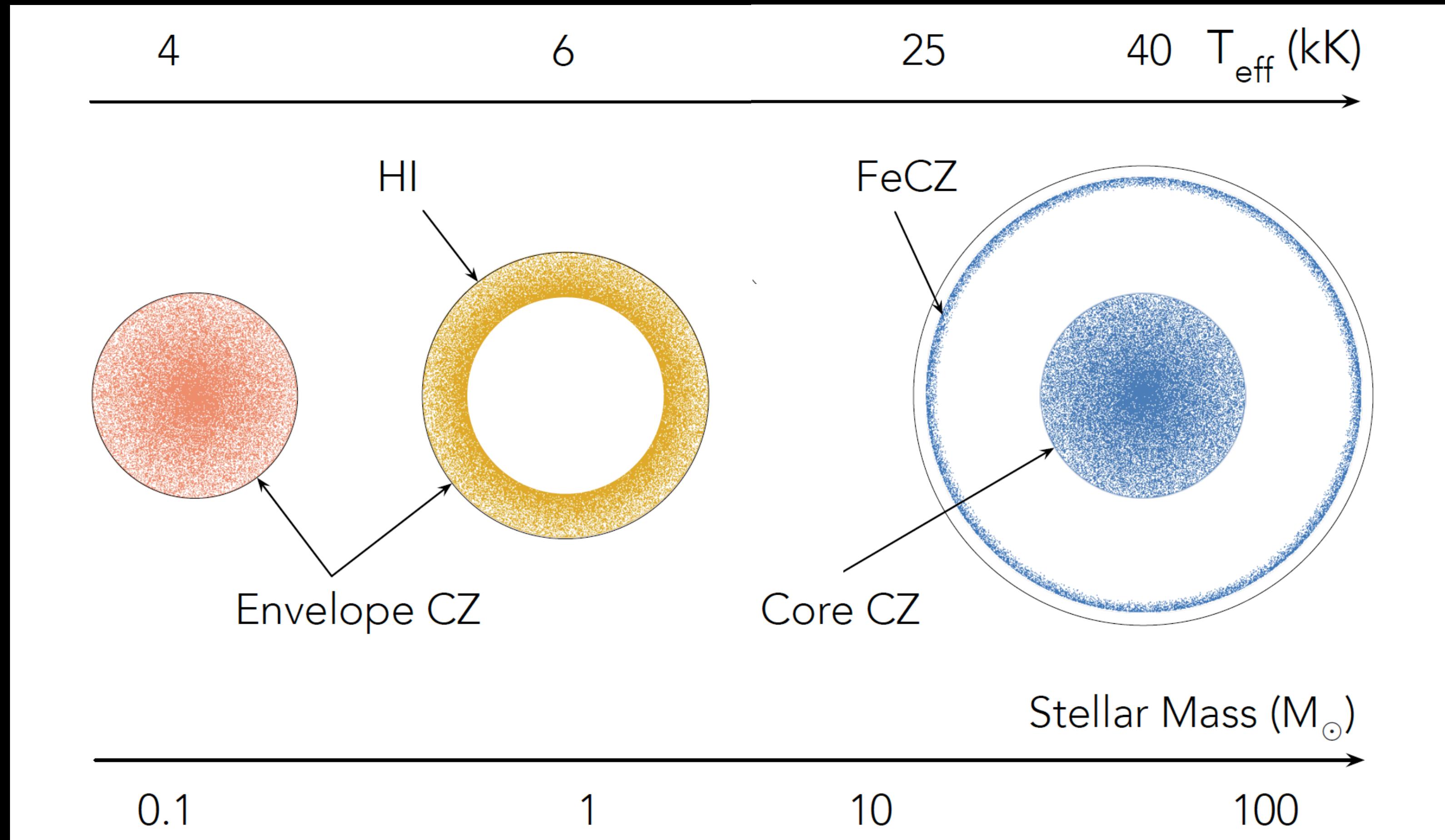
Convection happens in stars.

Shaded areas  
are convection



# Stellar convection is diverse

It spans a broad range of your favorite fluid number.



E.g., the Rossby number

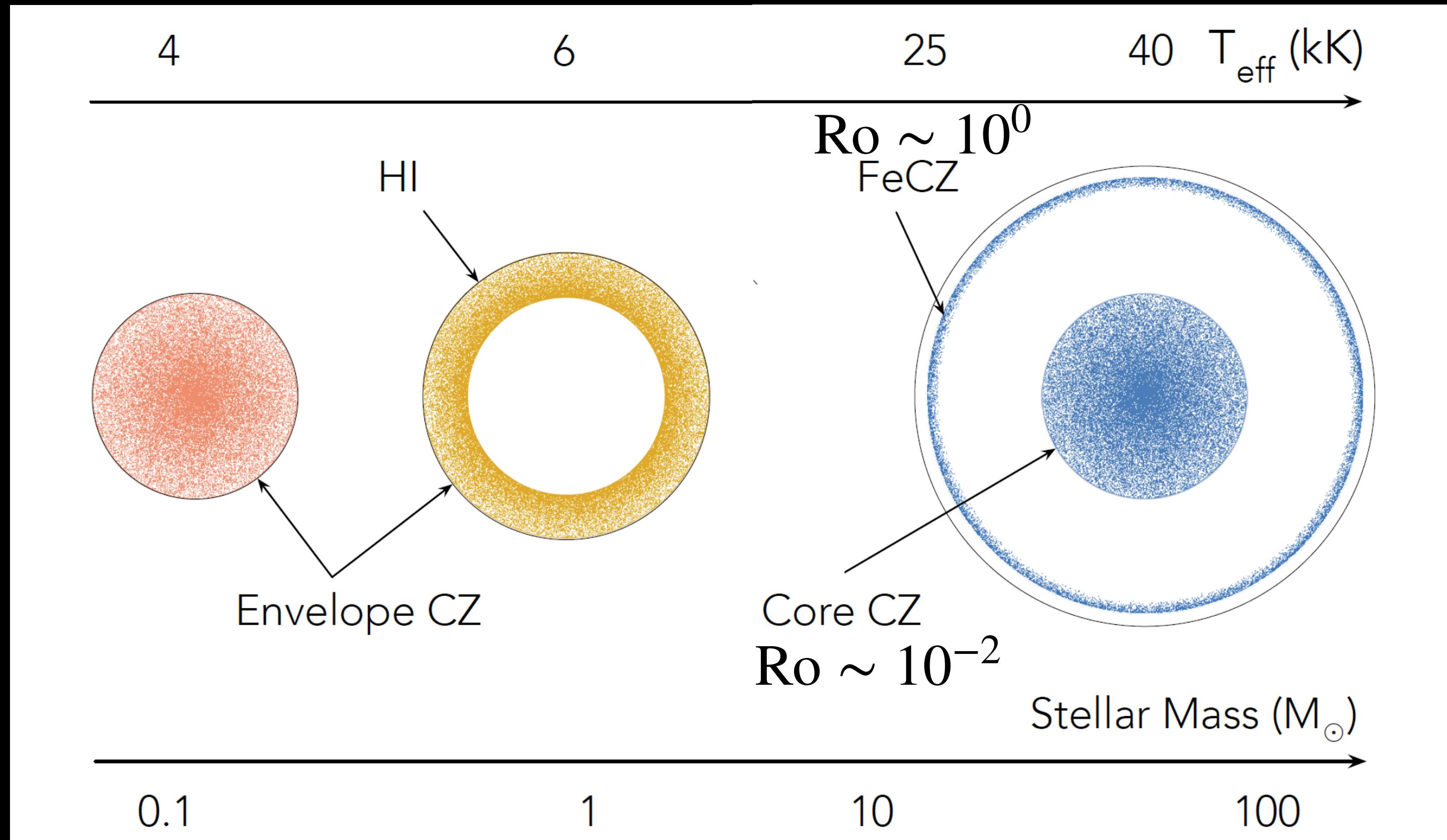
$$Ro = \frac{\tau_{\text{rot}}}{\tau_{\text{conv}}}$$

Ro measures  
Rotational constraint.

Smaller Ro = rotation  
matters more.

# Stellar convection is diverse

It spans a broad range of your favorite fluid number.



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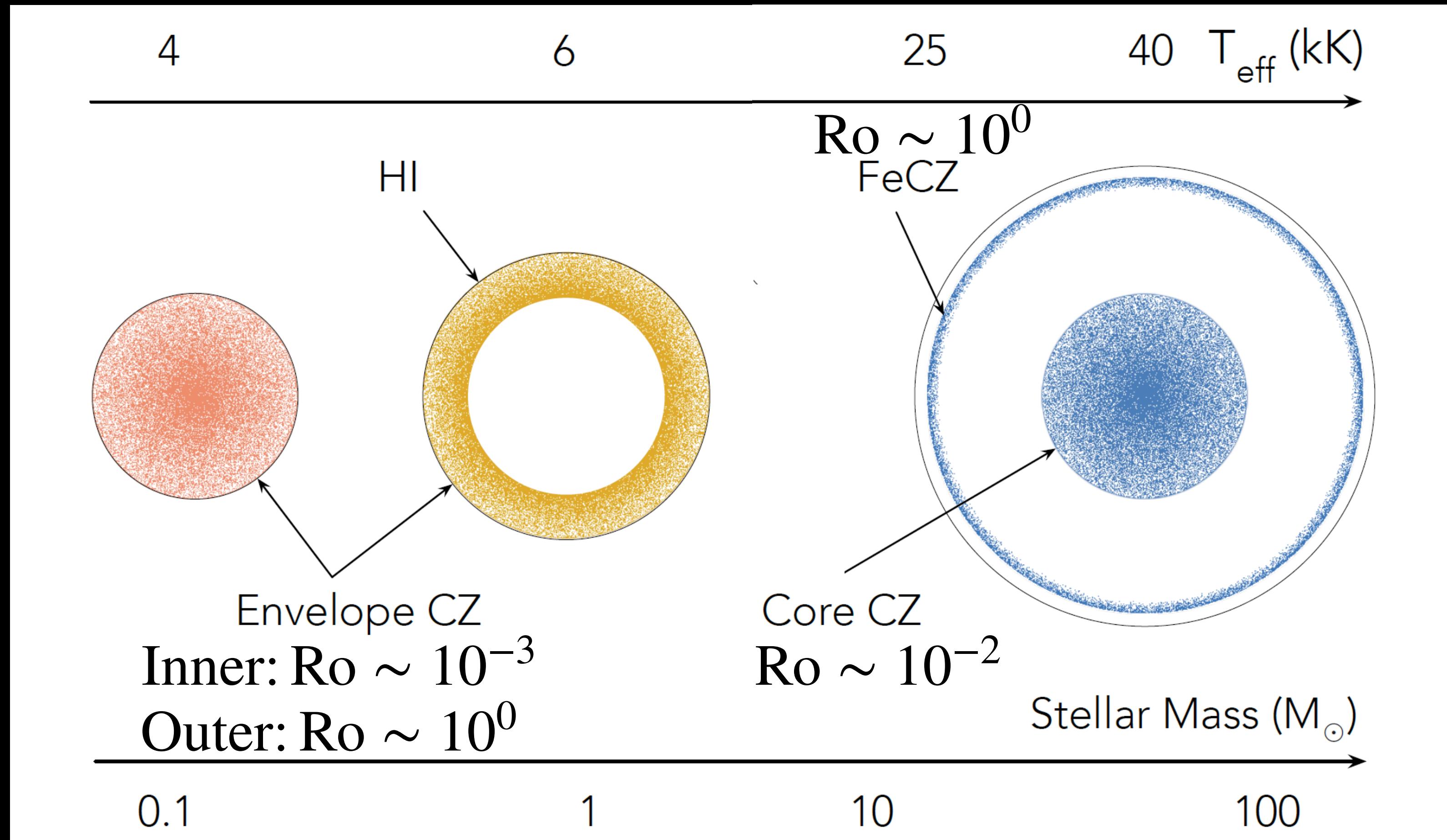
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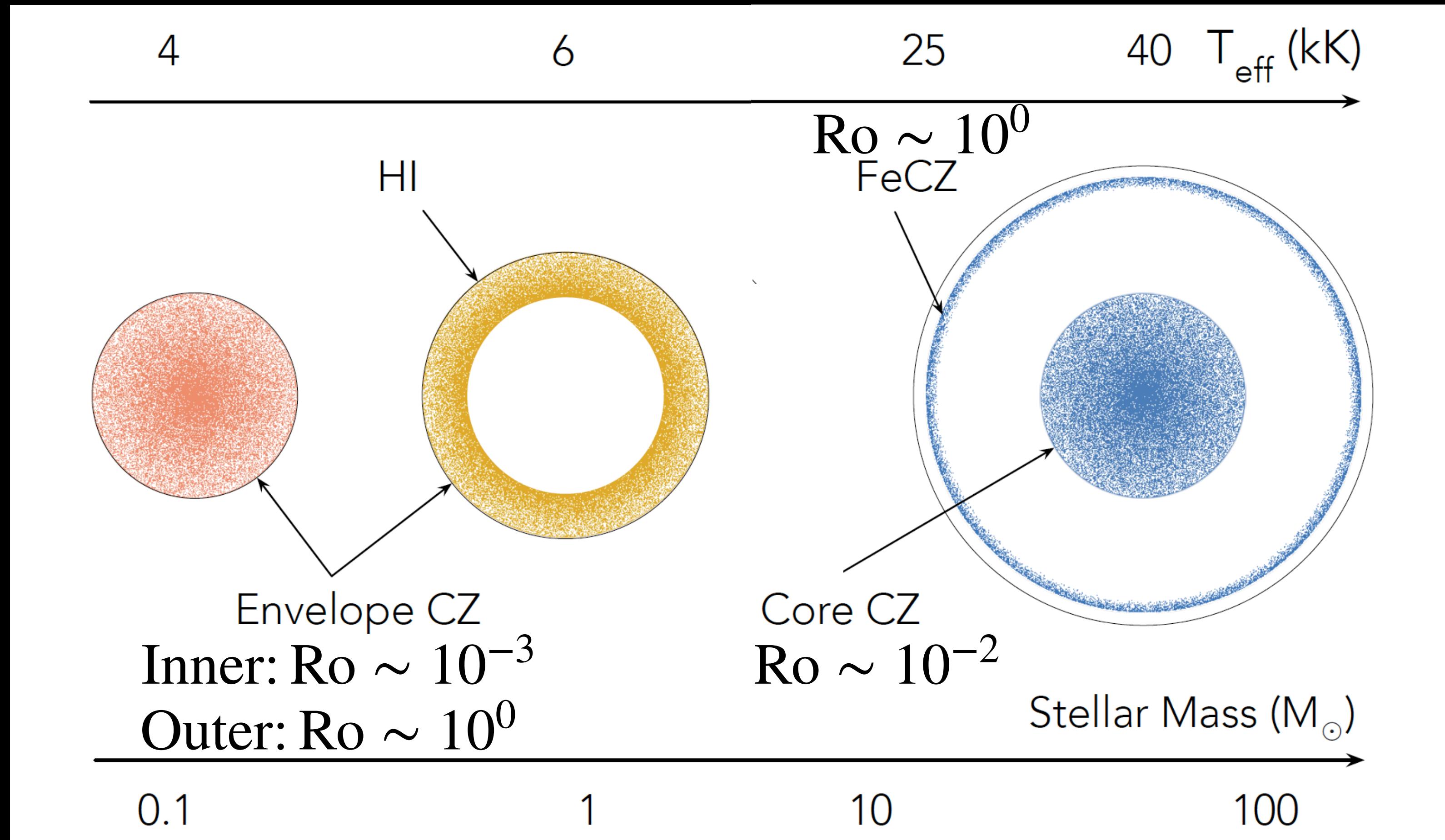
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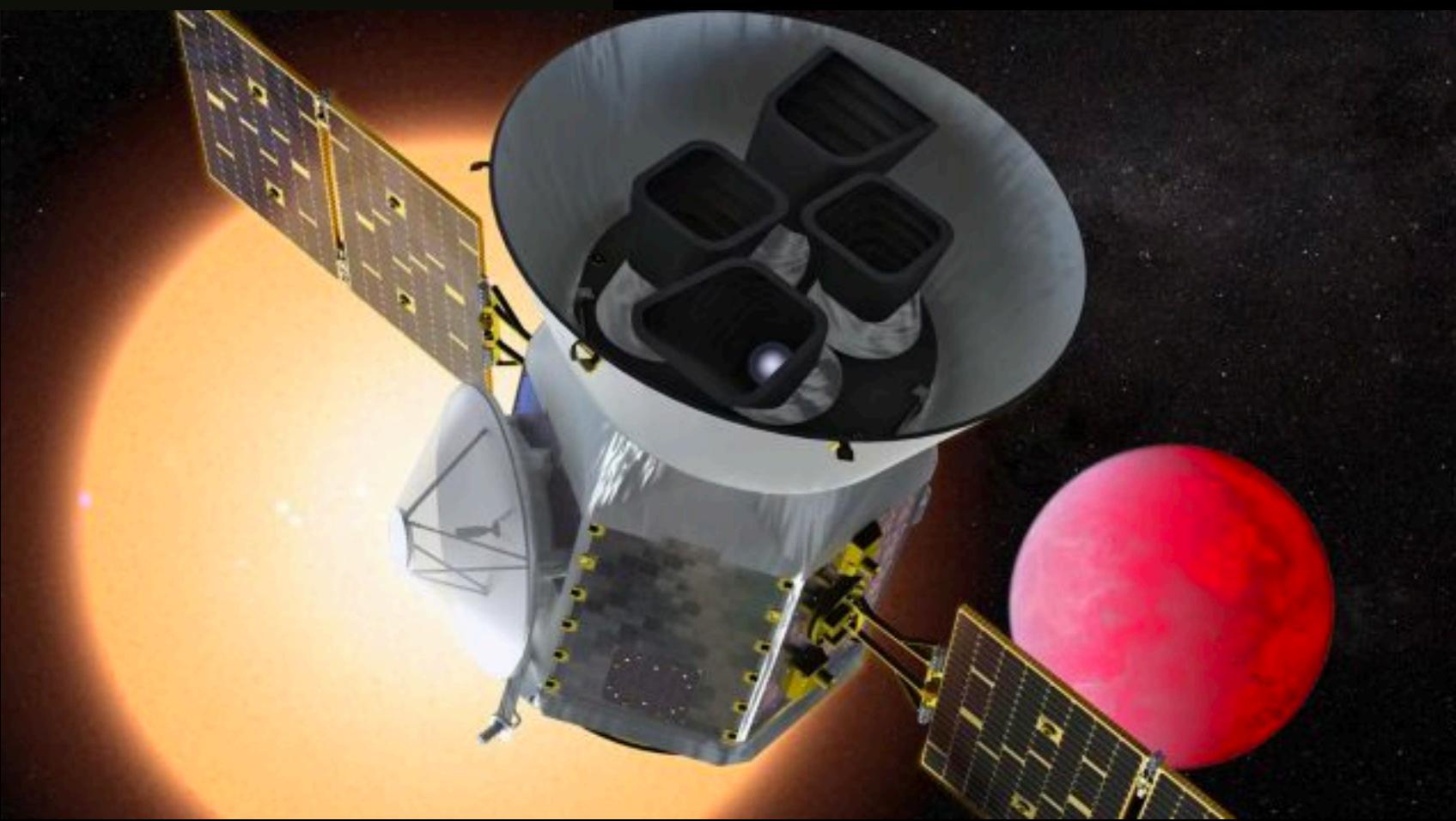


You can play this game with any non dimensional number.

Almost every regime is important in some star.

# Why now?

Observations: Kepler, TESS, and (soon! 2026) PLATO



# Why now?

But they're looking at *more* stars

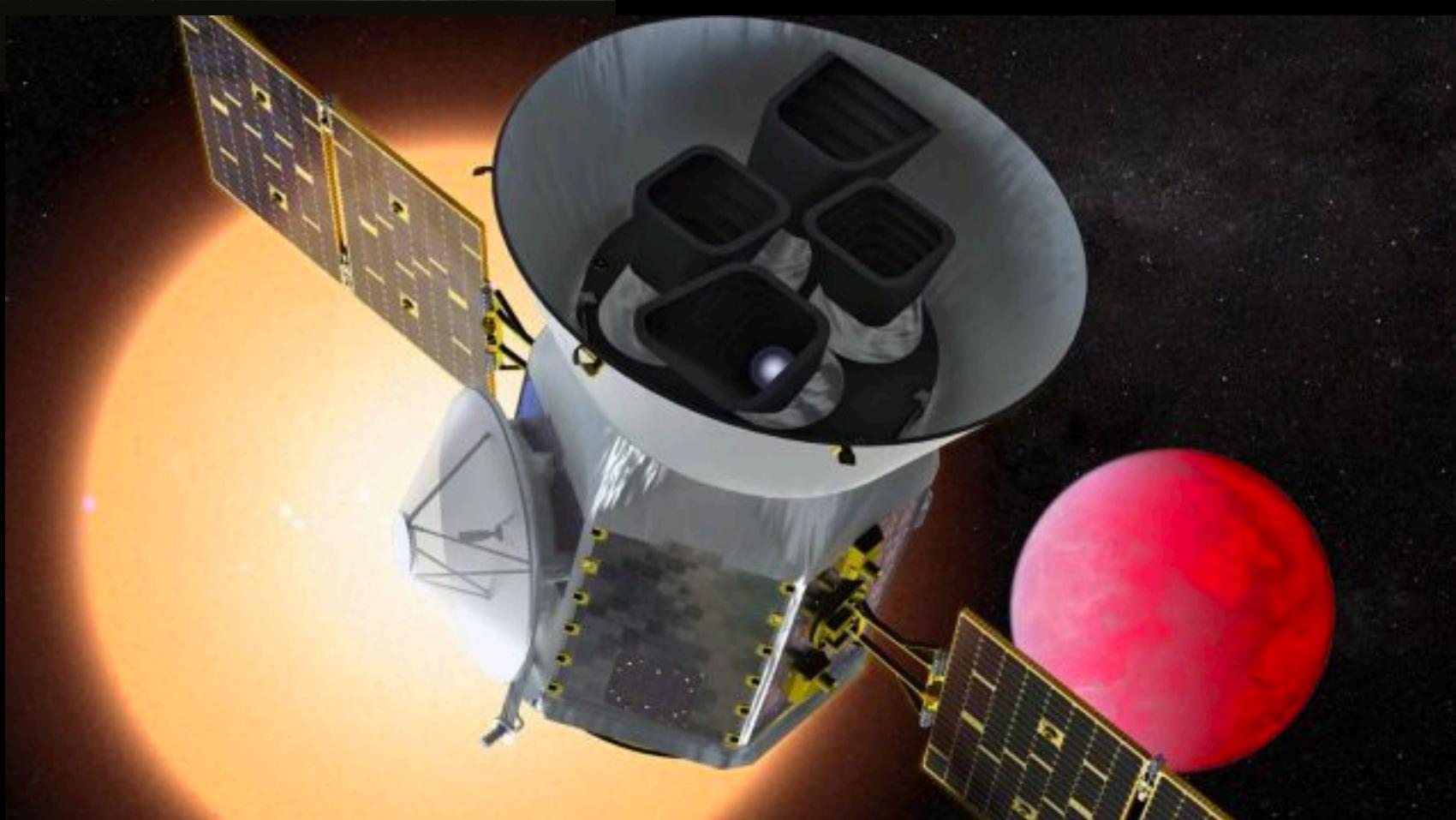
Kepler: ~150,000 stars



PLATO: ~1 million stars

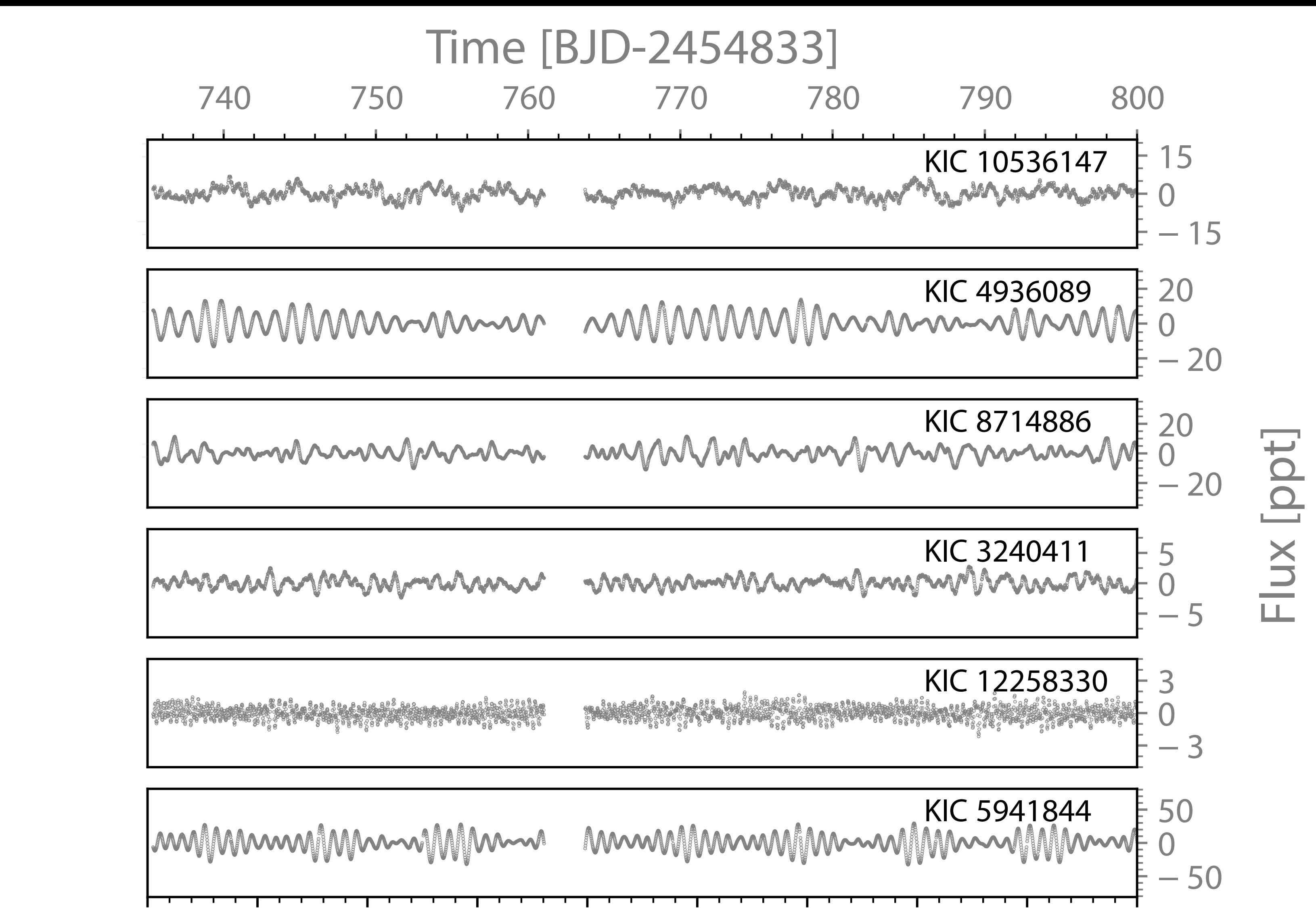


TESS: ~200,000 stars



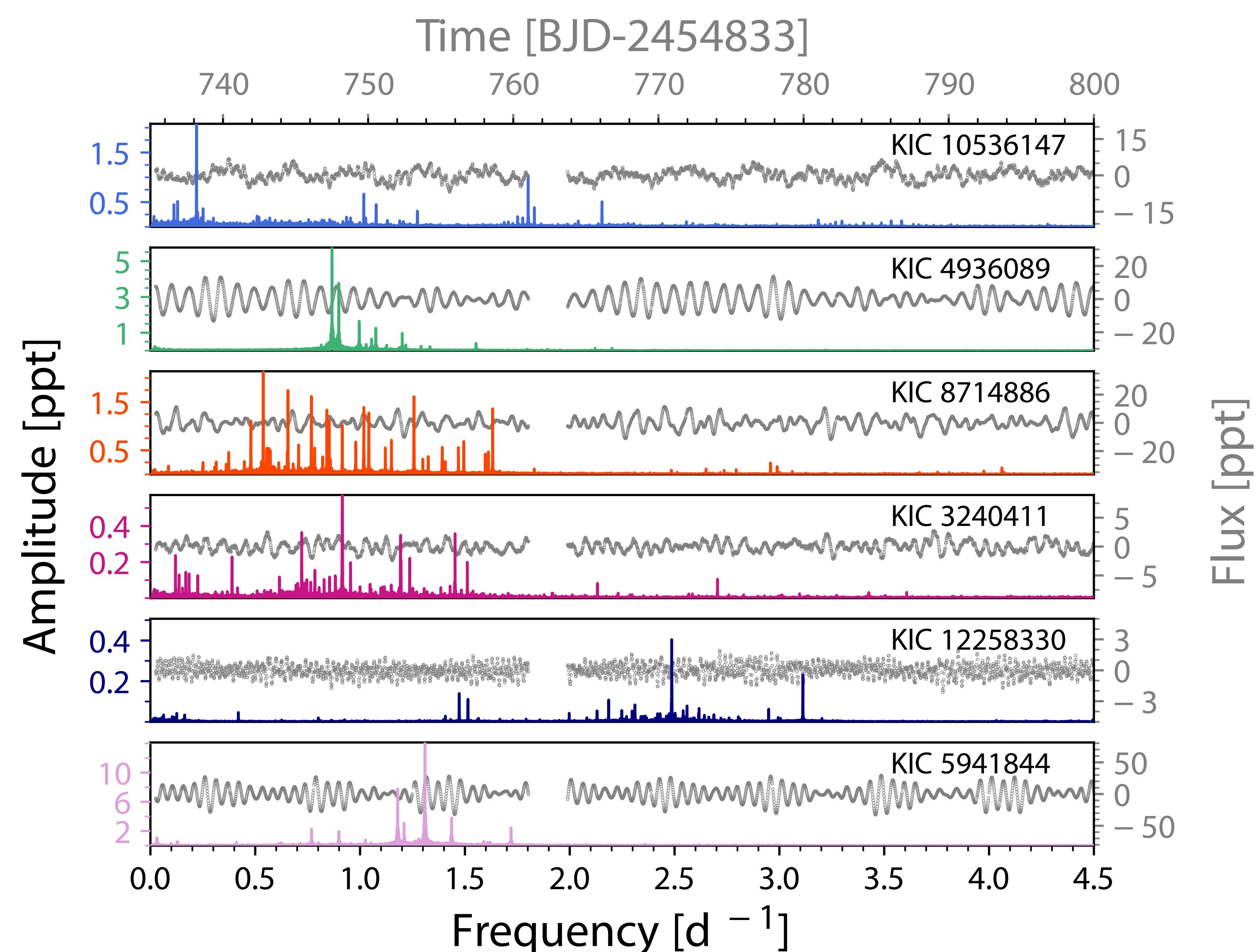
# Asteroseismology

Stars “wiggle”



# Asteroseismology

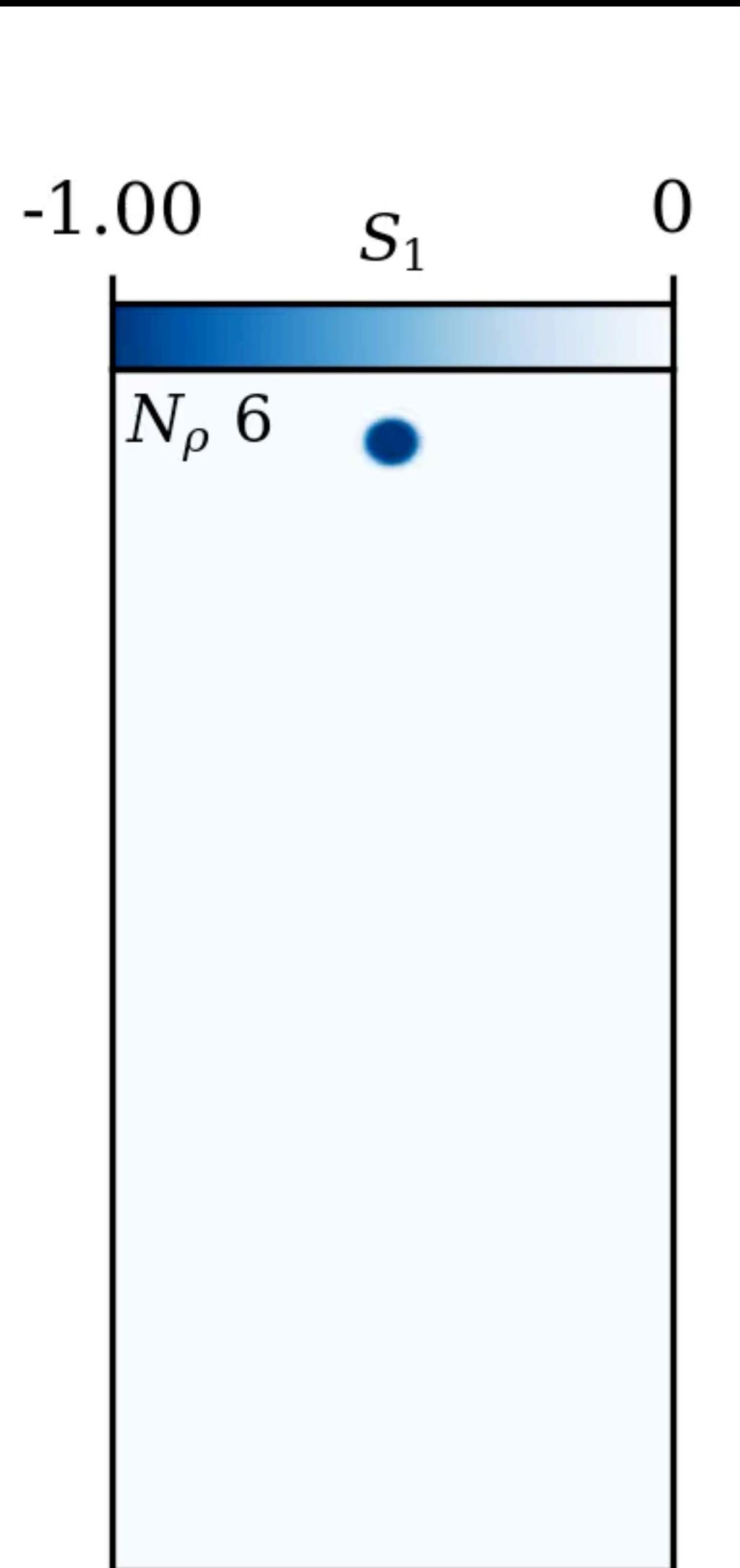
These wiggles are from waves that we can use to look inside of stars and discover "missing physics".



# My research studies the “missing physics”

Smallest length scales

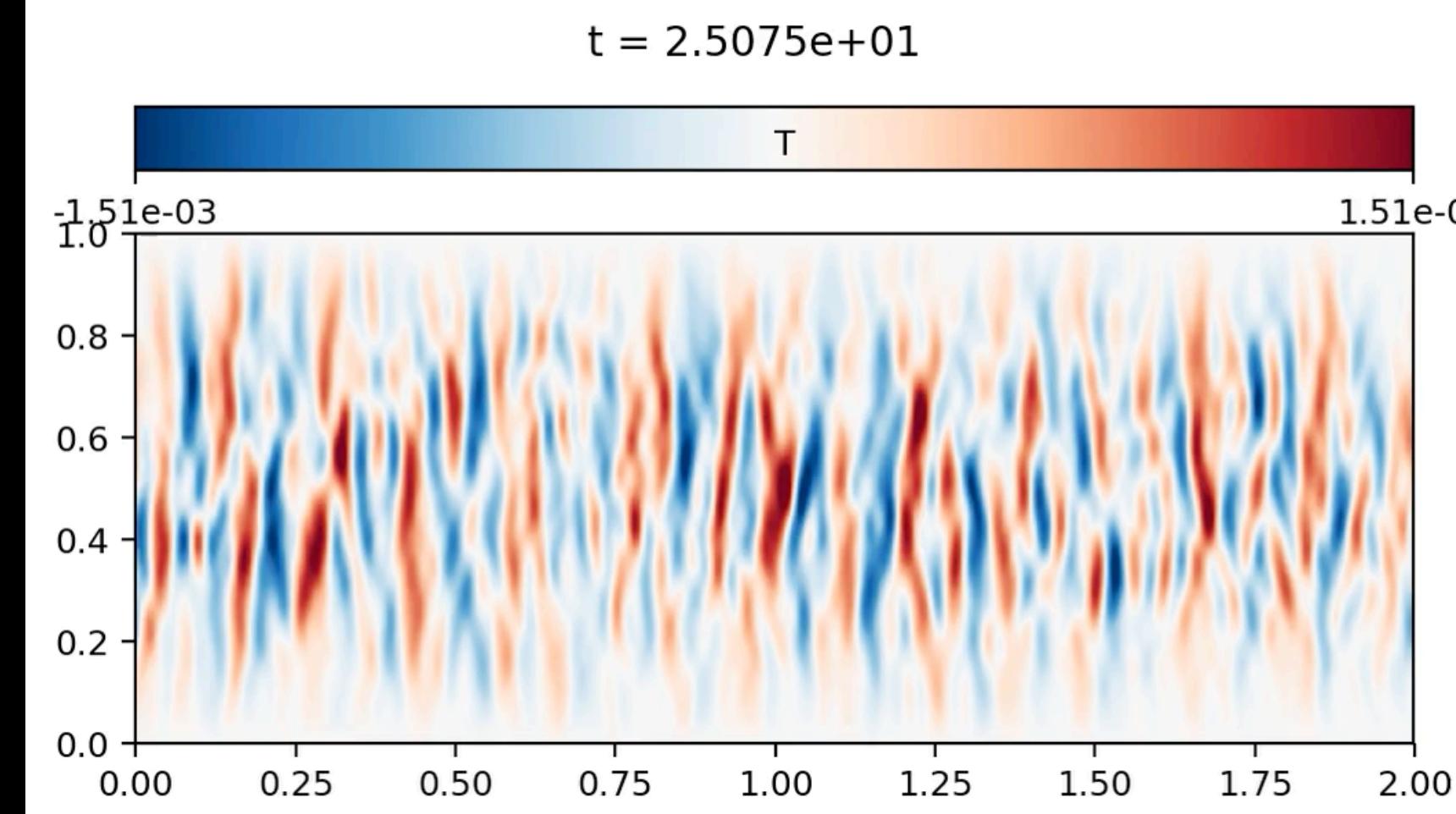
[Anders et al 2019 ApJ 884 65]



Disparate time scales

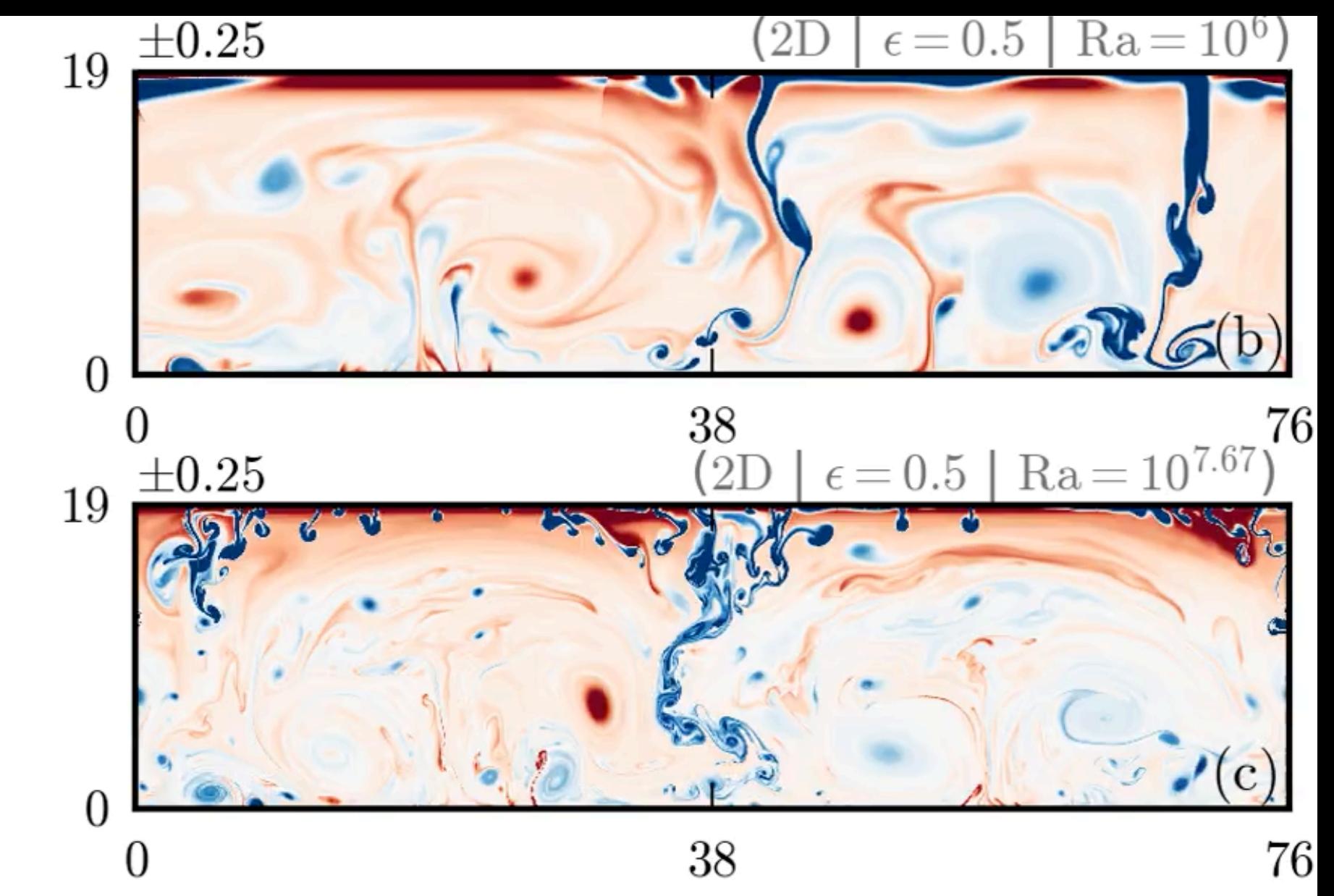
[Anders et al 2018 PRF 03 083502]

[Anders et al 2020 PRF 05 083501]



Density stratification

[Anders & Brown 2017 PRF 02 083501]





Methods paper: [Burns et al 2020 PRR 02 023068]  
Eigenfunctions: [Oishi et al 2021 JOSS 06 30790]

# Dedalus is a programming language for PDEs

Very flexible; includes IVPs, BVPs, EVPs.

$$\nabla \cdot \vec{u} = 0$$

$$\partial_t \vec{u} + \vec{u} \cdot \nabla \vec{u} + \nabla \varpi = -T \vec{g} + \nu \nabla^2 \vec{u}$$

$$\partial_t T + \vec{u} \cdot \nabla T = \kappa (\nabla^2 T + Q)$$

Internally heated,  
Boussinesq convection

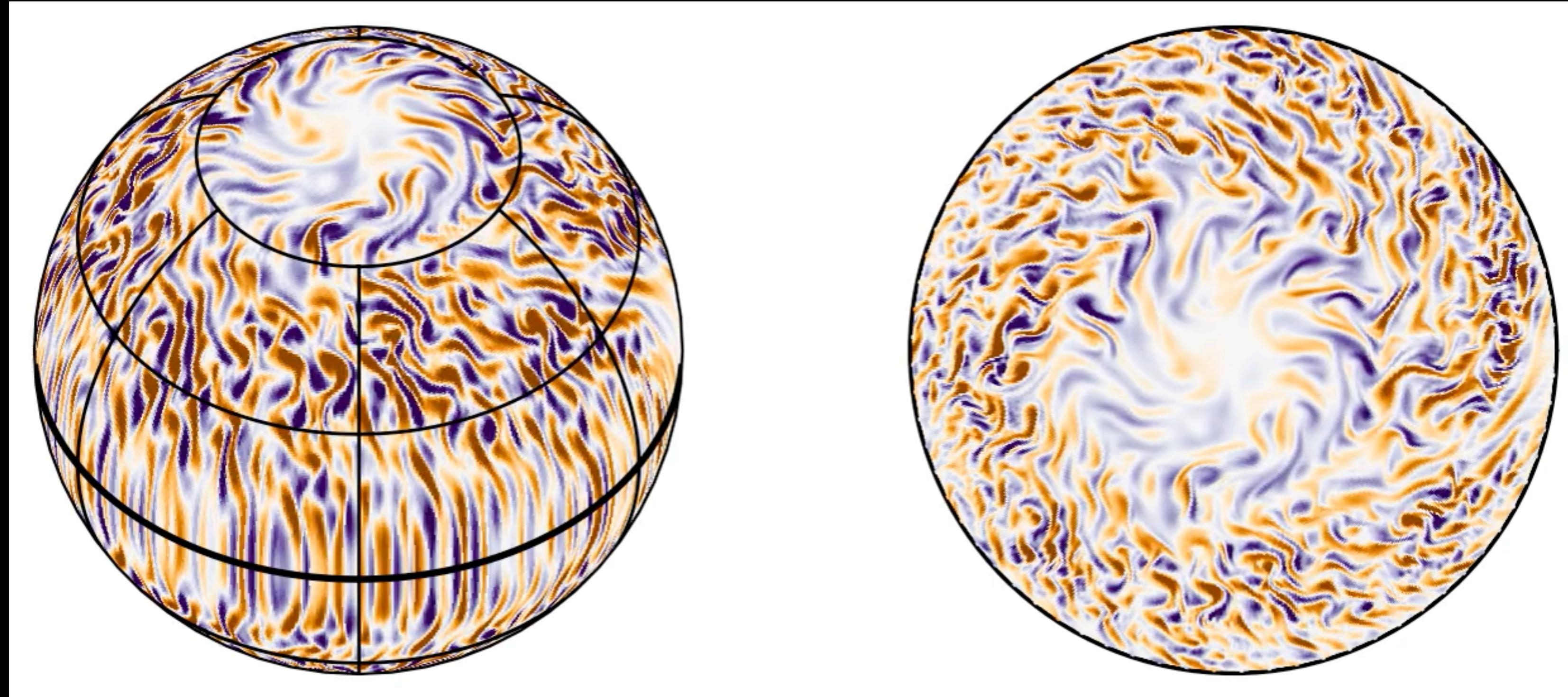
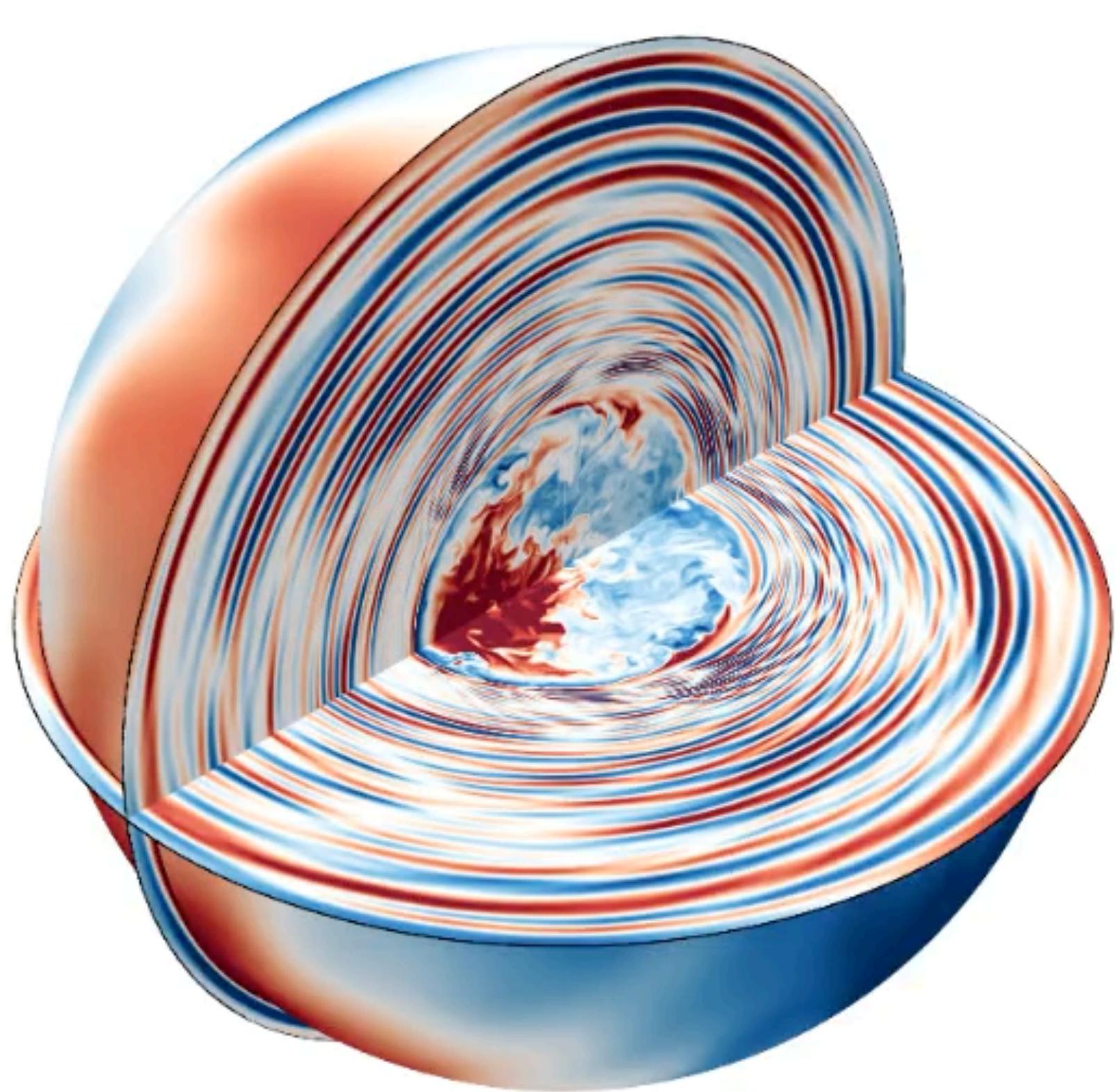
```
problem.add_equation("div(u) + tau_p = 0")
problem.add_equation("dt(u) - nu*lap(u) + grad(p) - r_vec*T + lift(tau_u) = - cross(curl(u),u)")
problem.add_equation("dt(T) - kappa*lap(T) + lift(tau_T) = - u@grad(T) + kappa*T_source")
```

Dedalus form

Methods paper: [Burns et al 2020 PRR 02 023068]  
Eigentools: [Oishi et al 2021 JOSS 06 30790]

# Dedalus' many applications: Geo/Astro Convection

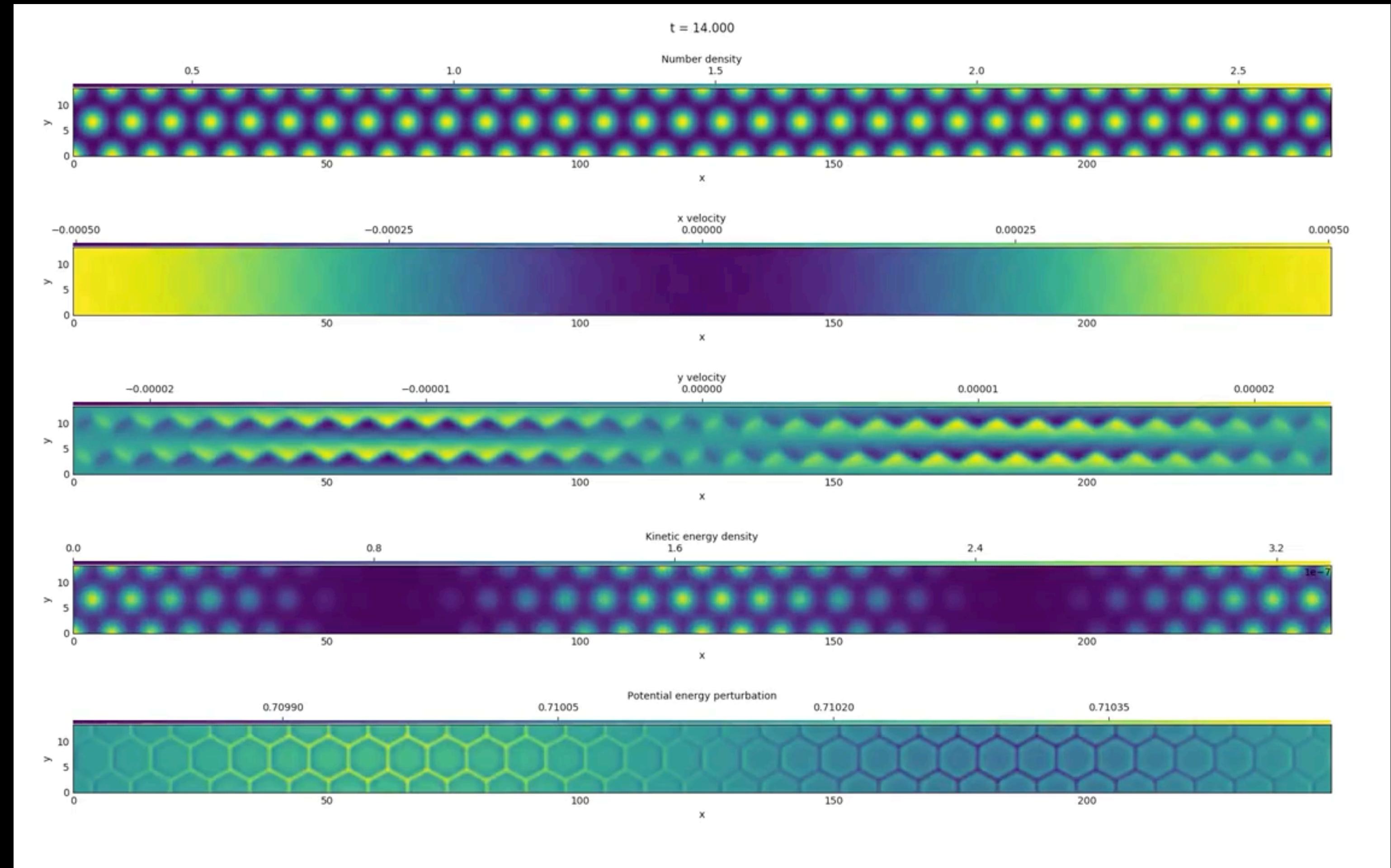
Convection / waves in stars



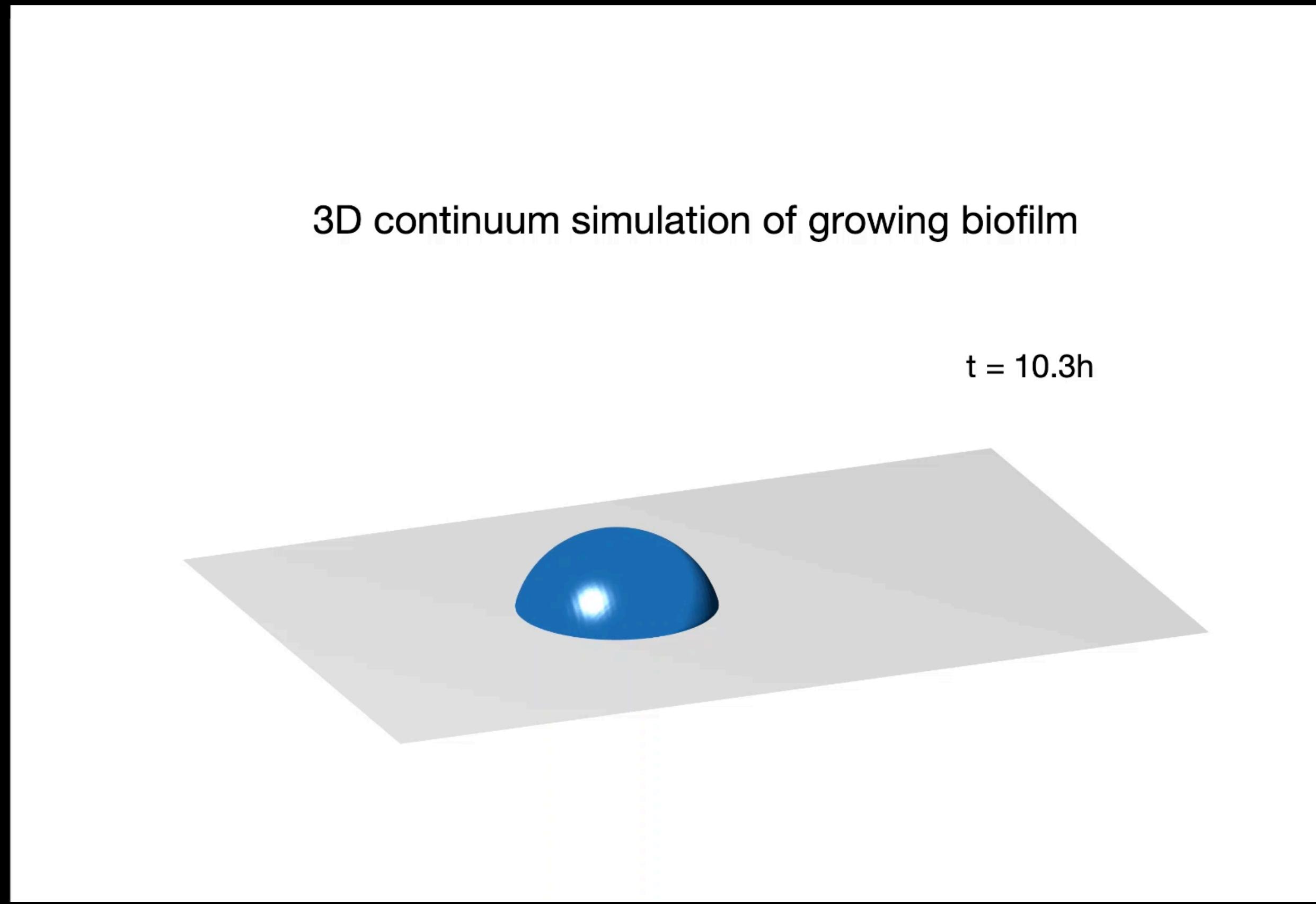
Rotationally-constrained convection in stars and planets

# Dedalus' many applications: Quantum Fluids

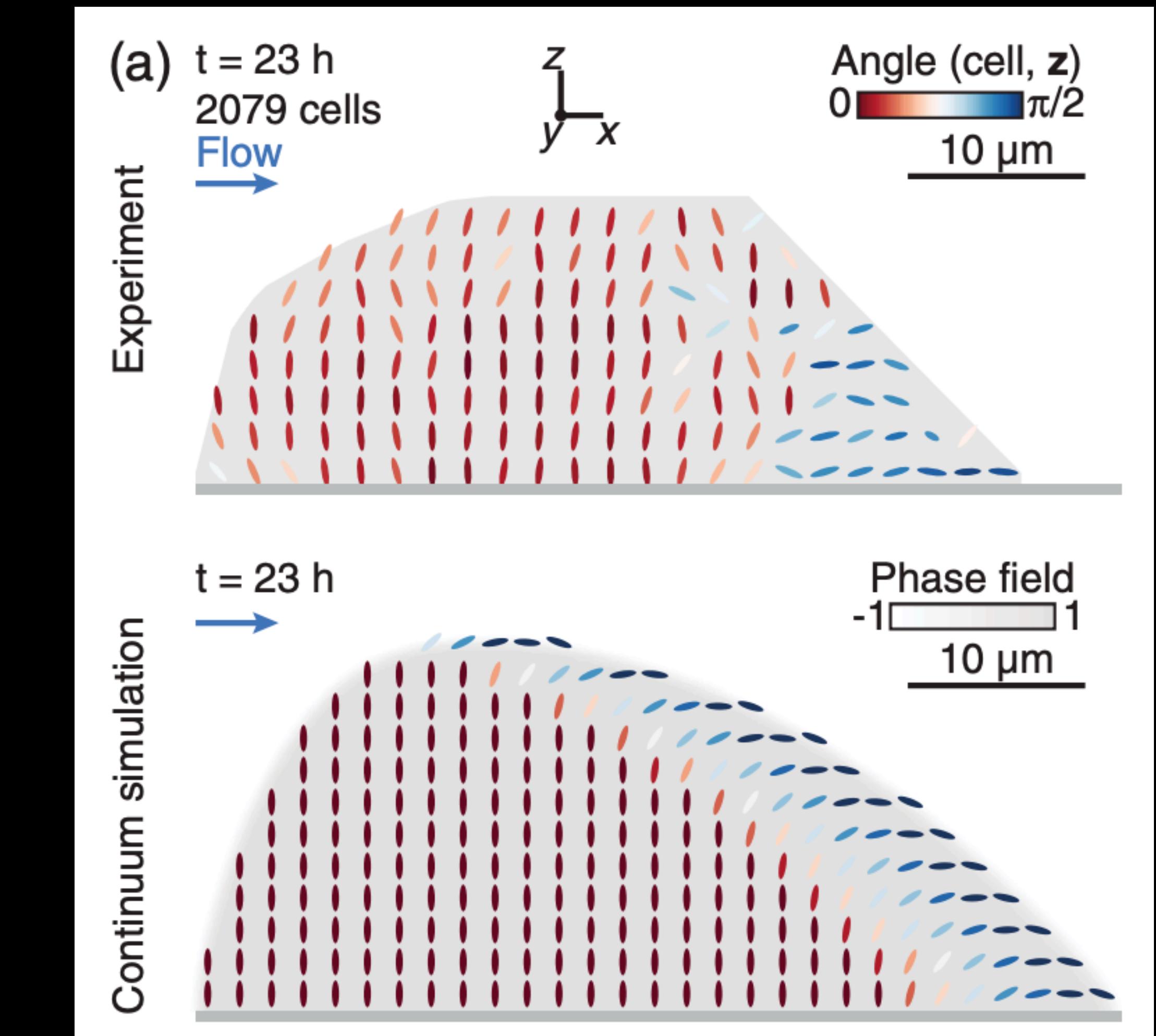
Sound wave dynamics  
in a supersolid



# Dedalus' many applications: Biofluids



Rod-shaped  
bacteria in a flow.

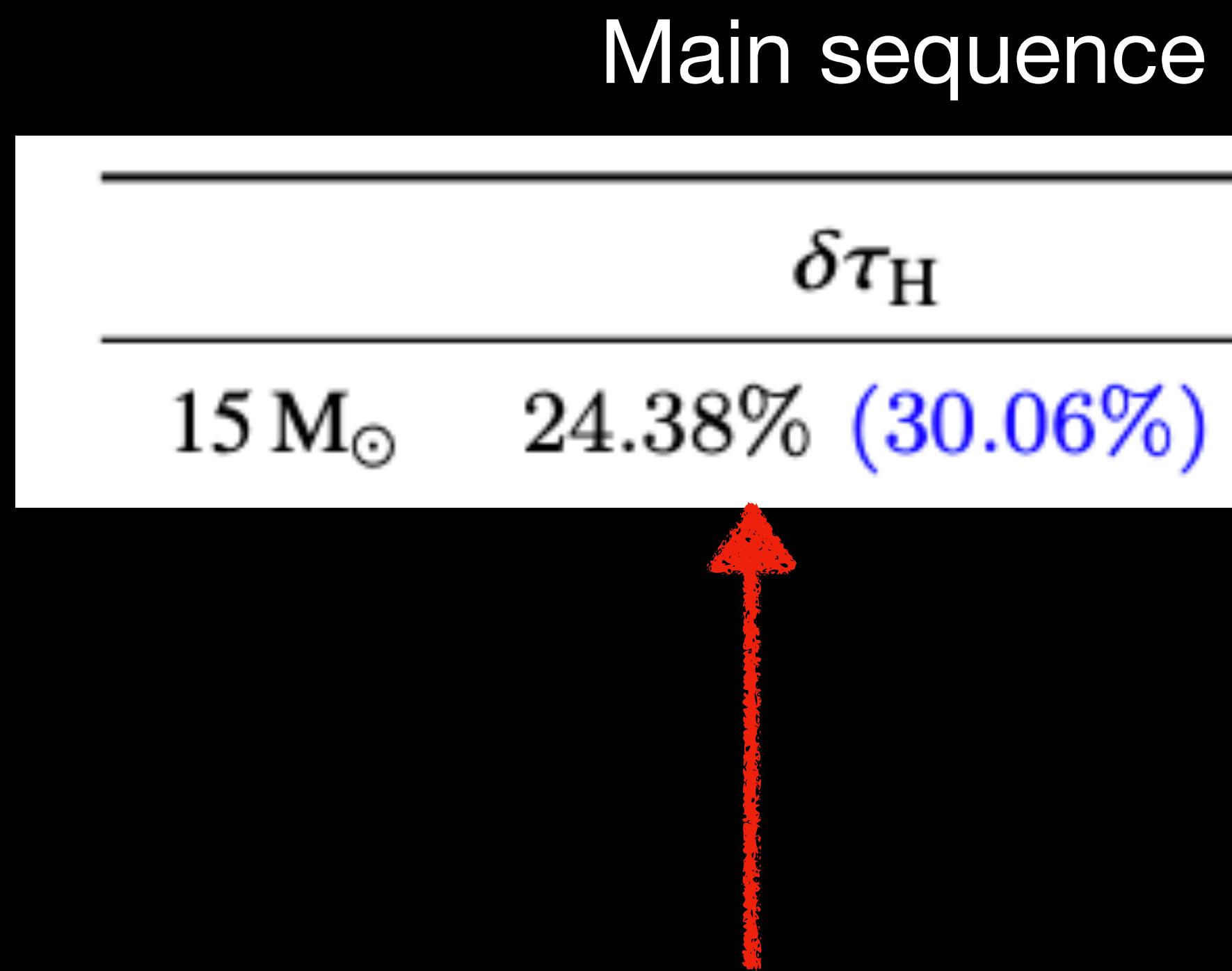


[Pearce et al 2019 PRL 123 258101]

I'm excited to use my expertise with  
this tool to work with all of you.

# Uncertainties from mixing are huge

# Uncertainties from mixing are huge



[Kaiser et al 2020, MNRAS 496 2]

Age uncertainty due to mixing prescriptions

# Uncertainties from mixing are huge

Main sequence

$\delta\tau_H$	
15 M <sub>⊙</sub>	24.38% (30.06%)

Blue supergiant

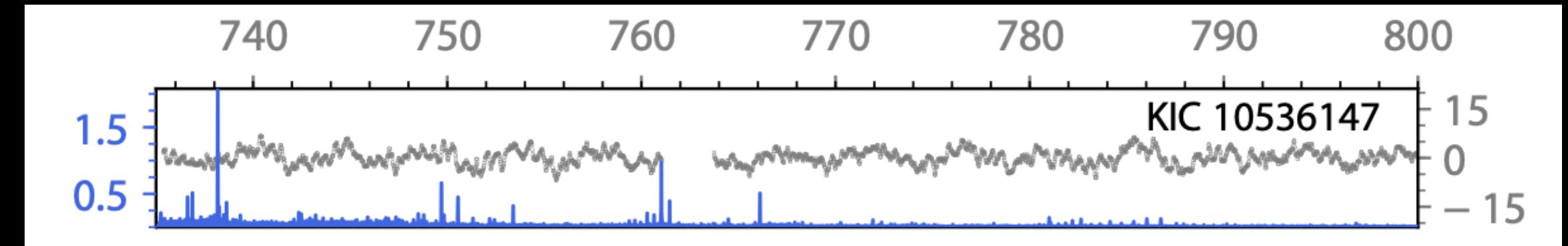
$\delta\tau_{BSG}$	
	5800.0% (5800.0%)

[Kaiser et al 2020, MNRAS 496 2]

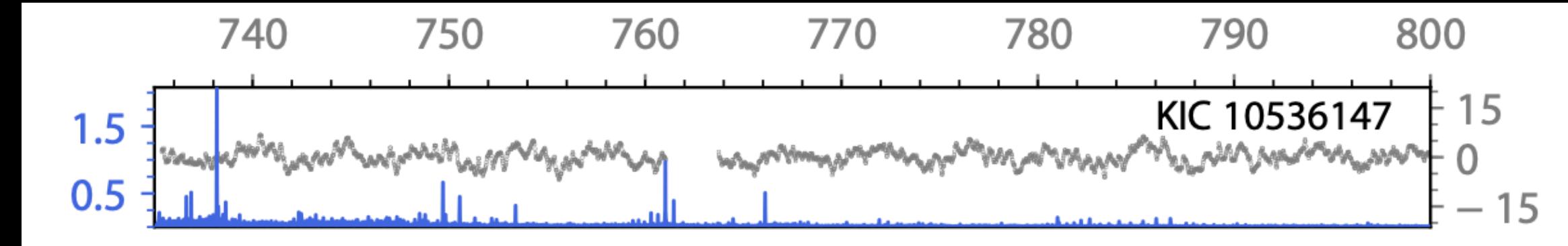
Age uncertainty due to mixing prescriptions

Ouch

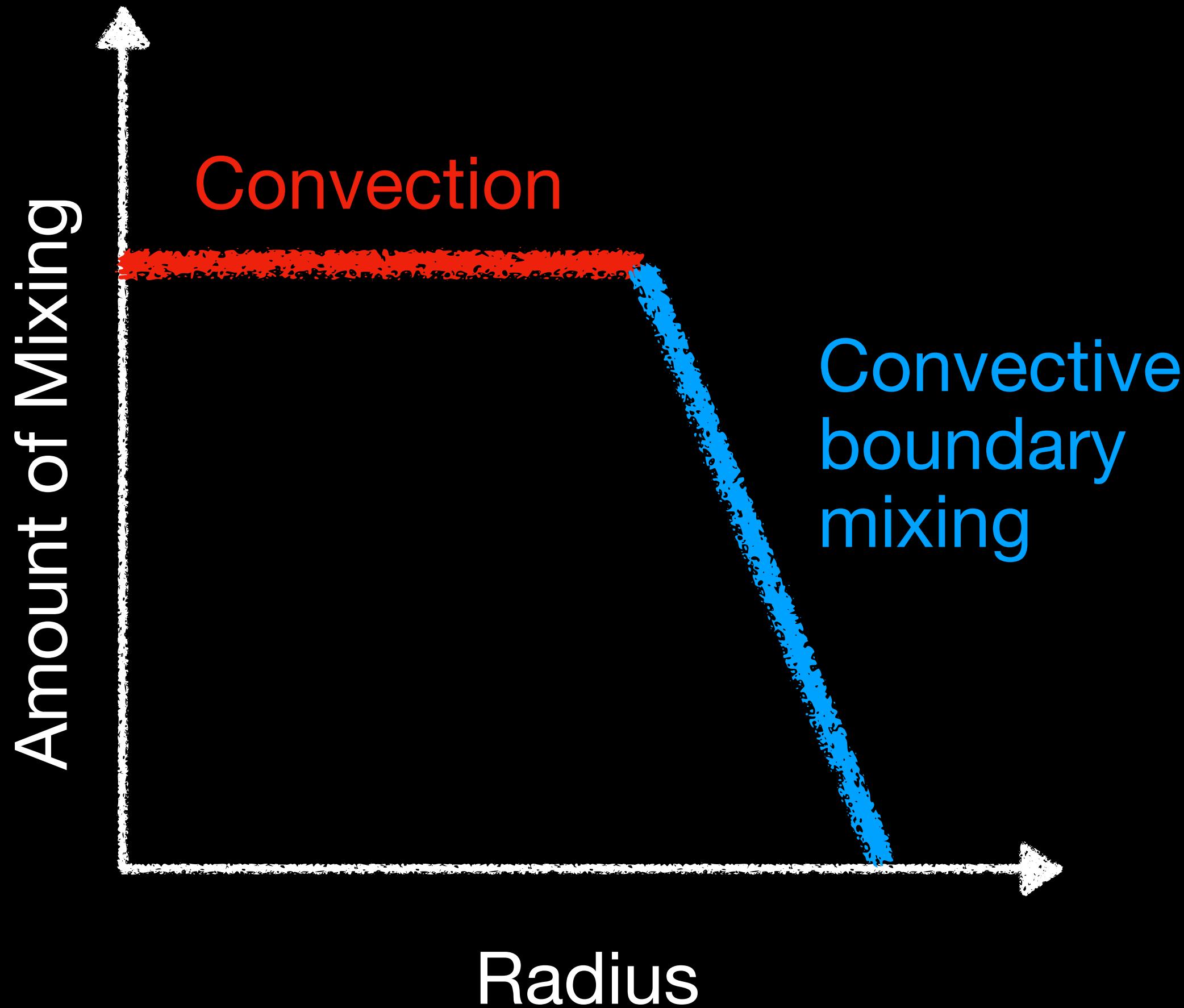
# Asteroseismology puts empirical constraints on mixing



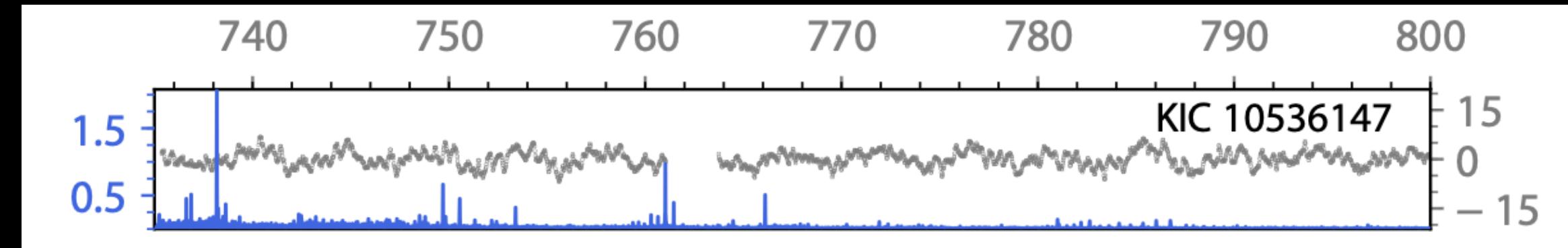
# Asteroseismology puts empirical constraints on mixing



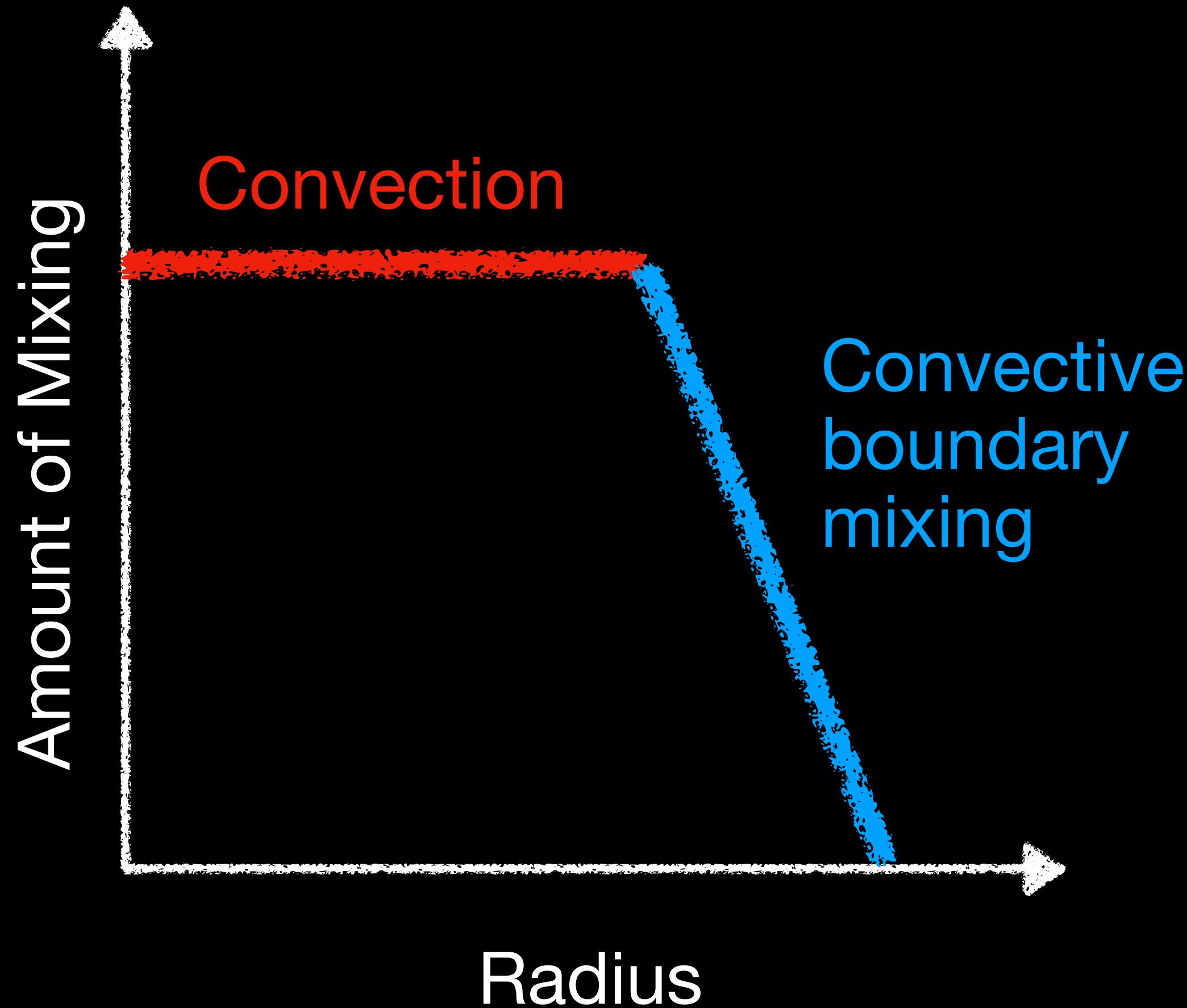
Model 1: Convective Overshoot



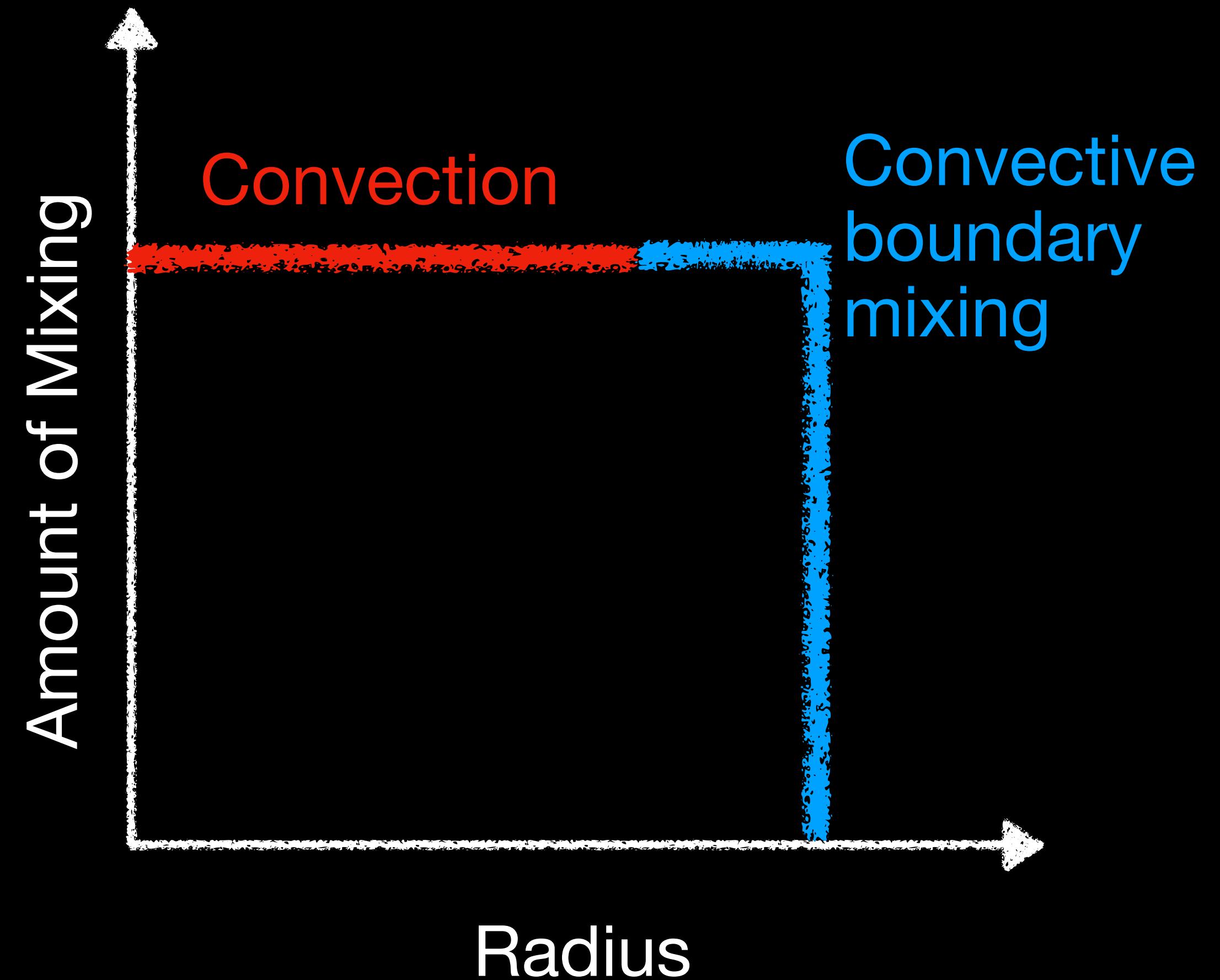
# Asteroseismology puts empirical constraints on mixing



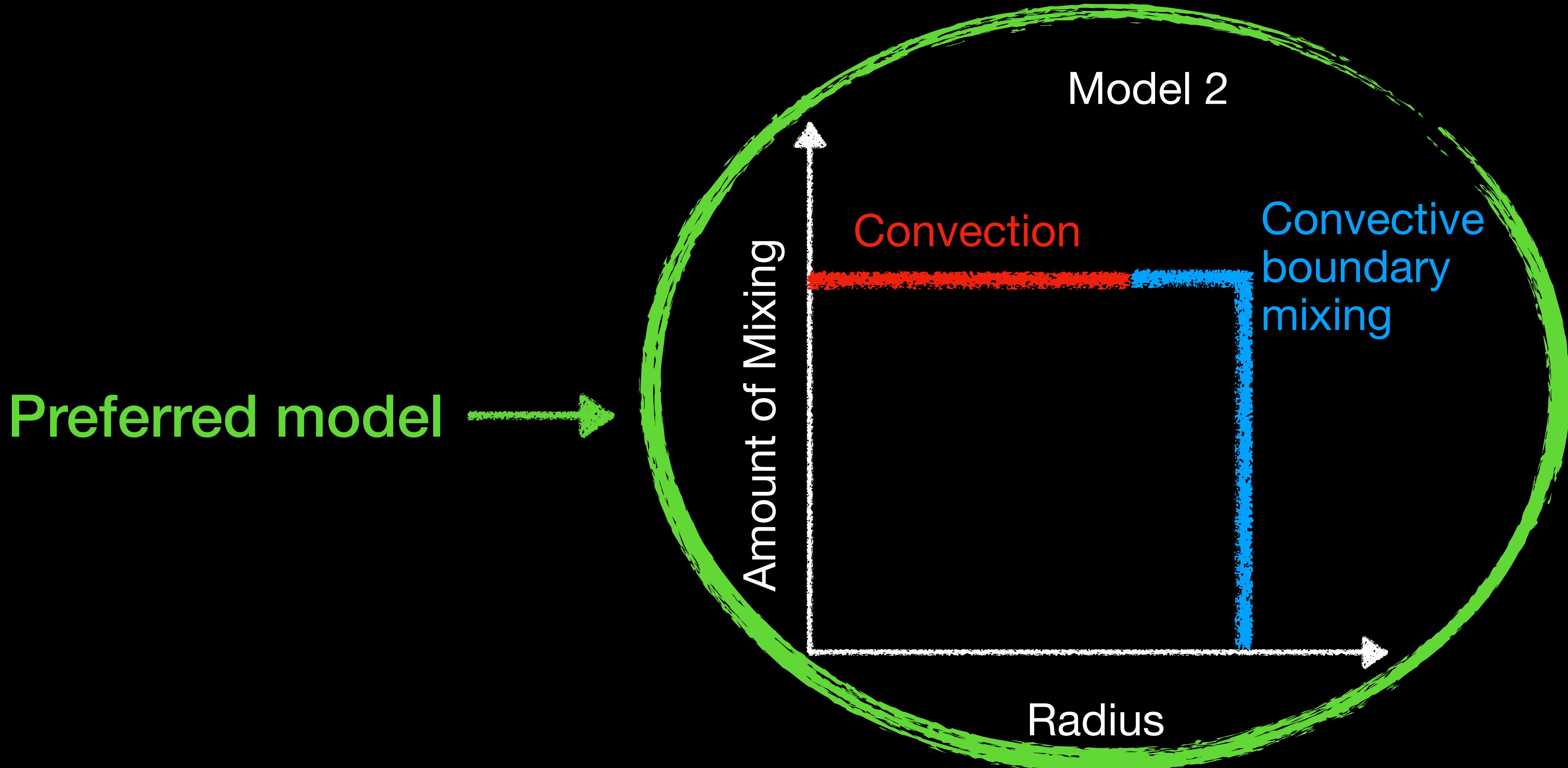
Model 1: Convective Overshoot



Model 2: Convective Penetration

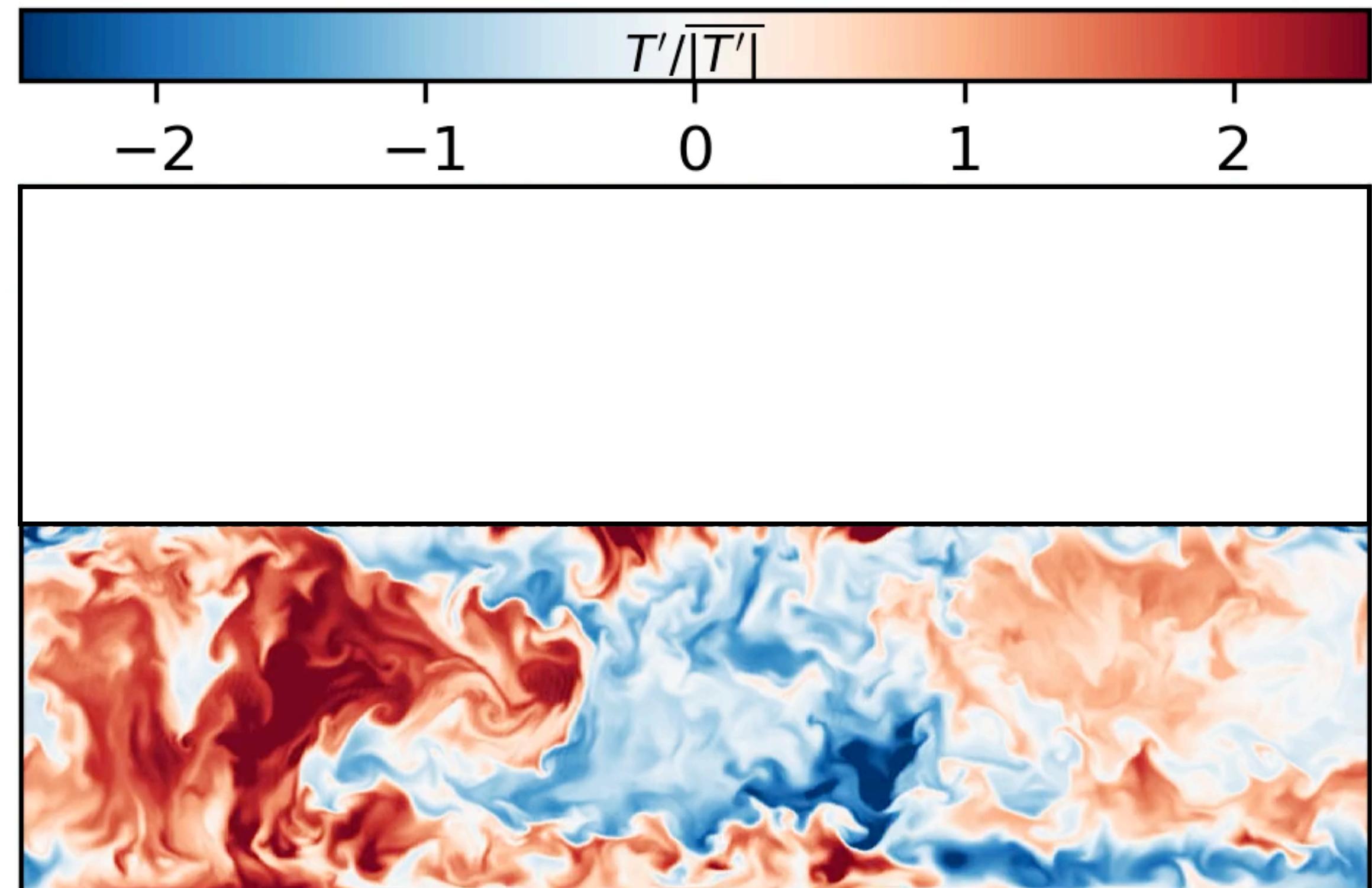
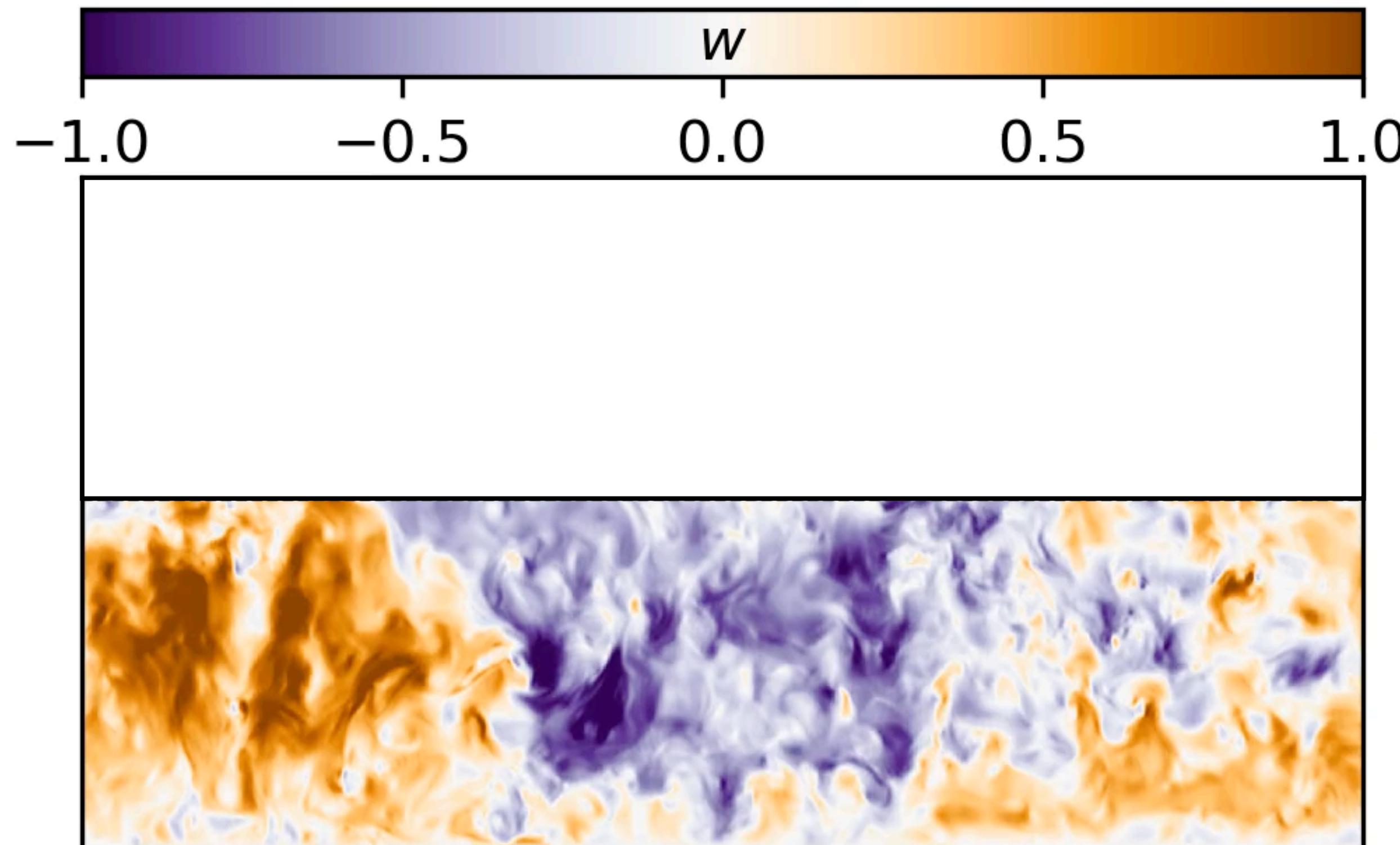


# Asteroseismology puts empirical constraints on mixing



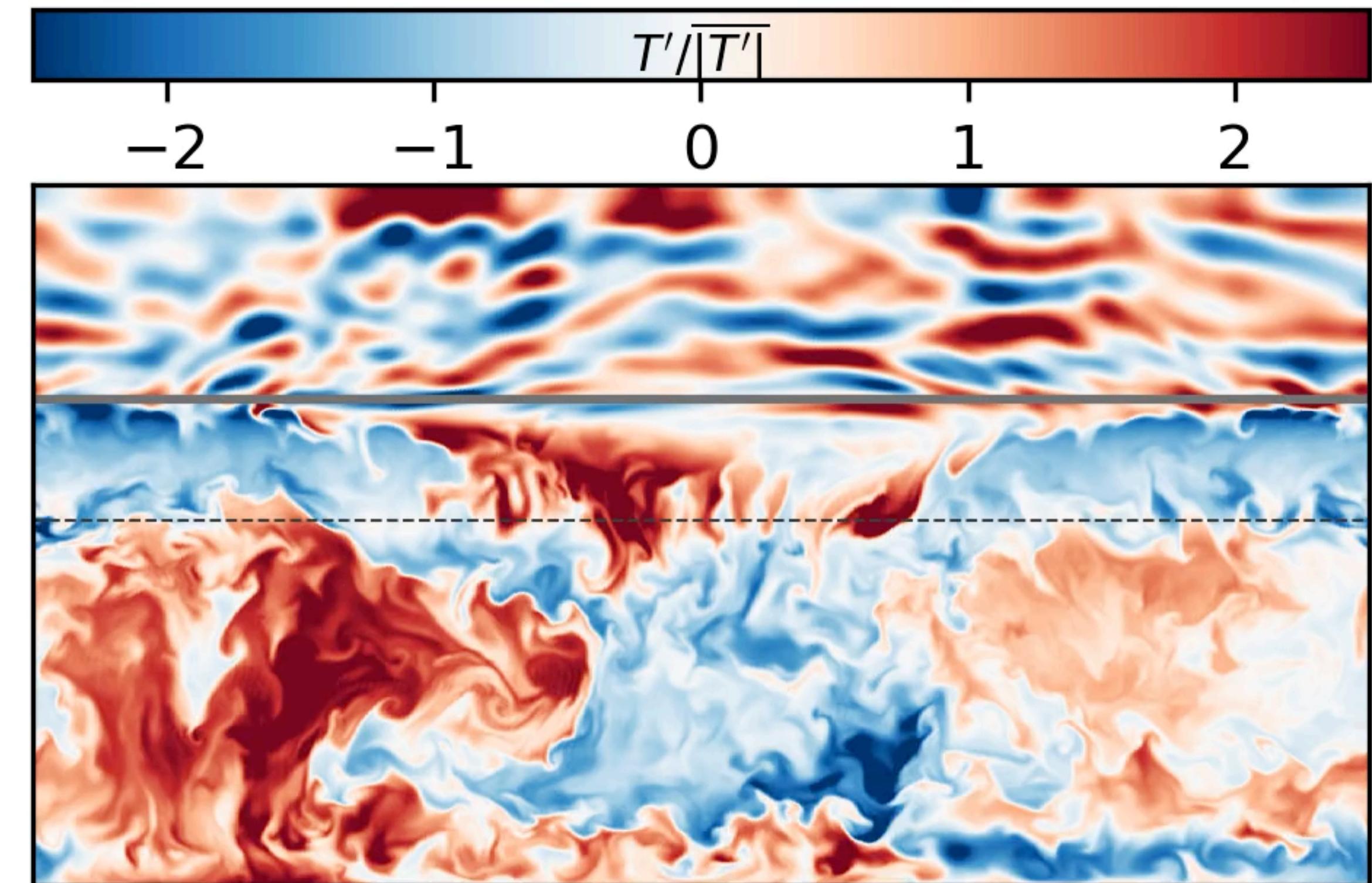
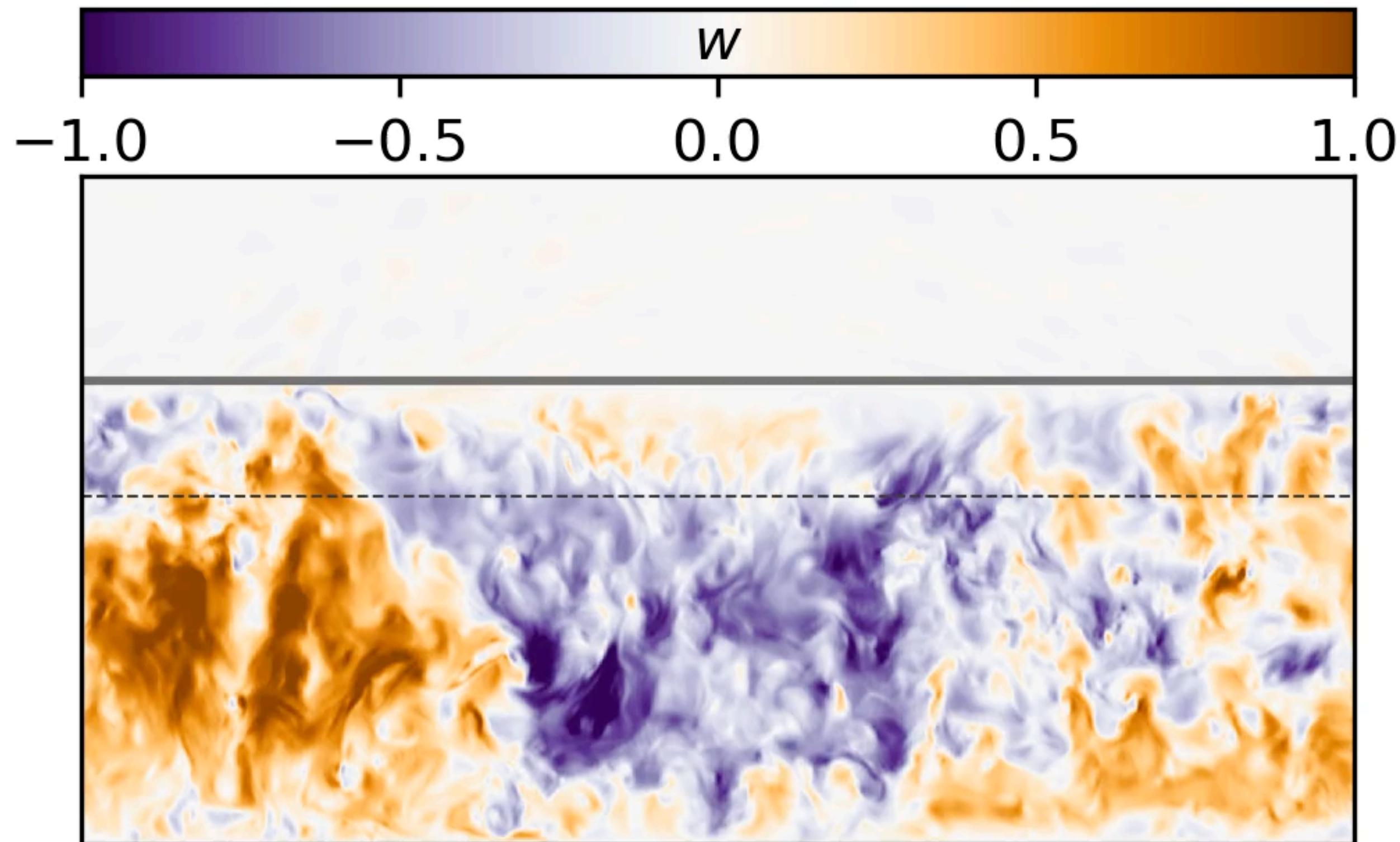
# Convection in a box

Widely studied, pretty well understood



# Convection eats into an adjacent stable region

## Our simulations look like what observers see in stars!



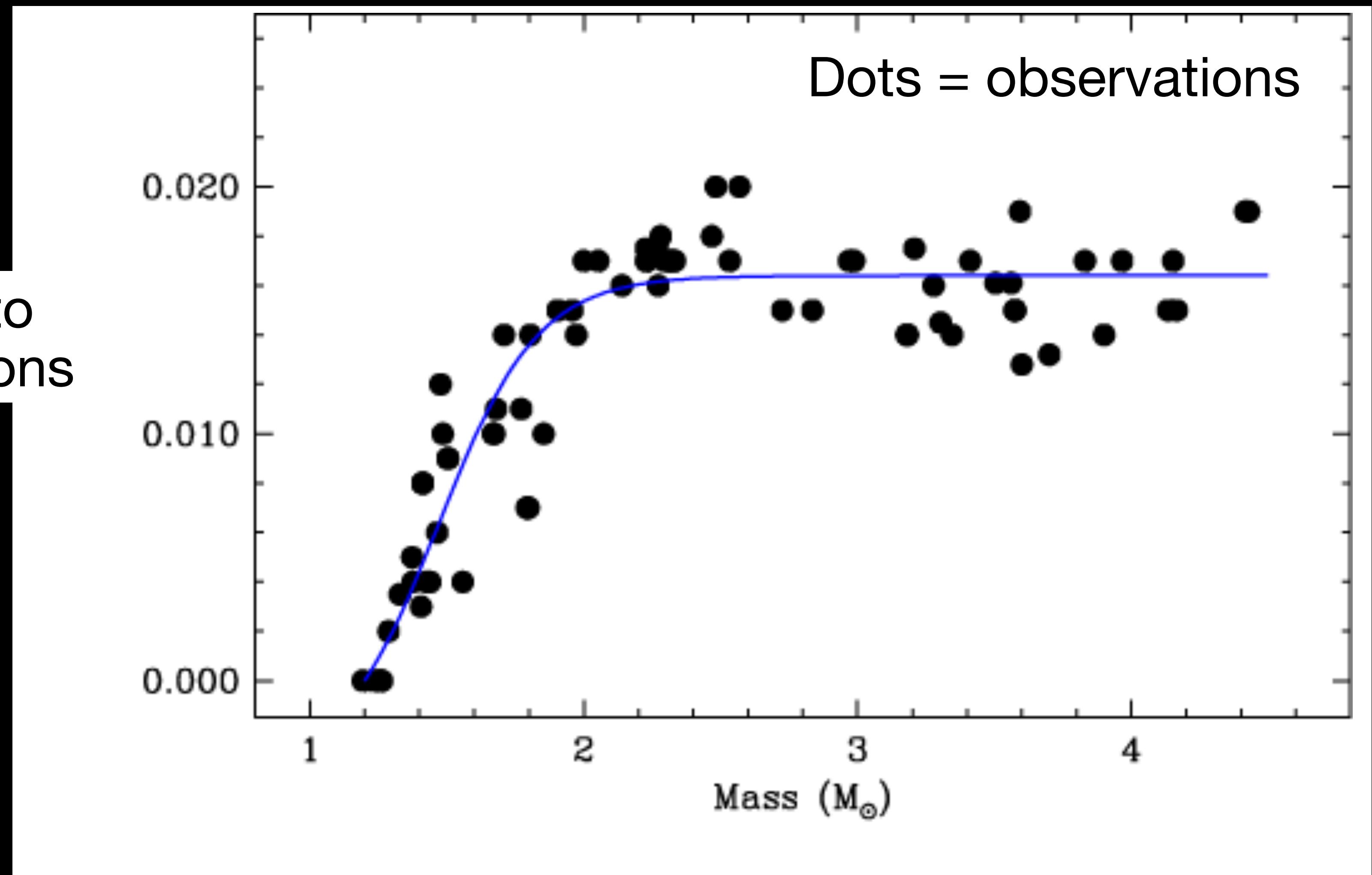
This is my most impactful work.

[Anders et al 2022 ApJ 926 169]

# Upshot: Mixing in Eclipsing Binaries

Observed mass dependence

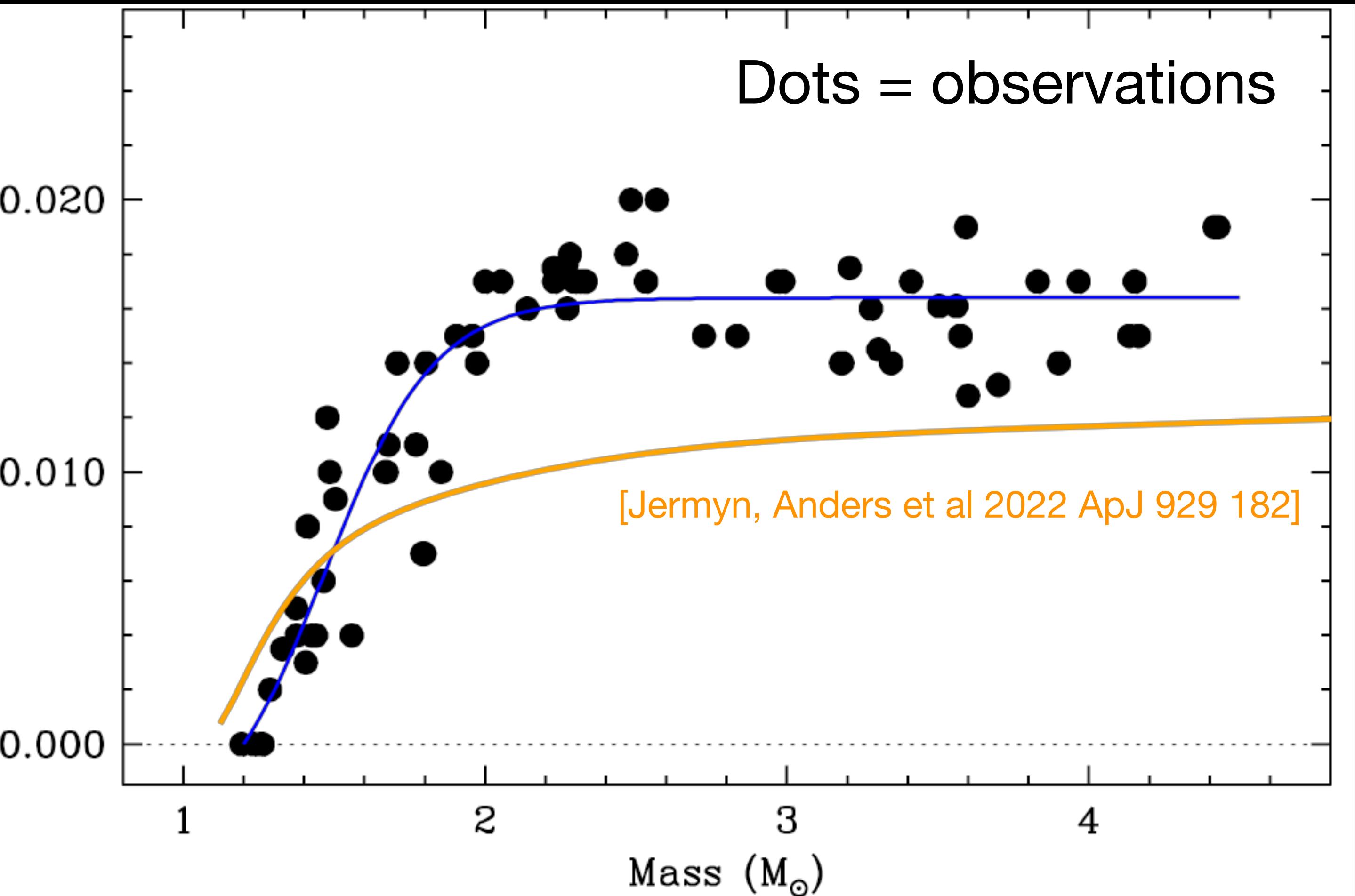
Mixing required to  
match observations



# Upshot: Mixing in Eclipsing Binaries

New theory matches better at low mass; still more work to do!

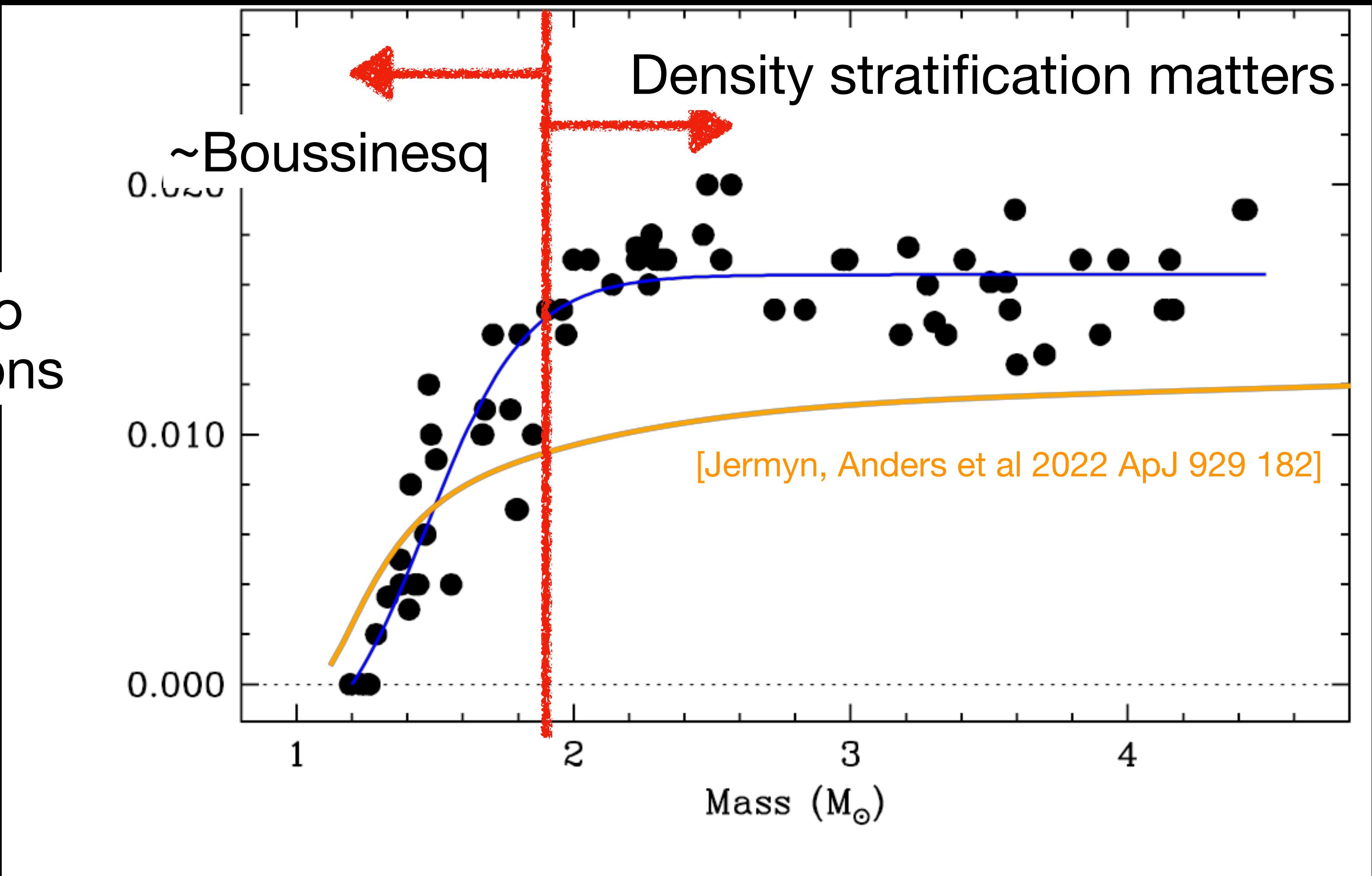
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# Upshot: Mixing in Eclipsing Binaries

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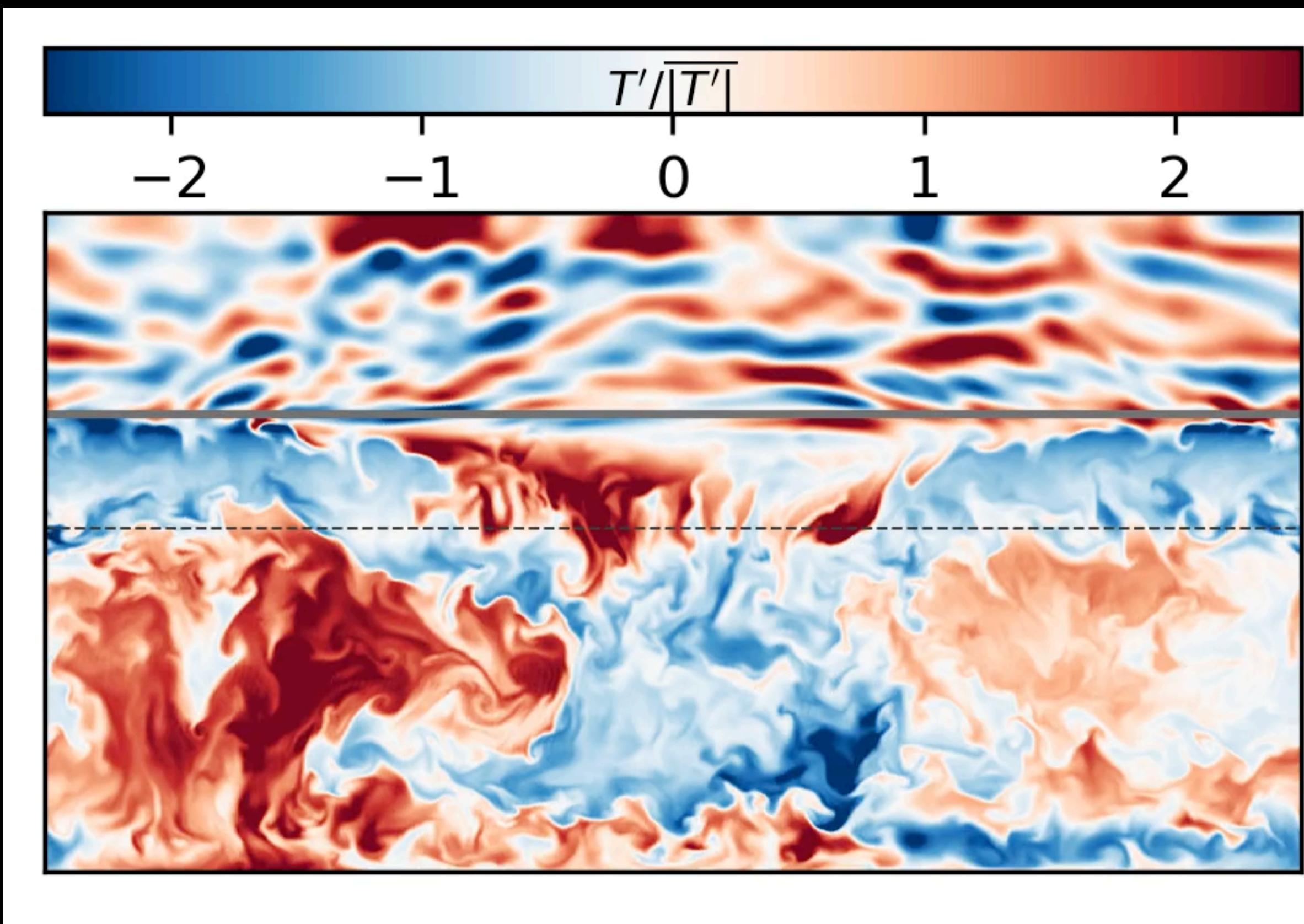
Mixing required to  
match observations



# What's next?

## 1. Improve the theory of penetrative convection with more ingredients

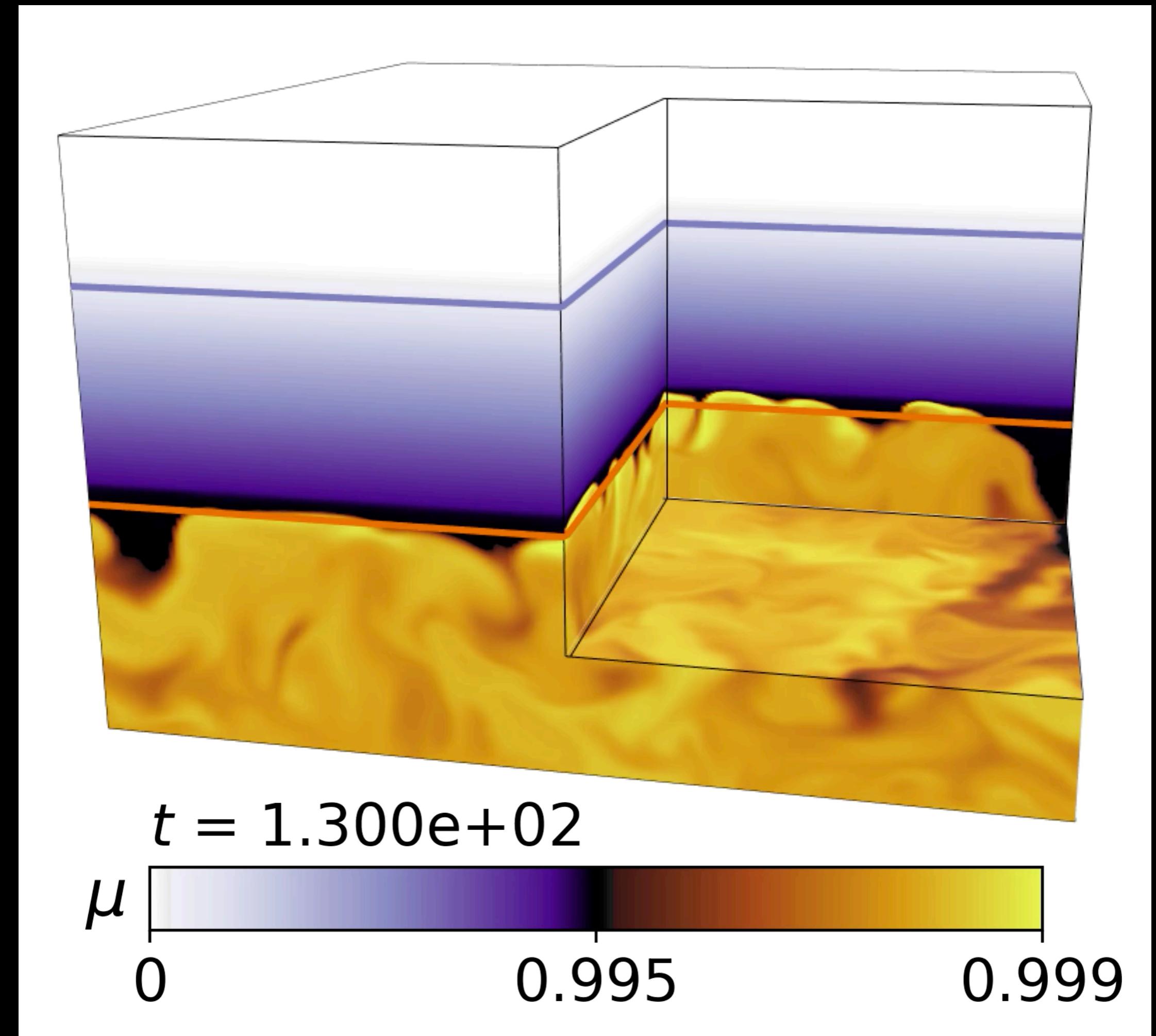
- Box -> sphere
- Boussinesq -> Compressible
  - (Sims, Theory)
- Self-consistent expected convective boundary



# What's next?

## 2. Answer other open boundary mixing questions

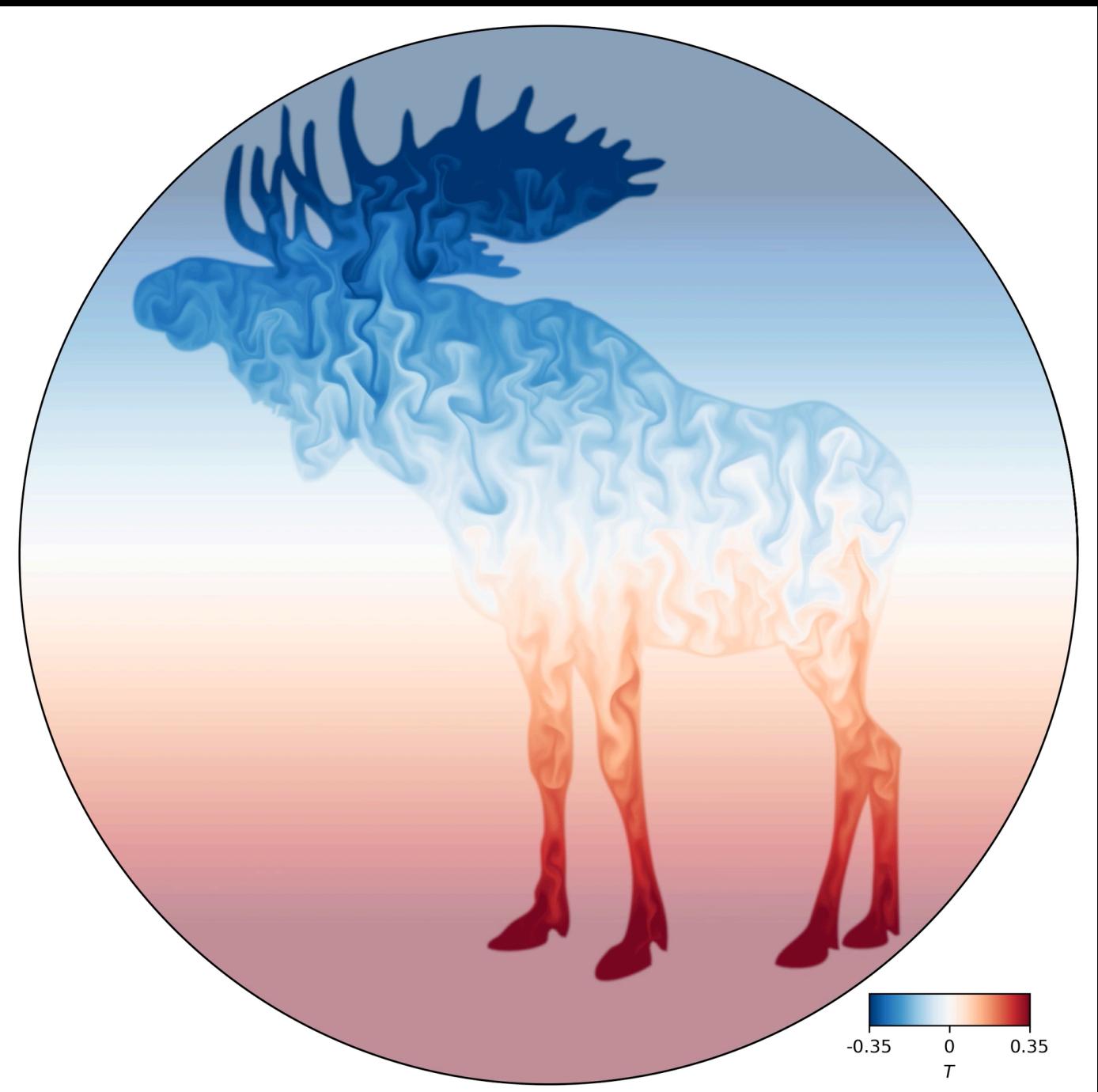
- Sometimes convection zones aren't equilibrated!
  - Need to better understand **entrainment**
  - Parameterize for 1D boundary modeling.
  - Move prior studies to 3D, compressible, spherical geometry, etc, etc, etc...



# Summary

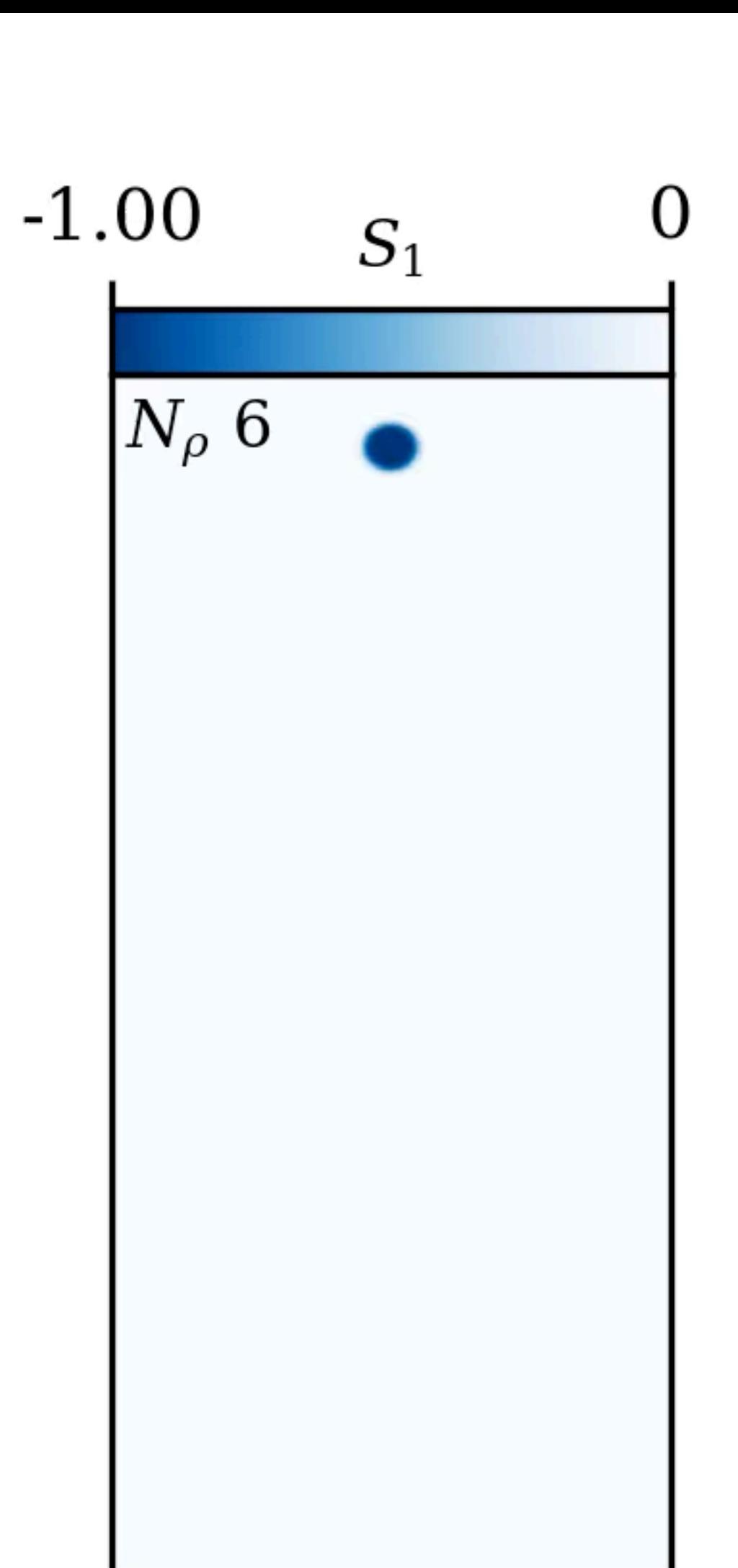
- It's an exciting time to be studying astrophysical fluid dynamics.
- I use Dedalus to study astro-fluids and am excited to collaborate with you all and apply it to other fields.
- My current and future research focus on improving theories of convective boundary mixing.

...And sometimes I study Moosinesq convection.  
[Anders et al 2022 arxiv: 2204.00002]



# My research studies the “missing physics”

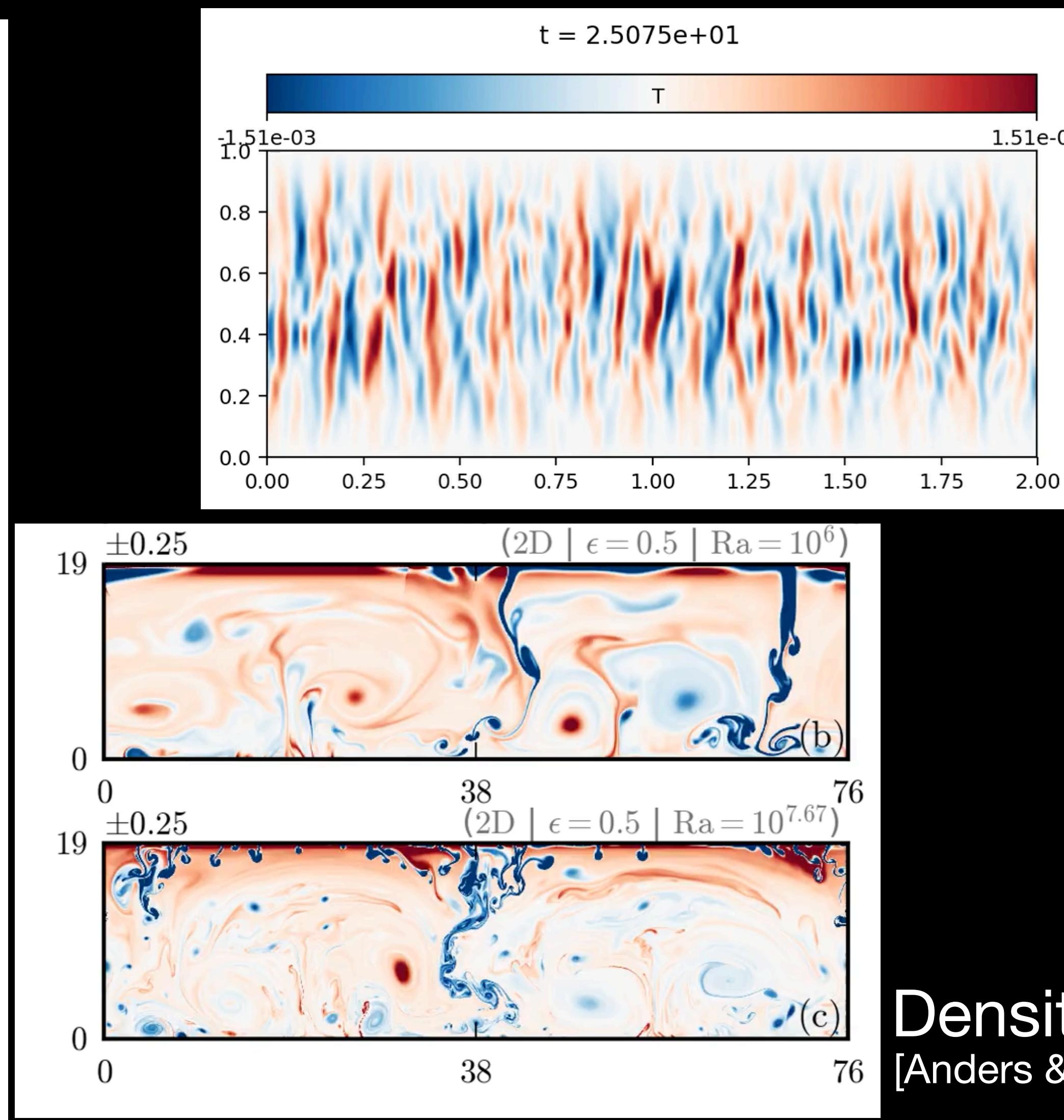
Smallest length scales  
[Anders et al 2019 ApJ 884 65]



## Disparate time scales

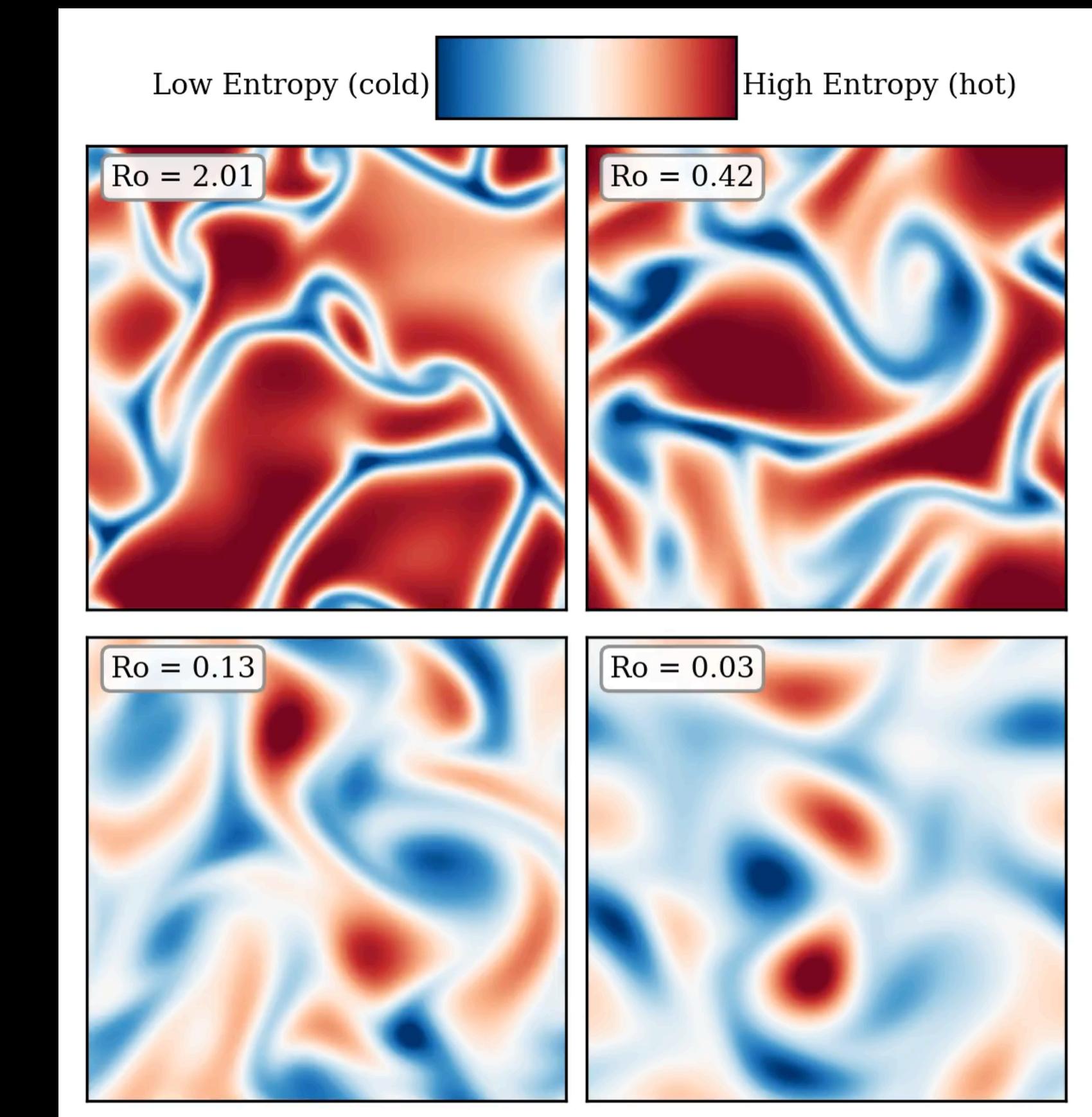
[Anders et al 2018 PRF 03 083502]

[Anders et al 2020 PRF 05 083501]



## Rotation + stratification

[Anders et al 2019 ApJ 872 138]

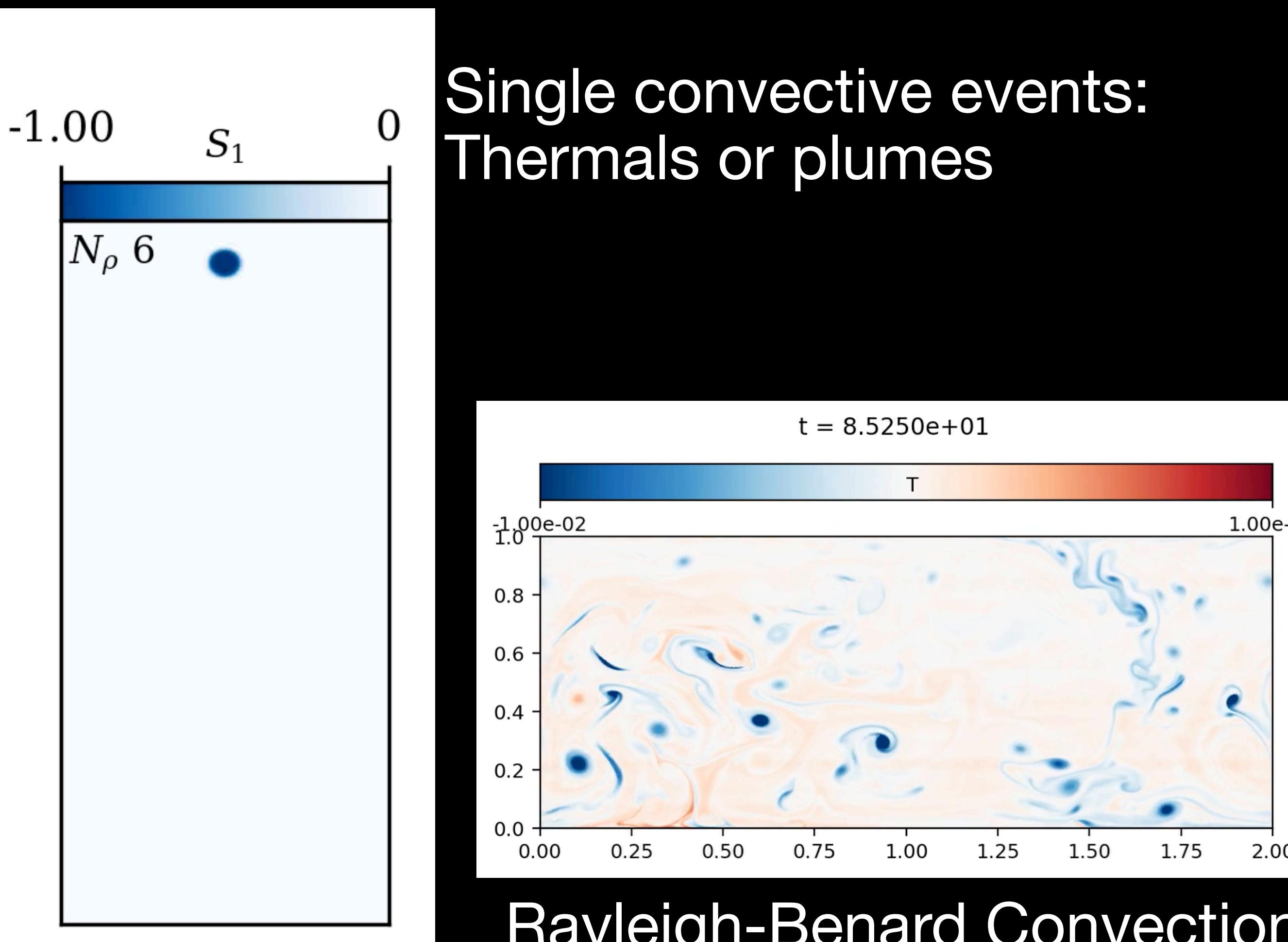


## Density stratification

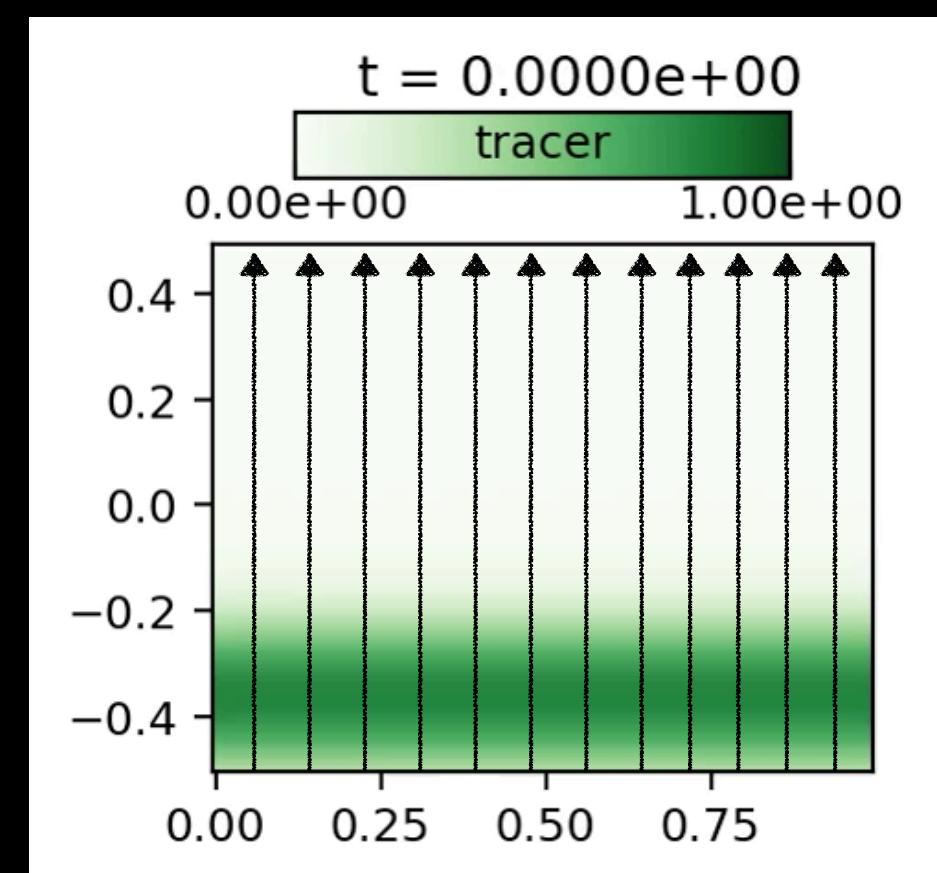
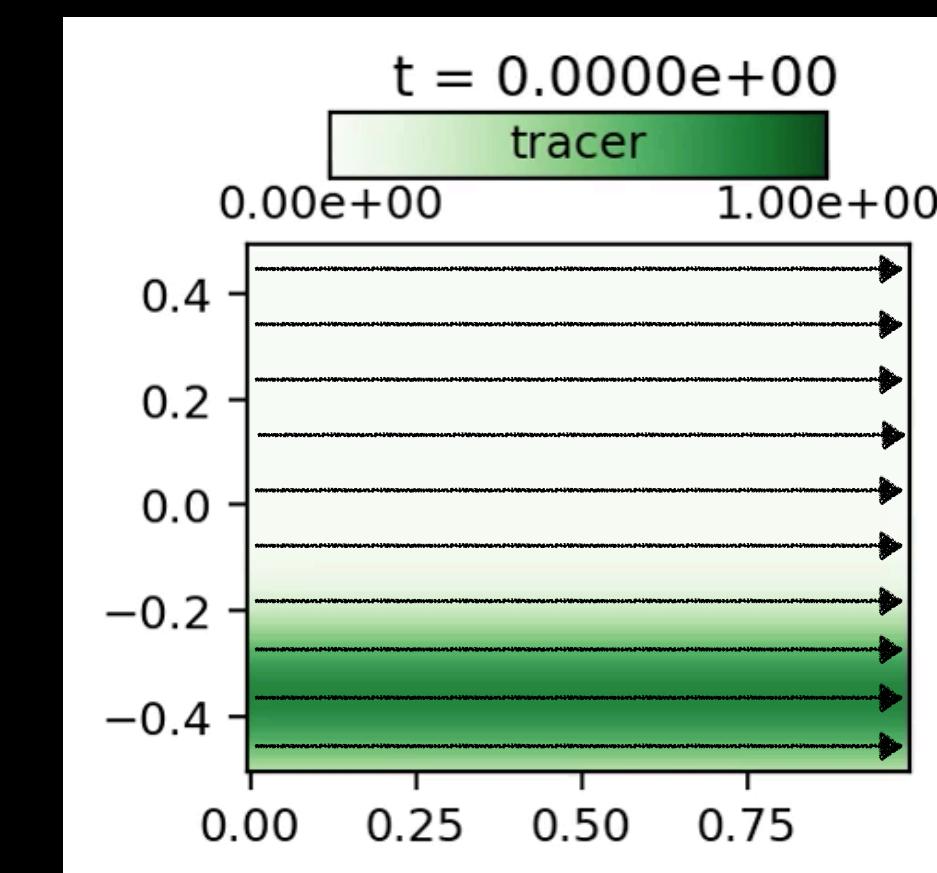
[Anders & Brown 2017 PRF 02 083501]

# Dedalus' many applications: Education

## MMath and MPhys projects



## Classroom examples



# What sets the size of a PZ?

Take the kinetic energy equation,

$$\frac{\partial \mathcal{K}}{\partial t} + \nabla \cdot (\overrightarrow{\mathcal{F}}) = \mathcal{B} - \Phi$$

[Zahn 1991 A&A 252 179]

[Roxburgh 1989 A&A 211 361]

# What sets the size of a PZ?

Volume average (over CZ + PZ) & assume time stationary

$$\cancel{\frac{\partial \vec{J}}{\partial t} + \nabla \cdot (\vec{J})} = \mathcal{B} - \Phi$$

[Zahn 1991 A&A 252 179]

[Roxburgh 1989 A&A 211 361]

# What sets the size of a PZ?

Volume average (over CZ + PZ) & assume time stationary

$$\cancel{\frac{\partial \mathcal{F}}{\partial t} + \nabla \cdot (\vec{\mathcal{F}})} = \mathcal{B} - \Phi$$
$$\iiint \mathcal{B} dV = \iiint \Phi dV$$

Buoyant work balances viscous losses.

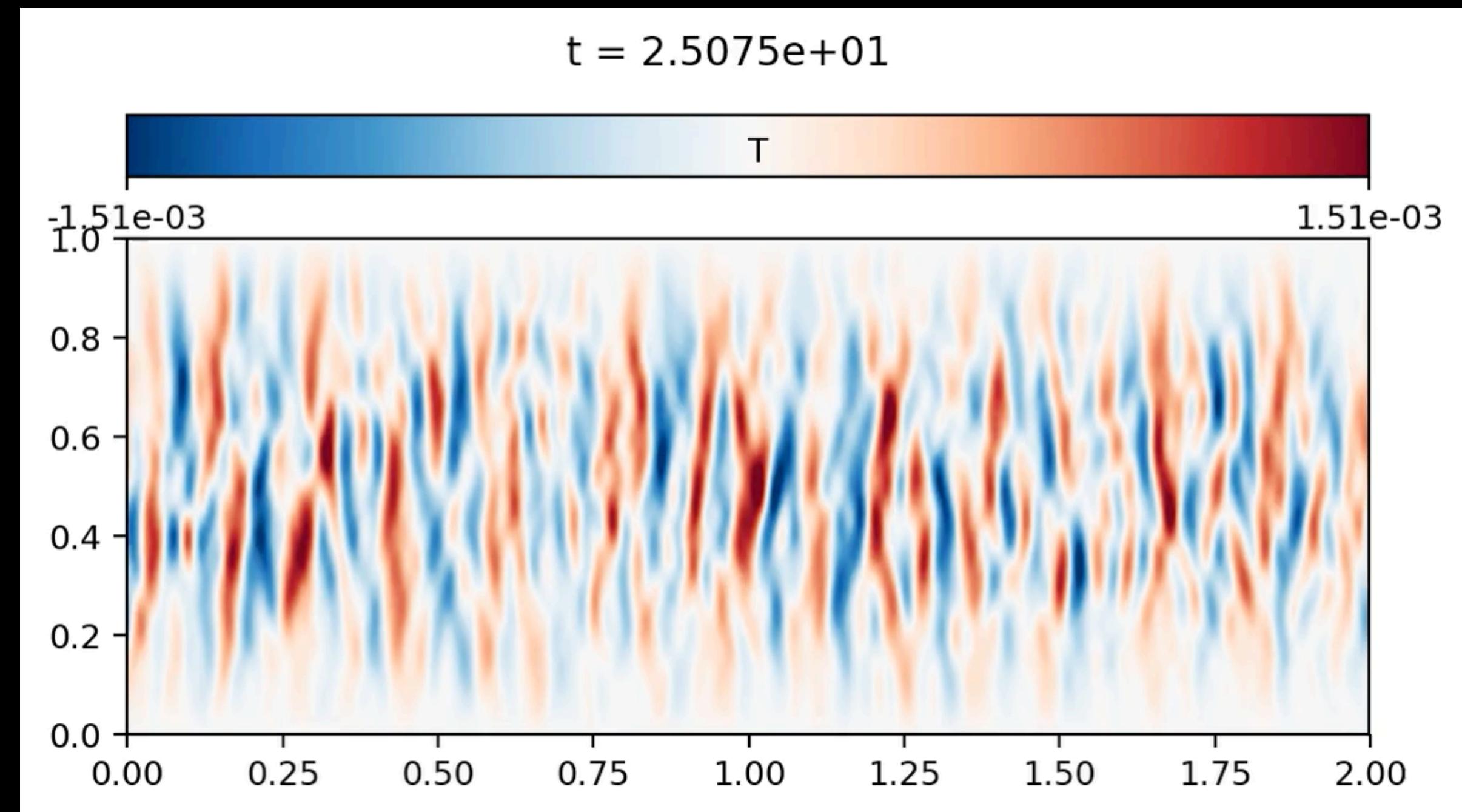
[Zahn 1991 A&A 252 179]

[Roxburgh 1989 A&A 211 361]

# What's next?

## 3. Expand accelerated evolution experiments

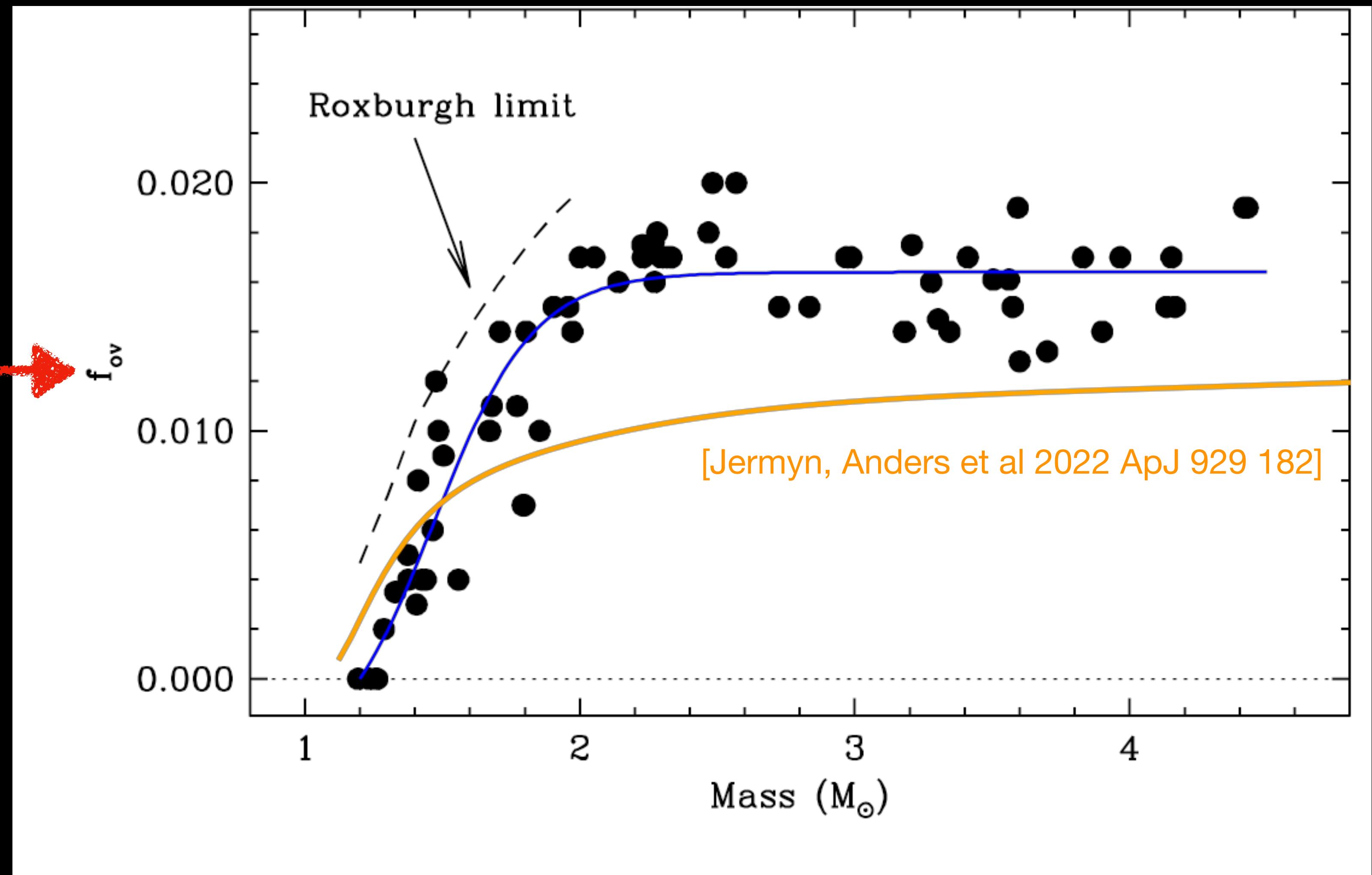
- Equilibrating a convection zone takes a long time. Unclear what happens when you have:
  - Convective layer + stable layer
  - Composition fields & irreversible mixing
  - Possibly can use these methods for mean flows (e.g., differential rotation) and vector fields (angular momentum)



# Upshot: Mixing in Eclipsing Binaries

Roxburgh limit uniformly over-predicts.

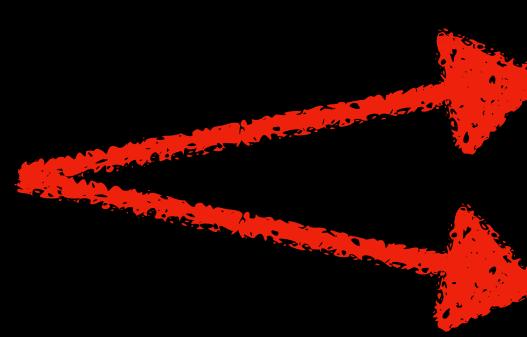
Mixing required to  
match observations



# What sets the size of a PZ?

Buoyant work

$$\iiint \mathcal{B} dV$$

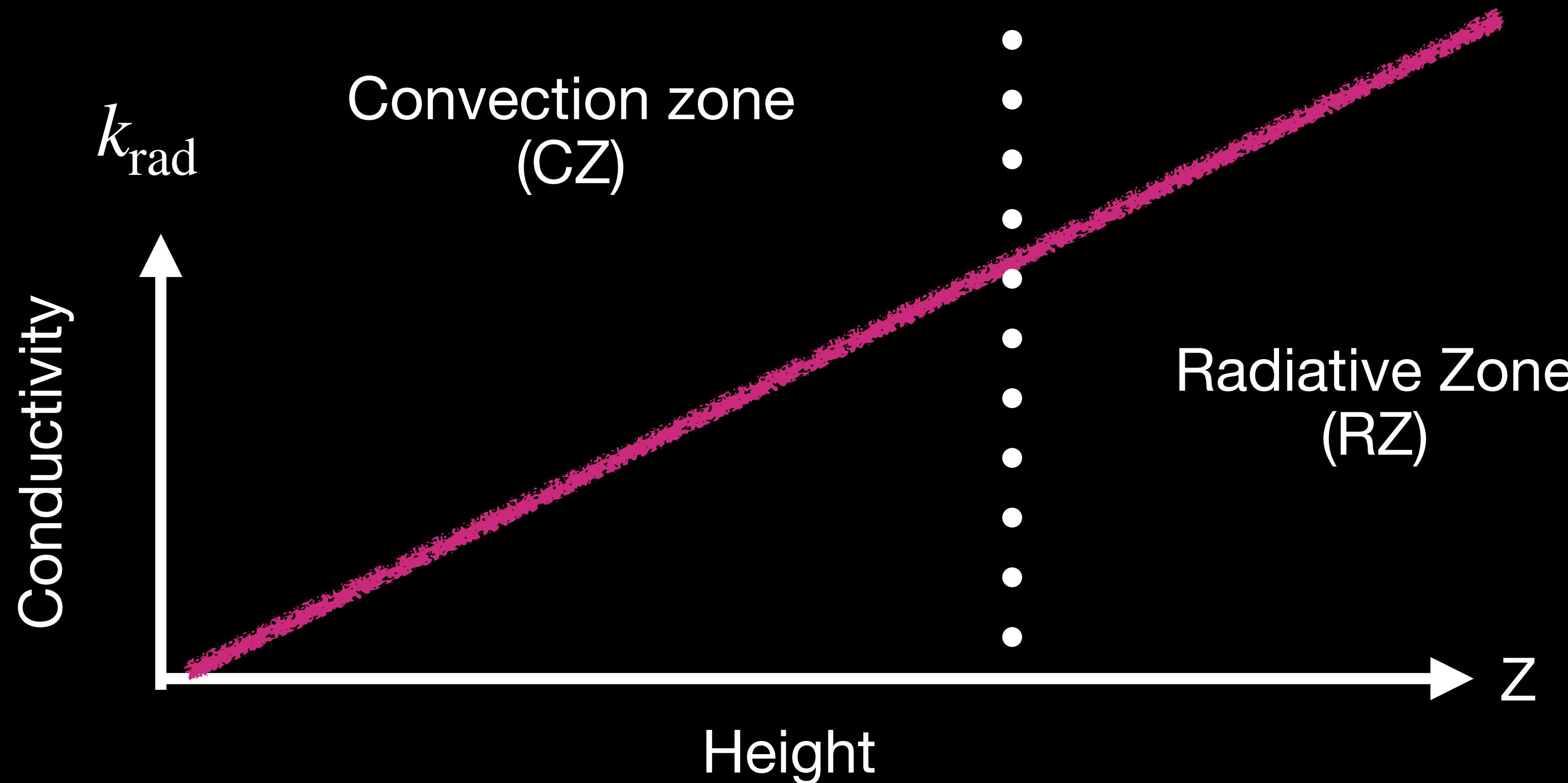
 Positive in the CZ  
Negative in the PZ

Viscous Dissipation

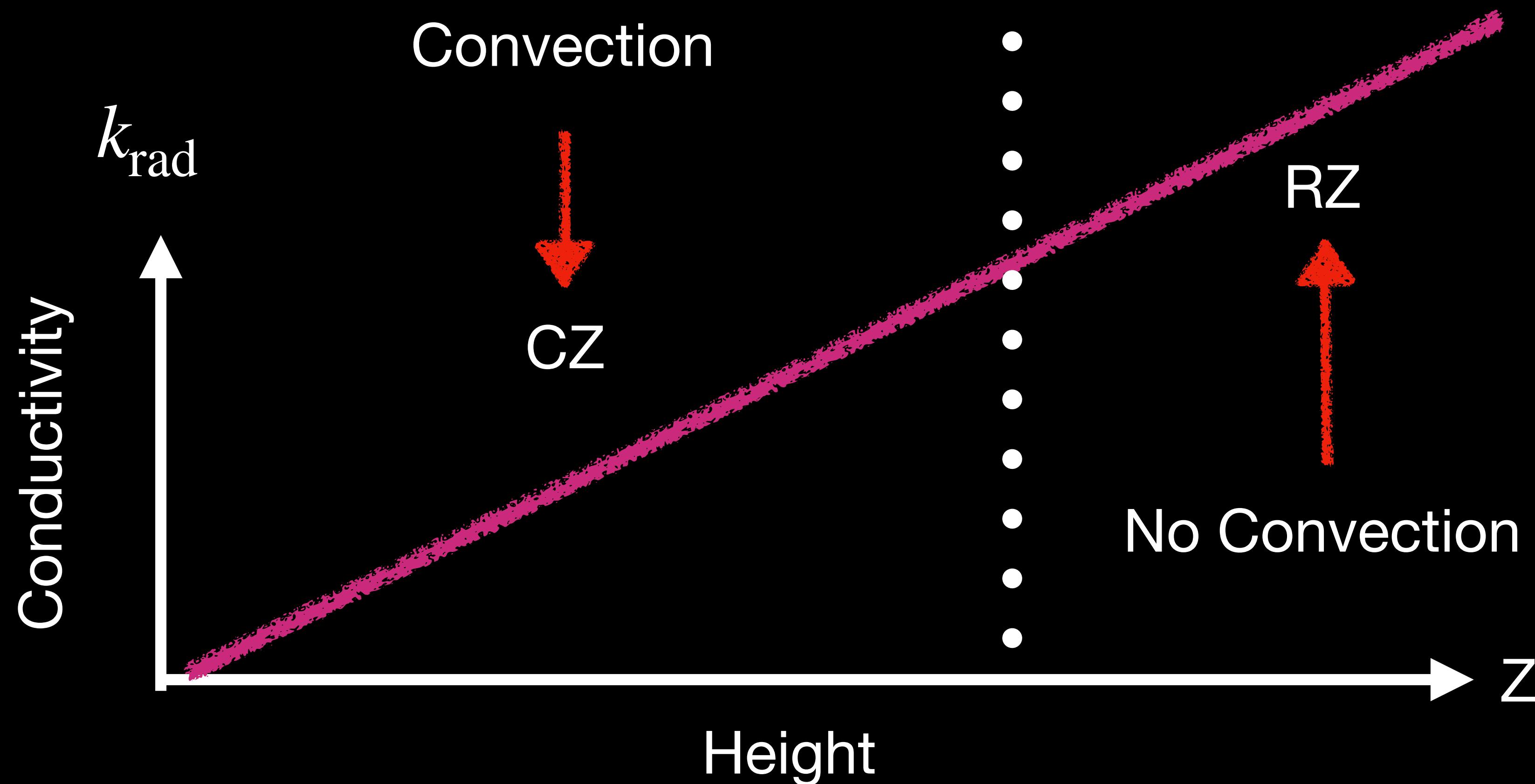
$$\iiint \Phi dV$$

 Positive everywhere

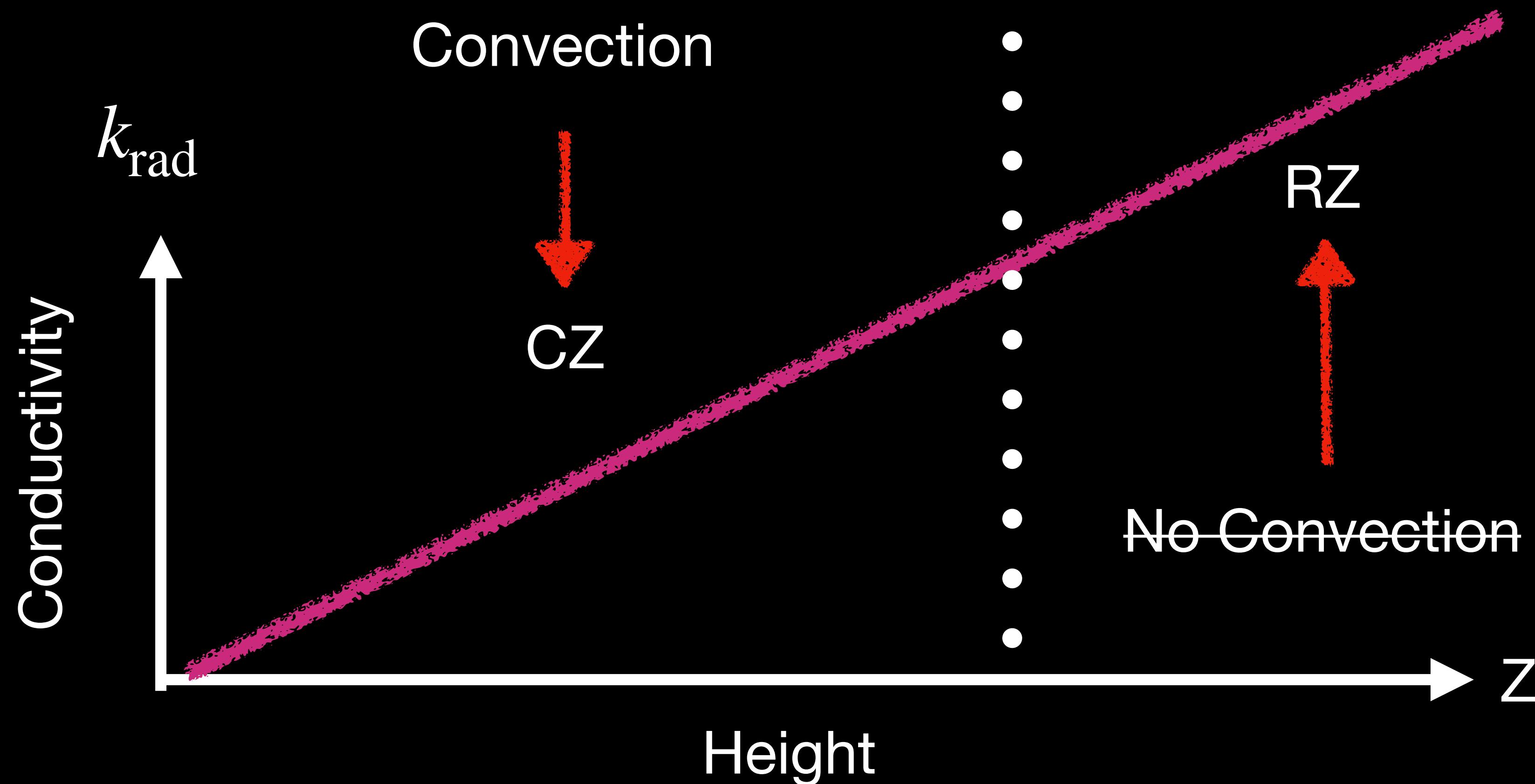
# Why is there a convective boundary?



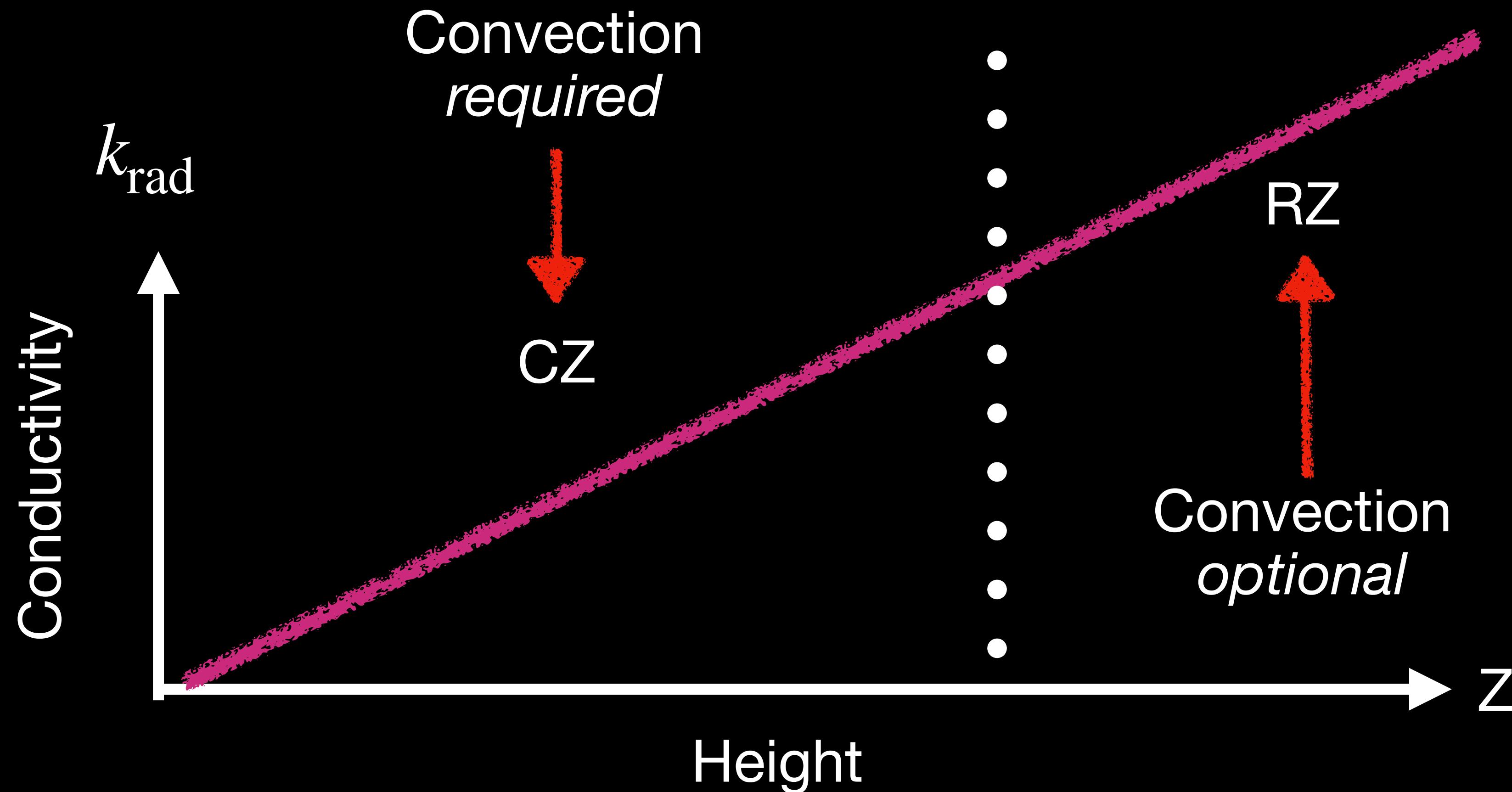
# Why is there a convective boundary?



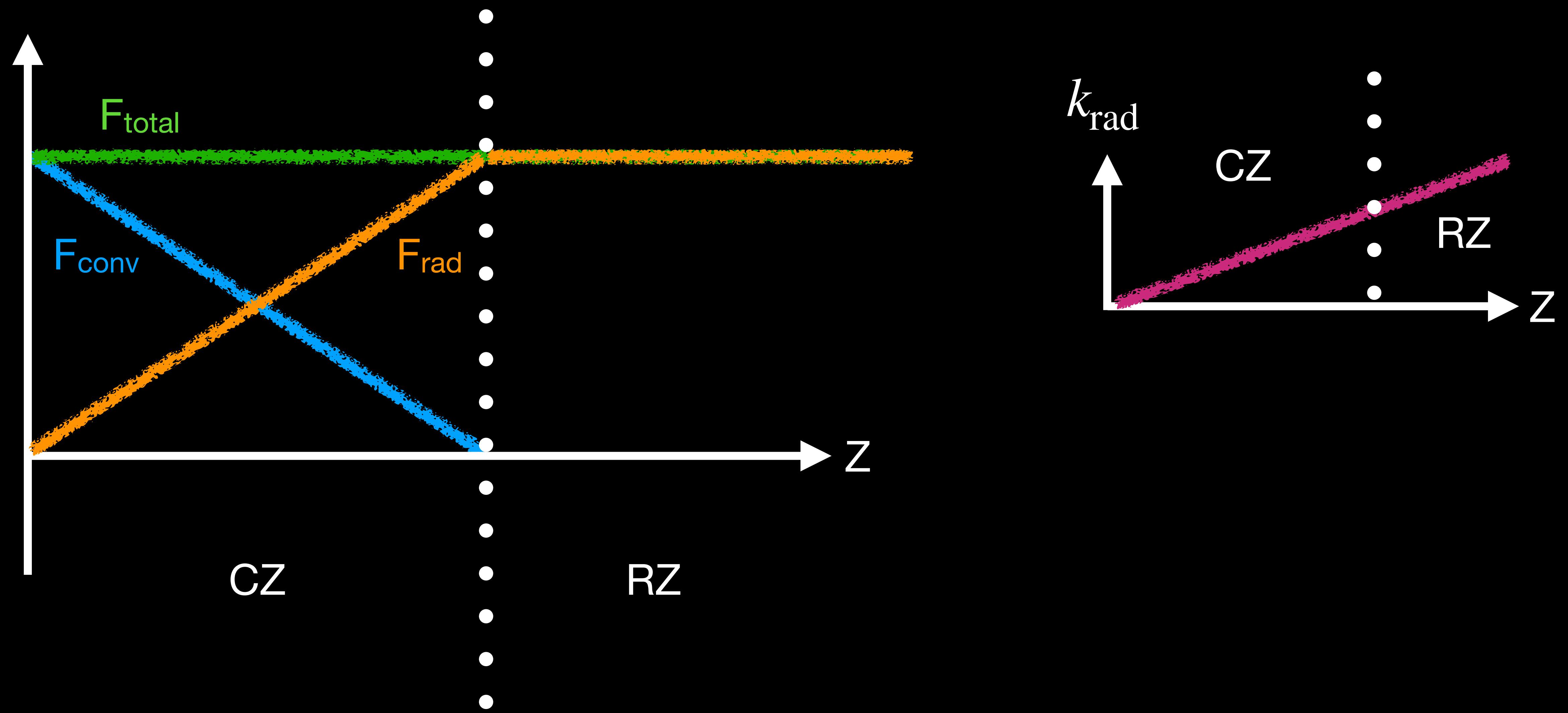
# Why is there a convective boundary?



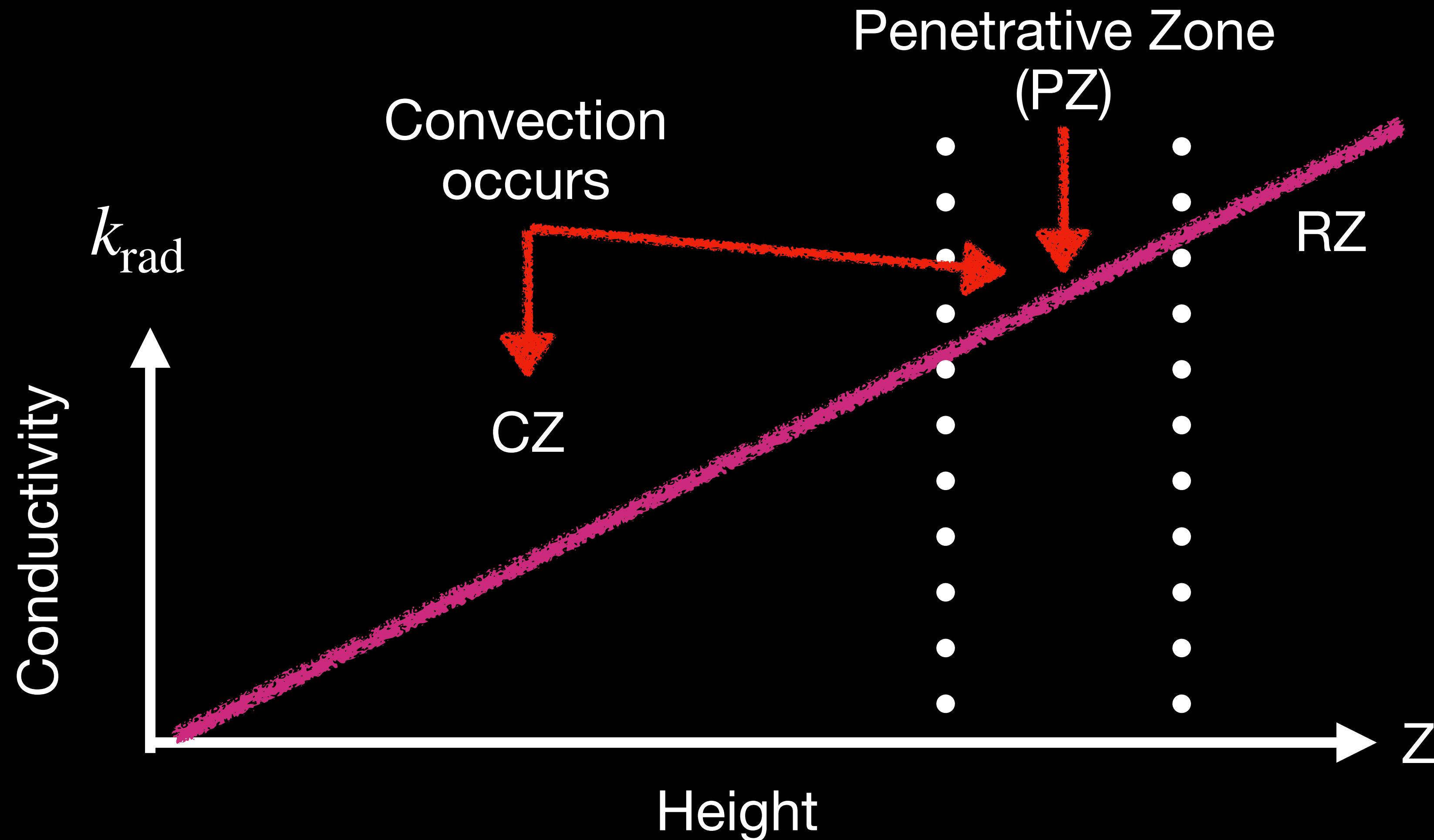
# Why is there a convective boundary?



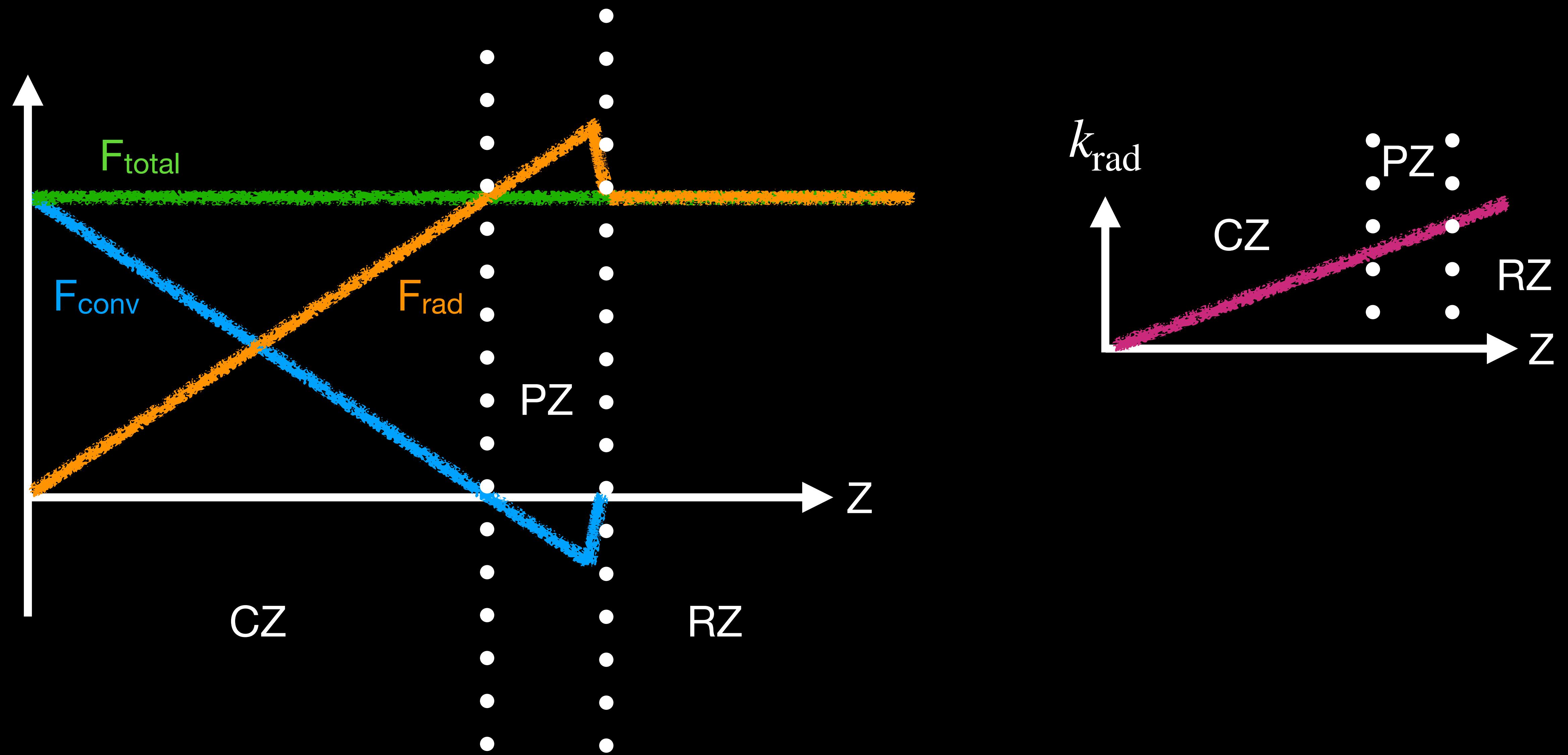
# Fluxes and Stability



# What is penetrative convection?



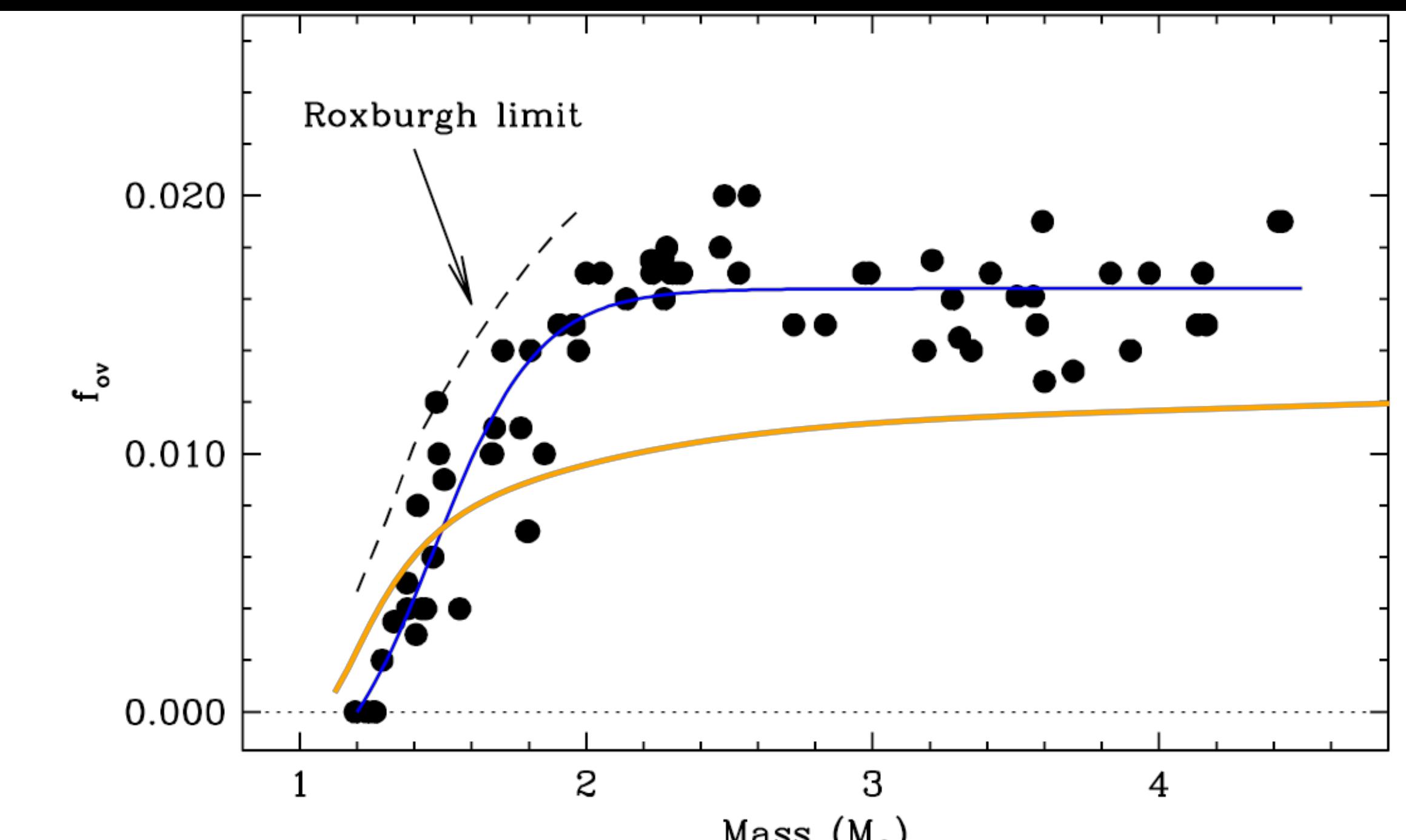
# Fluxes and Stability - with a PZ



# What's next?

## 2. Understand penetrative convection in stars

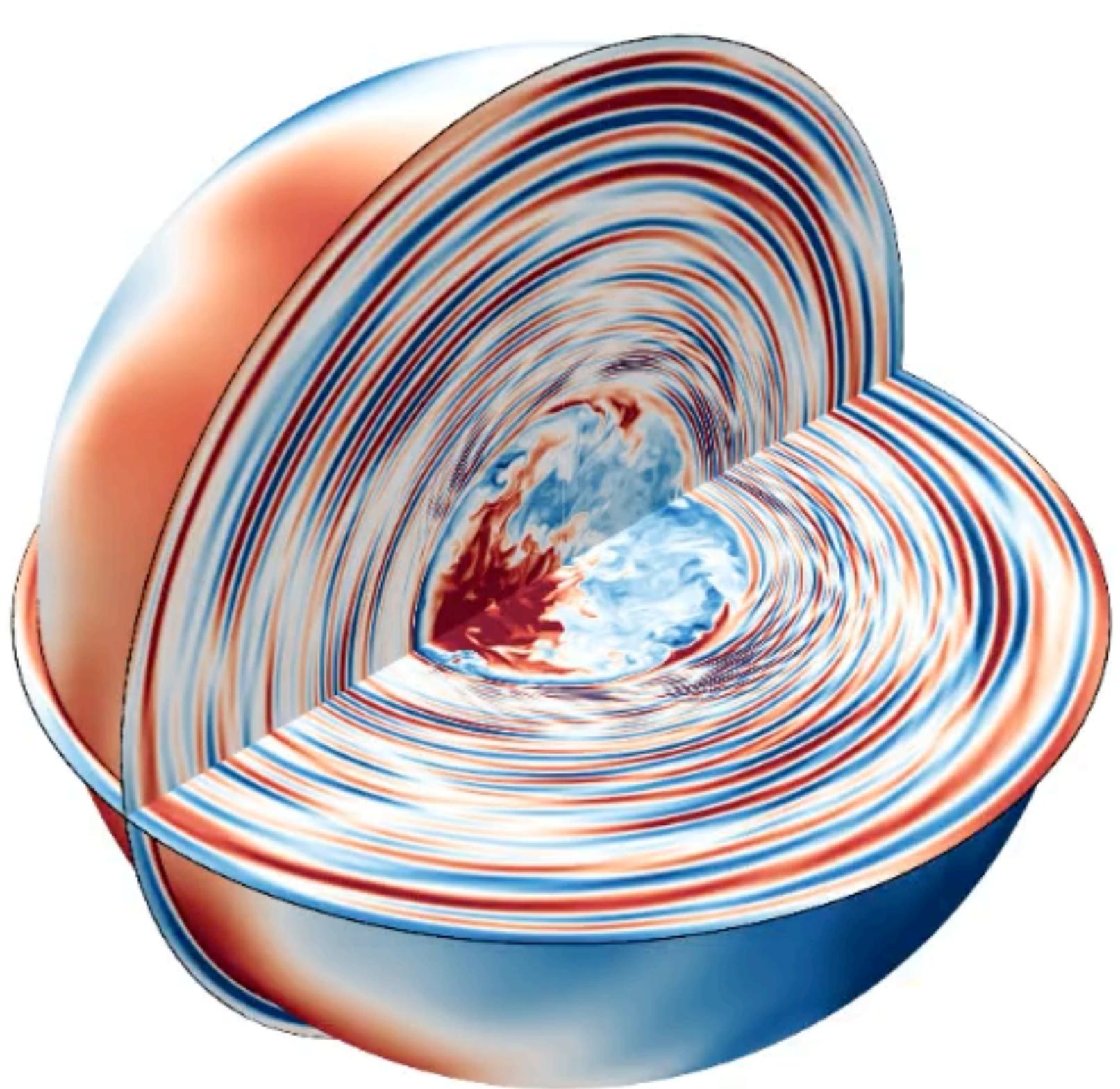
- Incorporate into 1D models self-consistently
  - Understand implications for stellar lifetimes, remnants, etc
  - Demonstrate validity of theory in a grid of 3D hydro models based on stars of different masses
  - Why does the mass change the fluid dynamics? / Rotation?



# What's next?

## 4. Internal gravity waves in massive stars

- First spectral fully compressible simulations that include  $r = 0$  coordinate singularity.
- Determine the effects of rotation on observed signal
- Understand how stiffness changes wave properties
- Understand how damping changes observable signals



$$S = \frac{N_{RZ}}{(u/\ell)}$$