Towards a more complete understanding of Stratified, Compressible Convection

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1 Background & Motivation

The Sun exhibits active magnetism which cycles in magnitude every 11 years. This magnetism arises from an organized dynamo seated in the turbulent plasma motions of the solar convective zone, which occupies roughly the outer 30% of the Sun's radius. This activity manifests itself in the collection of phenomena generally referred to as solar activity, including magnetic storms and coronal mass ejections. Such activity propagates towards Earth, threatening disruption of power grids and aircraft operations as well as endangering astronauts and satellites. The motivation to understand the Sun's magnetism in our increasingly technological society is great. A critical step in protecting our society from the Sun's magnetism is to gain an understanding of the nature of the dynamo that generates the Sun's magnetic fields Nordlund et al. (2009); Charbonneau (2014).

To understand the dynamo that drives the Sun's magnetism, we must understand the convection which powers that dynamo. Numerical studies of convection in stratified domain have a rich history in the past decade. The early work of Graham (1975) and Hurlburt et al. (1984) and others in simple domains provided rich insight into the nature of stratified convection and provided a basis in a field which now regularly creates both complex, 3D global models of convection (e.g., Brown et al. (2010), Guerrero et al. (2016), and many others) and smaller scale local area models with more complex physics (e.g., Stein & Nordlund (2012), Rempel (2014)). From these efforts we have learned a great deal about the nature of convection, and have even created beautiful simulations which even *look* like the convection we see on the surface of the Sun.

Unfortunately, the great advances made in computational prowess within the solar physics community seem to have surpassed our fundamental knowledge in the field. There is currently a so-called "Solar Convective Conundrum" which has two components. Both components of this conundrum are present in the recent observations by Hanasoge et al. (2012) (Fig. 1a). Here, solar convective velocities are two orders of magnitude smaller than theory predicts, and large length scales, which should have the greatest power due to their powerful driving in deep layers of the Sun, have less power than small length scales. These two problems – low convective amplitudes and a lack of "giant cells" at large length scales – *are* the convective conundrum. More recent work by Greer et al. (2015) (Fig. 1b) argue that the convective velocity amplitude is perhaps not so bad as previous reported, but there is still a distinct lack of giant cells imprinting on the near-surface flows. Even simpler doppler measurements of the velocity fields at the solar surface which are not muddied by complex helioseismic inversions do not exhibit the presence of giant cells (e.g., Hathaway et al. (2015) & Fig. 1c). The motions of surface granules and the slightly deeper supergranules are clearly present, but no larger length scale is distinct.

Lord et al. (2014) showed that unstable, convecting layers drive motions on a length scale which is proportional to the local density scale height, which increases with depth. According to our standard picture of convection in which the entire convective zone is unstable, the motions driven at high density deep in the convective zone should be large scale, and should imprint strongly at the solar surface. This paints a troubling picture. Years of simulation results – and the Mixing Length

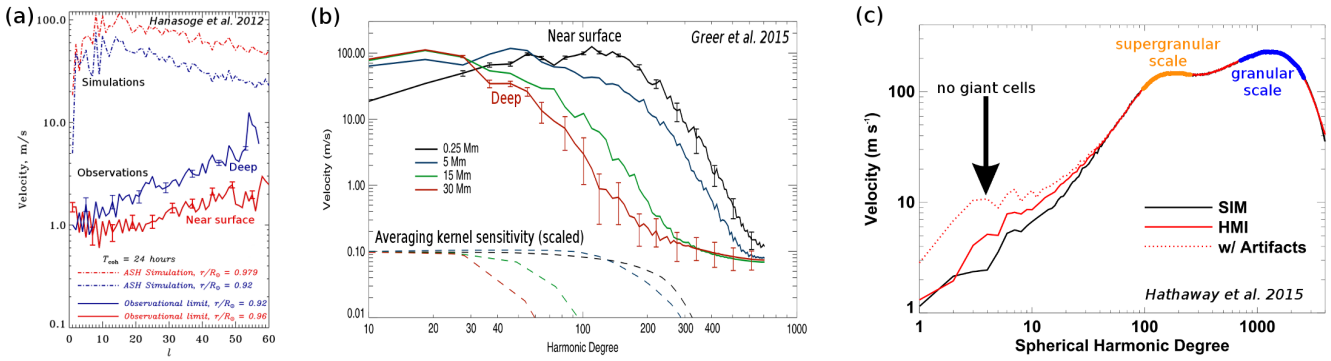

Figure 1: (a) Power spectra of solar convective velocities are shown for both observations and simulations, and both near the surface and deeper Hanasoge et al. (2012). Observations are obtained using time-distance helioseismology, and show velocities roughly two orders of magnitude lower than those predicted by simulations, and a decrease in power approaching larger length scales rather than the opposite. (b) Further observations of solar velocity power using ring-diagram helioseismology. Here, velocity magnitudes are roughly in line with those predicted by simulations, but show decreasing power as larger scales are approached, unlike what is expected from simulations Greer et al. (2015). A simple spectrum of horizontal velocities at the solar surface, obtained using line-of-sight Doppler velocities Hathaway et al. (2015). The length scales of surface granules and deeper supergranules appear as distinct features, but the hypothesized giant cells are not observed at low wavenumber.

Theory (MLT) of convection that they inform – seem to be wrong. Recent work by Brandenburg (2016) Brandenburg (2016) has sought to incorporate nonlocal effects in convection into MLT, but recent simulations Käpylä et al. (2017) have shown that regions which carry a great amount of flux convectively do not necessary locally drive the convection. It is the tendency of simulators in the field to ensuring that the physics in our simulations are correct, but this often comes at the price of having testable hypotheses which can be meaningfully explored. When running complex, 3D simulations, computational cost often limits the number of simulations which can be run, and parameter space studies are often out of the question.

Drawing on the knowledge and expertise of those in the physics community who study incompressible, unstratified Rayleigh-Bénard convection, we have recently examined hydrodynamic, compressible convection in simple stratified domains Anders & Brown (2017). We discovered that these somewhat complicated systems *transport heat in the same way as incompressible systems*, and that the Mach number of convection can be easily specified *a priori*. By setting up a simple experiment, we were able to learn something concrete about the fundamental nature of convection.

Here we propose a trio of similar, small experiments. In each experiment, we test the effects of one new piece of physics in order to gain an understanding of how they influence the convective motions and in order to gain a frame of reference for how to understand these effects in simulations with more complicated, "more realistic" physics.

2 Proposed Project

I propose a series of small projects which will probe and quantify the effects of key elements of solar convection on the dynamics of the evolved flows. Each of these pieces of physics is often included in the more complex global simulations or local area models.

In the interior of the Sun, there is a positive radial gradient of the opacity. This means that as one looks further outwards from the center of the Sun, radiation is less efficient at carrying flux and so convection becomes increasingly essential to carry the solar luminosity. This convection is strongly

driven near the solar photosphere, helped in large part thanks to the ionization and recombination of hydrogen atoms. We will study both of these phenomena in small pieces.

The divergence of the radiative flux in the solar interior acts like an internal heating term. Flux is deposited as it ceases to be carried by radiation, and this deposition of flux either drives convection or requires convection driven nonlocally to carry it. We will thus study simple atmospheres which are driven by internal heating rather than driven by boundary conditions.

The driving of convection at the solar surface by hydrogen ionization and recombination can be added to a simple study of convection through the use of a phase change at a specific temperature. We will study atmospheres where one of these phase changes occurs within the domain, and examine its effects on the dynamics of convection.

Then, we will examine the effects of using a realistic opacity profile, rather than a constant opacity throughout the depth of the box. With our knowledge of internally heated convection, we will understand how the divergence of the radiative flux drives the convection. With our knowledge of the hydrogen ionization and recombination, we will be able to employ a mechanism in the upper atmosphere which efficiently releases the large amount of energy carried by the convection. By combining these experiments with a more realistic profile of opacity, we will then

The first two projects involve understanding mechanisms of driving convection that are not frequently used in simple simulations. The third project involves a careful study of a more complicated piece of physics which is often included, but whose effects have not been studied in careful detail.

2.1 Mini Project 1: Kramer's Opacity

While it is convenient to use a simple form of the radiative flux of the form of Fourier's law of conduction Lecoanet et al. (2014) (flux $\propto -\kappa\nabla T$), as such a formulation makes it very simple to understand what regions in the atmosphere are stable and which are unstable, and exactly *how* unstable they are, things are not so simple in nature. While the general form is correct, κ is, in general, not a constant value through the depth of the atmosphere, but rather takes the form of a Kramer's opacity, such as $\kappa \propto T^3/\rho$. Building upon our previous work, we will determine the proper adiabatic gradient for a system with this form of radiative flux, understand how to quantify heat transport and determine how to set the Mach number of the experiment. Then, we will run experiments at low and high Mach number. We will compare a κ which is allowed to change with time and a κ that changes with height, but not time. We hypothesize that, for low Mach number flows, the time variance of κ, which makes it a fully nonlinear term, is unimportant, as low Ma flows are by definition very small compared to the background. Once we understand which regimes it is appropriate to use a fluctuating verses non-fluctuating κ, we will as further questions.

We are curious to see if this more realistic form of the conductivity significantly changes the transport properties of the overall convection: does it affect the heat transport (Nu)? How does the change in the adiabatic temperature gradient under this formulation change our intuition? We can likely naturally have heavily driven regions and lightly driven regions of naturally-occuring, internally heated convection. Our studies from project 1 will help inform how to interpret the divergence of this flux term as an internal heating source.

2.2 Mini Project 2: Hydrogen recombination

Most studies of convection are driven either by the choice of fixed boundary conditions (hot at bottom, cold at top) or through the volumetric depoisition of energy throughout the domain (internal heating). Here we propose a project in which we utilize a nonlinear equation of state to represent the ionization / reionization of hydrogen near the surface of the Sun. We examine the effects that this has on the

driving boundary layer region near the top of the simulation, and we do blahblahblah.

In order to carefully study the effects of the ionization of hydrogen, or other similar phase changes, we must first understand the reference state for such an atmosphere. It will no longer simply be polytropic, as the nonlinear EOS rules out the simple polytropic assumption. Once the appropriate state which is in hydrostatic and thermal equilibrium despite this phase change is discovered, we will then carefully study the effects of the temperature of ionization (which will in turn determine its depth), and also the ionization energy of the transition. In standard simulations of convection, the interactions of the flows with a hard boundary forms a thin boundary layer, which scales downwards as the diffusivities shrink, and this boundary generally drives convection. We suspect that we can create atmospheres with larger or smaller boundary layers than the natural thermal boundary length scale, which will nominally drive convection on different length scales than the natural one. We are interested in seeing how these "too large" convective flows interact with their surroundings, and in seeing how the atmosphere naturally evolves in the presence of these flows.

We are particularly interested in determining the effects of *where* in the atmosphere the transition from ionized to neutral Hydrogen occurs, and also *how much energy* is involved in the ionization process. These are two simple controls which can be examined in full detail through a suite of simulations. We hypothesize that, for sufficiently energetic ionization processes (such as that of neutral hydrogen), a natural boundary layer will form between an overlying stable layer and underlying convecting region in the atmosphere. We are further interested in determining how the ionization energy determines the length scale of the boundary layer.

Using these two control knobs, we are very interested in determining the startification of the evolved solution. This will tell us how large of a region is driving convection (is it a small layer near the reionization, or does it extend to a great depth below that? We are also very interested in the filling factor of convection compared to simple boundary-driven convection.

3 Numerical Tools and Feasibility

I will use the open-source Dedalus[1] pseudospectral framework Burns et al. (2016) to carry out my simulations. Dedalus is a flexible solver of general partial differential equations, making it extremely easy to study diverse sets of equations under many different atmospheric constraints. I have already published one paper using this tool Anders & Brown (2017), have submitted another paper, and am now adept at using it to create suites of simulations in short timeframes.

I will primarily study 2D convective solutions in plane-parallel atmospheres in order to gain intuition about the mean behavior of vertical profiles within the atmosphere. Once I have a grasp on how my measurements vary in 2D across parameter space, I will run select 3D simulations to verify whether or not that behavior holds in 3D, as I did in my previous paper Anders & Brown (2017). In cases where 2D and 3D diverge, I will quantify how and why they do so, but most questions I am asking are quite basic, and most of the systems I propose to study here have not been studied in the compressible context, at least not recently. By primarily studying in 2D, and by carefully selecting my 3D runs once I know which parameters I must examine more carefully, I can complete a full suite of simulations, such as those in my previous paper Anders & Brown (2017), using roughly 3 million CPU-hours. Through my advisor, I have access to an allocation on NASA Pleiades of roughly 20 million CPU-hours/year, so one- or two- of the following projects of the scope I am proposing can be completed each year.

Furthermore, my recent work (Anders, Brown, & Oishi 2018, submitted to PRFluids) has shown that properly constructed boundary value problems, coupled with initial value problems, can fast-

[1] http://dedalus-project.org/

forward the slow thermal evolution of these convective simulations. This work was done in Boussinesq, Rayleigh-Bénard convection but can be easily extended to stratified convection, and will greatly extend both the number of simulations we are able to complete and the level of turbulent driving (while attaining converged atmospheres) that we are able to solve.

3.1 Timeline of proposed work

Year 1 (Fall 2018 - Summer 2019):

- Finalize work on Internally Heated convection, which will be started Spring-Summer 2018. Submit a short paper summarizing the methods and results of this work to the Astrophysical Journal by mid-Fall 2018. Release code upon submission such that the community can use it.
- Delve into literature on past work regarding ionizing convection and moist convection. Understand the pieces of physics necessary to correctly implement a nonlinear equation of state. Begin to develop Dedalus simulations of ionizing convection by end of 2018.
- Fully Develop ionizing convection code. Determine the range of parameter space to be studied and execute the simulations within this range. Analyze data, and have a short paper written and submitted on them to the Astrophysical Journal by the end of summer 2019.

Year 2 (Fall 2019 - Spring 2020):

- Begin literature review on Kramer's opacity late summer 2019-early fall 2020. Understand past work done, and implementing both time-dependent (fully nonlinear) and time-independent versions of the Kramer's opacity in simple atmospheres. Run a suite of simulations at high- and low- Mach number by end of year 2019.
- Analyze data from Kramer's opacity results and prepare a small paper to submit to the Astrophysical Journal Letters by end of Winter 2020.
- Combine work from five published papers into a thesis, to be defended at the end of Spring 2020.

4 Relevance to NASA

The proposed work fits with NASA's 2014 Strategic Plan objective 1.4: "Understand the Sun and its interactions with Earth and the solar system, including space weather." Specifically, I aim to help answer the fundamental question, "What causes the Sun to vary?" This work also aims to answer one of the three overarching science goals in chapter 4.1 of NASA's 2014 Science Plan: "Develop the knowledge and capability to detect and predict extreme conditions in space to protect life and society and to safeguard human and robotic explorers beyond earth." In order to understand how to predict space weather appropriately, we need to understand the processes that cause this weather. It is clear from recent work that our understanding of the fundamentals of convection is not as perfect as we once thought, and now is an exciting time to clarify our theory and determine which parts of it fail and which parts hold true under more examination. Only once we understand the fundamental nature of stratified, compresible convection can we begin to understand how it drives the dynamo in the Sun in the presence of many complications such as differential rotation, shear layers near the base and top of the convection zone, and magnetism.

The work has been motivated by data from the Helioseismic and Magnetic Imager (HMI) onboard the NASA Solar Dynamics Observtory (SDO) spacecraft Hanasoge et al. (2012); Greer et al. (2015); Hathaway et al. (2015), and will continue to be informed by new helioiseismic measurements made from SDO data, and from the new measurements which will be made possible by the upcoming joint NASA-ESA Solar Orbiter's Polarimetric and Helioseismic Imager (PHI).

5 Summary

Recent observations call into question our fundamental understanding of stratified convection in systems such as the solar convection zone Hanasoge et al. (2012); Greer et al. (2015). Here we present three focused, scoped studies of stratified convection which probe the specific effects of individual elements of convection in the Sun. Convection must carry flux in the Sun's convective envelope, because the radiative flux becomes too small and deposits energy there, acting essentially like internal heating. Convection is driven at the surface of the Sun due to the ionization and recombination of hydrogen near the solar photosphere. The magnitude of the flux that must be carried by convection varies greatly throughout the depth of the convection zone due to the increase of opacity with height.

We propose to probe the first of these elements by studying simple internal heating systems, the second of these elements by studying convection with a nonlinear equation of state / phase change, and the third of these elements by studying convection with a realistic opacity profile in the context of our knowledge from the first two experiments. Due to the developed nature of our computational tool, Dedalus, the simulations for these projects can be implemented and carried out on short timescales, and the body of work suggested here should be finished within two years, by the end of the spring of 2019.

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