Novel observations of convectively-driven flows in stars constantly prove that our theories of convection are incomplete. For example, observations of massive stars demonstrate unexpectedly high mixing at the core convection boundary (Johnston, 2021), so theories underestimate the sizes of convective regions. A recent observation has claimed the detection of waves driven by massive star core convection (Bowman et al., 2019). Such a wave signal could resolve disagreements about convective excitation mechanisms and the transfer of waves through stellar interiors (Rogers et al., 2013; Lecoanet et al., 2021). Observations of the Sun show a convective conundrum: "giant cells" predicted by theory and simulations are not detected by many observations (Hanasoge et al., 2012; Proxauf, 2021), suggesting that we do not understand the how the Sun's convection is driven.

A major problem in applied mathematics is developing theories that explain these observations. In the next five years, **I will conduct research and create community tools to help resolve these and other modern mysteries in applied mathematics.** People: Matt, Isabelle,

Task 1: Convective boundary mixing As I mentioned in my cover letter, mixing at convective interfaces is a compelling problem in astrophysics. My recent work on penetrative convection may help to resolve discrepancies between observations and theory (Anders et al., 2022). My previous work focused on a minimal model: it employed the incompressible Boussinesq approximation, and it studied a Cartesian domain. There remains a great deal of work to be done in developing a robust theory of penetrative convection that works broadly in stellar evolution. For example, the effects of rotation, composition gradients, magnetic fields, density stratification, and geometry have not yet been studied in detail. Furthermore, use of the fully compressible equations subtly changes the theoretical constraint that I derived, and must be explored.

Need to mention COBOM erc stuff that Isabelle has done. Mention that this is a *complementary* approach; Here I focus on long term evolution and saturation; they focus on rare event statistics; put these together and boom. Matt's expertise in dissipation modeling.

Task 2: Open-source, easily accessible code for multi-D stellar simulations There are no open-source, user-friendly codes for running dynamical simulations of stars. Current stellar dynamics codes require help from an expert to run, are restricted to limited equation sets, or only focus on a portion of the stellar domain. Over the next five years, I will build an easy-to-use Python module that uses the Dedalus pseudospectral framework (Burns et al., 2020) to create dynamical simulations of full stars. It is possible to include the full stellar domain in Dedalus, because flows that pass through the coordinate singularity at $r = 0$ can be resolved (Vasil et al., 2019; Lecoanet et al., 2019), and radial discretization can be fine-tuned so that resolution elements are localized in turbulent regions. Background stellar stratifications will be imported from MESA stellar models (Paxton et al., 2011). Planned evolutionary equation sets will include both hydrodynamic and magnetohydrodynamic for-

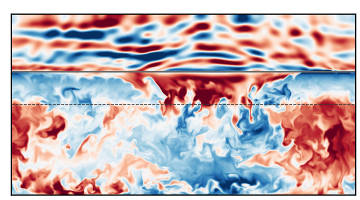

Figure 1: A vertical slice of the temperature anomaly in a 3D simulation of penetrative convection.

mulations of both the anelastic and fully compressible equations. All of these formulations must be verified in the various dynamical regimes occurring in stars. I will need to build a robust scheme for radial discretization that handles arbitrary stars at various stages in stellar evolution. My 7 years of experience using Dedalus has given me the expertise to implement and debug all of these features. Once complete, my proposed open access module will enable researchers to carry out experiments to understand observations like those discussed in my introduction.

Need to be careful here and mention that Matt will be crucial due to his expertise with Massive star simulations (but with a core boundary condition) and Isabelle's experience with music and envelope convection.

Task 3: The parameter space of convection Running simulations of stellar convection is difficult, because something is idealized. This idealization can throw off the detailed force balances felt by the convective flows in unexpected ways. In the fully constrained regime we generally understand what happens. However in the transitional regime it's less clear (predictive rossby). I will study simulations of rotating convection in a solar-type star and measure the force balances directly and determine how the rossby number scales as a function of stratification, convective driving, and rotation rate.

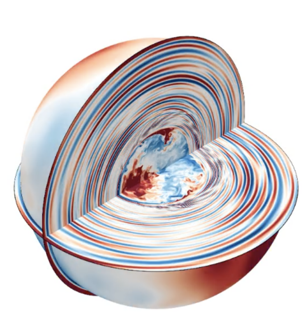

Figure 2: A volume rendering of scaled entropy fluctuations in a Dedalus simulation of a 40 $M_{\odot}$ star.

Task 4: Convection at the smallest scales What is the convective conundrum. What is the entropy rain hypothesis. What did we learn in our previous entropy rain paper. What work is left to do on the thermal model of entropy rain. What work is left to do on the plume model of entropy rain. (tarshish, romps).

References

Anders, E. H., et al. 2022, The Astrophysical Journal, 926, 169

Bowman, D. M., et al. 2019, Nature Astronomy, 3, 760

Burns, K. J., et al. 2020, Physical Review Research, 2, 023068

Hanasoge, S. M., et al. 2012, Proceedings of the National Academy of Science, 109, 11928

Johnston, C. 2021, Astronomy & Astrophysics, 655, A29

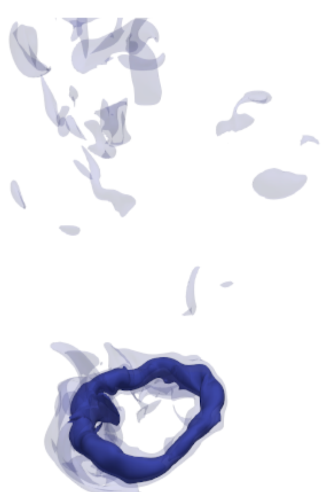

Figure 3: A volume rendering of an evolved thermal in its buoyant vortex ring state.

Lecoanet, D., et al. 2021, Monthly Notices of the Royal Astronomical Society, 508, 132

Lecoanet, D., et al. 2019, Journal of Computational Physics: X, 3, 100012

Paxton, B., et al. 2011, The Astrophysical Journal Supplement, 192, 3

Proxauf, B. 2021, arXiv e-prints, arXiv:2106.07251

Rogers, T. M., et al. 2013, The Astrophysical Journal, 772, 21

Vasil, G. M., et al. 2019, Journal of Computational Physics: X, 3, 100013