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Moosinesq Convection in the Cores of Moosive Stars

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ABSTRACT

Did you know moose exist? 'Cause they do, and they're on the loose!

Keywords: I don't know.

1. INTRODUCTION

Convection is important in all sorts of natural systems[1]. In particular, most stars have convection zones. This can be determined by looking at the Sun[2] or by running 1D stellar evolution models. It is crucial that we gain a better understanding of convection since it is such a ubiquitous and poorly understood process in astrophysics[3].

Convective motions are three-dimensional, so many modern convective experiments utilize multi-dimensional fluid dynamical simulations. However, modern computational resources are too limited to time-evolve the Navier-Stokes equations in their most complex form (Landau & Lifshitz 1987), including all of the important physics present in stars (Paxton et al. 2011, 2013, 2015, 2018, 2019). As a result, numericists make simplifying assumptions to create tractable experiments, for example by simplifying the geometry or the equations. The most widely-used simplified set of equations is called the *Boussinesq approximation* (Spiegel & Veronis 1960), and it has been used in thousands of studies (see e.g., Ahlers et al. 2009).

In this Letter, we present the first ever simulations which use an oft-overlooked simplified equation set: the *Moosinesq* approximation. This approximation is similar to the Boussinesq approximation in every way, except it allows for the study of convection inside of a moose (*Alces alces*). While this approximation has many[4] applications in the terrestrial context, the authors of this paper are largely astronomers. Therefore we are most interested in the application of this approximation to *moosive stars*, that is, stars with masses $M \gtrsim 1.3 M_\odot$ which have convective course on the main sequence. In section 2, we describe the Moosinesq equations and our numerical methods. In section 3, we demonstrate the nature of flows in two-dimensional Moosinesq Convection. Finally, in section 4, we discuss applications of Moosinesq Convection to stars, human society, and beyond.

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[1] Trust us, we're experts
[2] Please do not look at the Sun.
[3] The financial wellfare of many of the authors depend on you agreeing with this statement.
[4] *many*

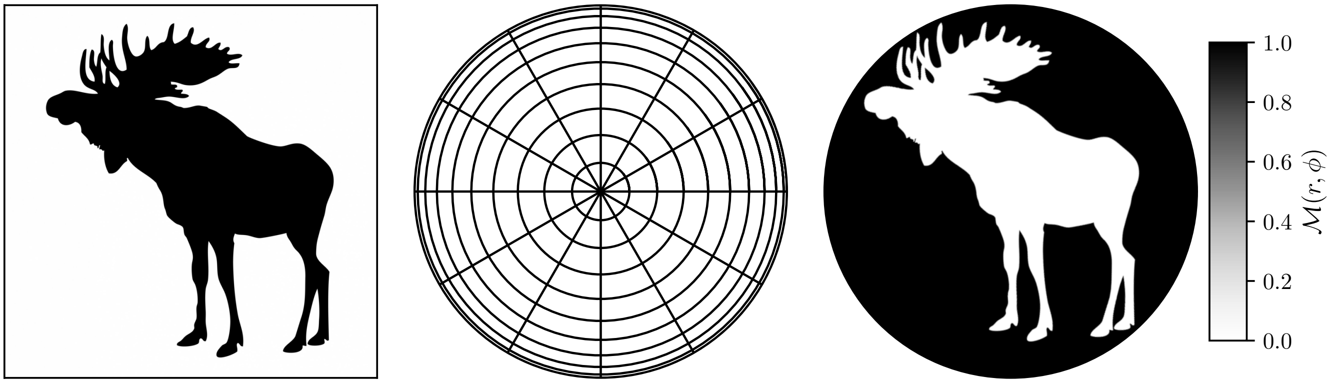

Figure 1. (Left) A public-domain silhouette of a moose. (Middle) A sparse representation of the polar-coordinate grid on which we represent fields in our simulation. (Right) The Moosinesq mask $\mathcal{M}$ felt by our equations; fluid motions are damped where $\mathcal{M} > 0$.

2. NUMERICAL METHODS

The Moosinesq Equations are

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0, , \tag{1}$$

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} \varpi - T\boldsymbol{g} + \nu \boldsymbol{\nabla}^2 \boldsymbol{u} - \gamma \mathcal{M} \boldsymbol{u}, , \tag{2}$$

$$\partial_t T + \boldsymbol{u} \cdot \boldsymbol{\nabla} T = \kappa_T \boldsymbol{\nabla}^2 \boldsymbol{u} - \gamma \mathcal{M} T. \tag{3}$$

In other words, they are the Boussinesq Equations (Spiegel & Veronis 1960) with crucial Moose $\mathcal{M}$ terms in the moosementum and temperature equations (Eqn. 2- 3). Here, $\boldsymbol{u}$ is the velocity, T is the temperature, ϖ is the reduced pressure, ν is the kinematic viscosity, κ_T is the thermal diffusivity, and γ is a frequency associated with the damping of motions. We solve these equations in polar (r, ϕ) geometry, because we are astrophysicists and this geometry is most applicable to moosive stars. Inspired by the groundbreaking work of Burns et al. (2019), we naturally choose to have gravity point down in a Cartesian sense, $\boldsymbol{g} = -g\hat{z} = -g(\sin\phi\hat{r} + \cos\phi\hat{\phi})$ for increased confusion and lack of clarity[5].

The Moose is implemented using the volume penalization method described in e.g., Hester et al. (2021). We first take an image of a moose from the internet (Fig. 1, left[6]). We convert this image into polar coordinates on the grid space representation of our basis function (Fig. 1, center). We then convert the image into a smooth mask $\mathcal{M}(r, \phi)$, see Fig. 1 right panel, which is fed into Eqns. 2-3. This is obviously a trivial exercise which we leave to the reader[7].

We nondimensionalize Eqns. 1-3 and evolve them in time using the Dedalus[8] (Burns et al. 2020) pseudospectral solver. Details of the nondimensionalization and simulation can be found in appendix B. The simulation presented in this work was run at a Rayleigh number of $\text{Ra} = 10^{11}$ and a Prantler number of $\text{Pr} = 1$. The code used to the run simulations is available online in a github repository[9].

3. RESULTS

We display the majesty of moosinesq convection in Fig. **??**. We visualize the temperature field T, so red is warm fluid that buoyantly rises, and blue is cold fluid that buoyantly falls. Plotted over the temperature field is the mask $\mathcal{M}$, which is fully transparent when it is zero and which is a low-opacity white when $\mathcal{M} = 1$. This allows us to show that, indeed, there are no appreciable motions outside of the moose and the mask is working properly.

The moose is filled with interesting dynamics[10]. The legs largely serve as thin, tall "tunnels" which are filled with Von Kármán vortices and which connect the hot moose feet to its neutrally-buoyant body. Cold fluid parcels from the body have managed to mix down one leg, and the moose would probably benefit from having that leg wrapped in a warm cloth or heat pack. The body of the moose exhibits dynamics familiar from classical 2D Rayleigh-Bènard

[5] It is unclear what mass distribution would produce this in a star. Oh well.

[6] Available online at https://www.publicdomainpictures.net/en/view-image.php?image=317077&picture=moose.

[7] Or see appendix A.

[8] Ironically we did not the MOOSE simulation framework (https://mooseframework.inl.gov/). Sorry.

[9] https://github.com/evanhanders/moosinesq_convection

[10] Likely due to a recent, delicious meal.

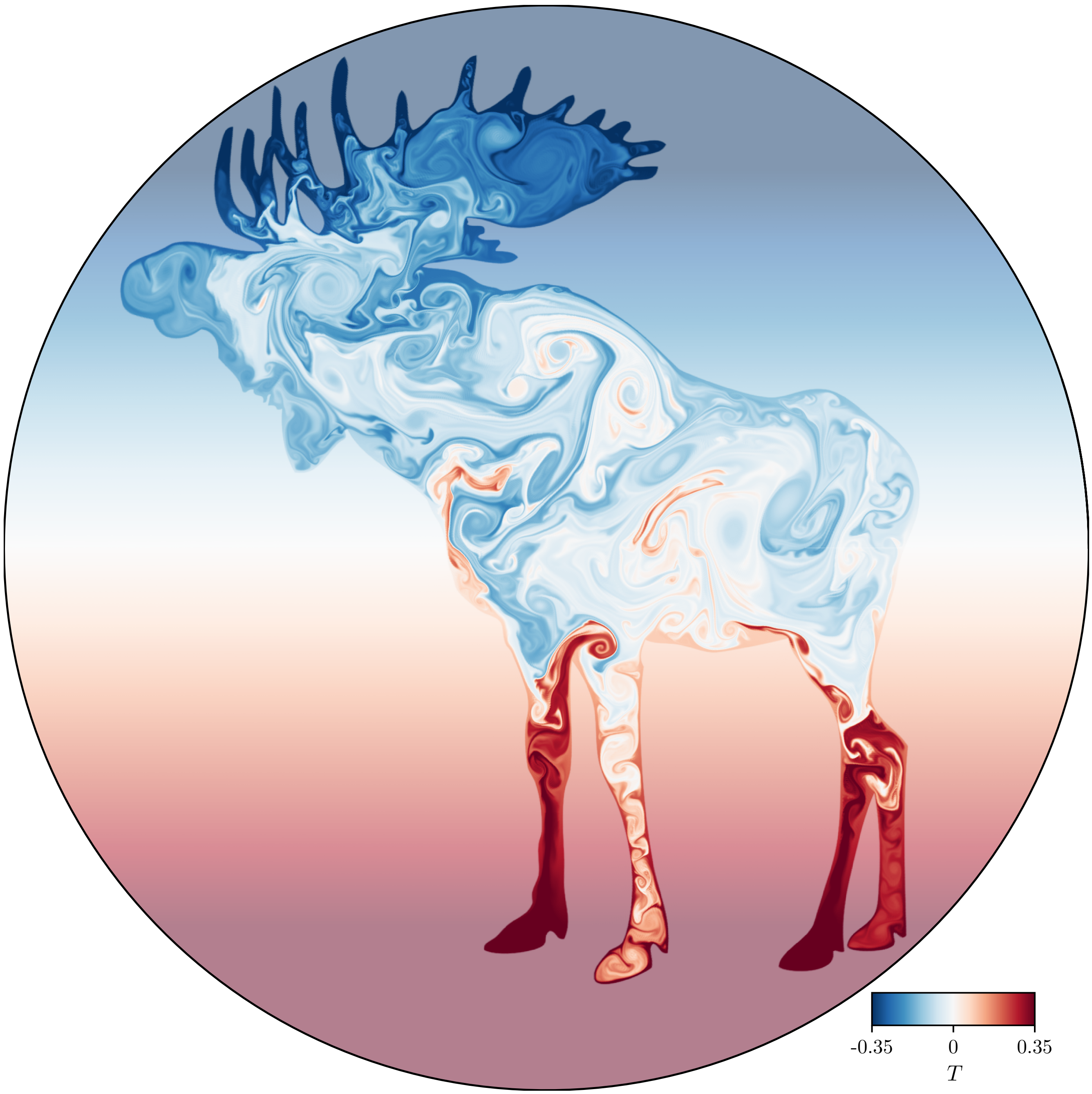

Figure 2. The beautiful, powerful moose.

convection. Hot and cold fluid swirl together, forming many vortices, and mix. Aside from the legs, the antlers are probably the most interesting part of the moose. Long-lived vortices of relatively hot fluid establish themselves there and turn for a few convective overturn times. Then, violent flows from the moose's body disrupt those vortices with fresh, hot fluid and this process repeats itself. Occasionally some of this hot fluid rises into the tips of the moose's antlers, which probably accounts for the growth of the moose's antlers.

Now that we have examined the dynamics in Moosinesq Convection in some detail, we turn our attention towards its astrophysical applications. Kaiser et al. (2020) note that "massive [sic] stars" are sensitive to "the details of their complex convective history" (Kaiser et al. 2020). We agree. One consideration that previous authors have ignored

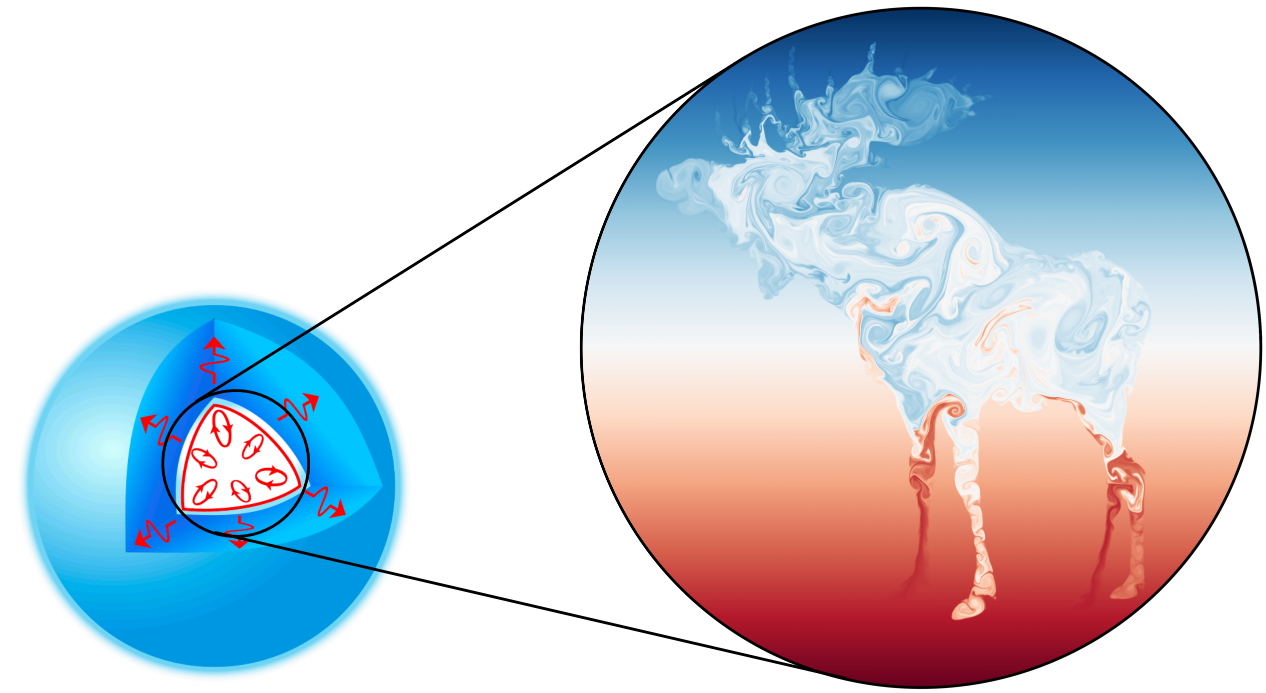

Figure 3. Moosive Stars.

when considering convective uncertainties in moosive stars is displayed in Fig. 3. That is, the cores of these stars are filled with Moosinesq Convection, which can have important consequences for moosive stellar evolution. it is unclear at this time how the complex flow morphologies associated with moosinesq convection would affect e.g., the magnetic fields, chemical profiles, and lifetimes of these stars. We leave these important considerations to future work.

4. CONCLUSIONS & DISCUSSION

Future work: Elkman number, Mooselt number.

The methods developed in this work may suggest a pathway toward unraveling the mysteries of other wildlife-related fluid phenomena, such as the powerful and mysterious otter of Schwab (2021), also known as the Papaloizou-Pringle Patronus. Lessons learned from this and future work on the moosinesq approximation may also be of interest to those working in the yet-underexplored field of *goosinesq* convection.

However, the Boussinesq approximation must be abandoned in situations where compressibility is an important consideration. This can occur in microbiology (Ravetto et al. 2014), and it has long been known that wildlife ecology is another such situation (e.g. Enright 1963). Such studies have often focused on domains in which animal compressibility is more pronounced (i.e. deep-sea life). However, motivated by a recent report of an unexpected occurrence of acute compression and deformation in a land-animal context (Gudmannsson et al. 2018)—as well as a delightful linguistic coincidence—we present ground-breaking work on fundamental fluid dynamics in the context of the moose (*Alces alces*), a domain which we dub the *moosinesq approximation*. This approximation is suitable for describing the active and dynamic inner lives and environments (both physical and mental; see Gibson 2015) of the moose.

The moose is a large mammal indigenous to North America and Europe, which can have a mass of up to 550 kg and a height of up to 2 m (CPW 2021). ... Our study is not the first time that moose have prompted significant scientific or technological development (see, e.g., Händel et al. 2009).

4.1. *Historical context*

The circumpolar distribution of Alces alces (moose) has led to multiple independent points of cultural connection to and subsequent efforts directed at harnessing the largest cervid with varying levels of success. Rock carvings in the Kalbak-Tash group, Altai Republic, Russia indicate that ab antuquo efforts to ride or cause moose to pull sleds have been documented and subsequently received comment since the Bronze Age and likely earlier (USEEV 2014). In North America, European efforts to colonize Canada have been intermittently aided by the capture of wild individual moose more suited to traversing long distances in snow and over boggy ground than domestic alternatives leading to practical applications such as the use of a team of moose to deliver mail by Mr. W.R. Day (of Alberta 2007). The exploitation of this practicality was limited, obviously, by the size and temperament of the moose which after about the age of 3 days is entirely antagonistic towards humans (Sipko et al. 2019). A few spectacular exceptions of moose under harness has kept hope burning for a future with a less fraught relationship. The most noteworthy partnership is widely remembered through the efforts of Albert Vaillancourt of Chelmsford, Ontario who exhibited moose pulling a surrey during the intermissions of horse racing (Chisholm 2019). His pair of racing mooses named Moose and Silver clearly demonstrated the majesty and potential of this species (Landry 1941).

This potential inspires the continuing quest for a fully domestic moose that is somewhat less likely to attack and kill humans. In Russia, moose domestication is the subject of investigations initiated by Prof. P. A. Manteifel who oversaw an effort to create moose nurseries across Russia for the purpose of creating a recognizably domestic animal from about 1934 through the present (Sipko et al. 2019). In an early, perhaps premature demonstration on December 1937, I.V. Stalin watched a military moose drill. He was, “particularly impressed by the moment when moose cavalry flew out of the forest, bristling with machine guns.” He did note that the moose were not yet trained to distinguish the Red Army soldiers from the White Finns (Pererva 2017).

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APPENDIX

A. MOOSE MASK CREATION

The Moose is implemented using the volume penalization method described in e.g., Hester et al. (2021). We first take an image of a majestic moose from the internet (Fig. 1, left[11]). We convert this image into polar coordinates on the grid space representation of our basis function (Fig. **??**).

We then interpolate from pixel values into polar coordinates (r, ϕ) sampled on the natural grid of our spectral bases (Fig. 1, middle). The resulting moose mask which is fed directly into our equations during timestepping is thus produced and shown in Fig. 1, right.

We next compute a signed distance function d_s at each pixel to determine how far that pixel is from the edge of the moose. We convert that signed distance function (whose range is [-0.5, 0.5]) into a profile that varies smoothly from 0 to 1 over the moose boundaries, $\mathcal{M} = 0.5(1 - \mathrm{erf}(\pi^{1/2} d/\delta))$.

[11] Available online at https://www.publicdomainpictures.net/en/view-image.php?image=317077&picture=moose.

B. NONDIMENSIONAL EQUATIONS, SIMULATION DETAILS & DATA AVAILABILITY

All of the python scripts and ipython notebooks used to create the simulation in this paper and the figures in this paper are available online in a github repository (https://github.com/evanhanders/moosinesq_convection).

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