Towards a more complete understanding of Stratified, Compressible Convection

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Blah blah blah. This project supports objective 1.4 of NASA's 2014 Strategic Plan and will assist in developing "the knowledge and capability to detect and predict extreme conditions in space" in accordance with NASA's overarching Heliophysics science goals.

I. BACKGROUND & MOTIVATION

The Sun exhibits active magnetism which cycles in magnitude every 11 years. This magnetism arises from an organized dynamo seated in the turbulent plasma motions of the solar convective zone, which occupies roughly the outer 30% of the Sun's radius. This activity manifests itself in the collection of phenomena generally referred to as solar activity, including magnetic storms and coronal mass ejections. Such activity propagates towards Earth, threatening disruption of power grids and aircraft operations as well as endangering astronauts and satellites. The motivation to understand the Sun's magnetism in our increasingly technological society is great. A critical step in protecting our society from the Sun's magnetism is to gain an understanding of the nature of the dynamo that generates the Sun's magnetic fields [1, 2].

To understand the dynamo that drives the Sun's magnetism, we must understand the convection which powers that dynamo. Numerical studies of convection in stratified domain have a rich history in the past decade. The early work of [3] and [4] and others in simple domains provided rich insight into the nature of stratified convection and provided a basis in a field which now regularly creates both complex, 3D global models of convection (e.g., [5], [6], and many others) and smaller scale local area models with more complex physics (e.g., [7], [8]). From these efforts we have learned a great deal about the nature of convection, and have even created beautiful simulations which even *look* like the convection we see on the surface of the Sun.

Unfortunately, the great advances made in computational prowess within the solar physics community seem to have surpassed our fundamental knowledge in the field. Recent observations by Hanasoge et al. (2012) [9] (Fig. 1a) show that velocities in the solar convection zone are much lower than we would anticipate from our knowledge of simulations. While the more recent observations of Greer et al. (2015) [10] (Fig. 1b) argue that the problem is perhaps not so bad as it appeared a few years before. However, Greer's observations still show a decrease in power at large length scales, which does not line up with simulations. Lord et al. (2014) [11] showed that unstable, convecting layers drive motions on a length scale which is proportional to the local density scale height, which increases with depth. According to our standard picture of convection in which the entire convective zone is unstable, the motions driven at high density deep in the convective zone should be large scale, and should imprint strongly at the solar surface. However, even doppler measurements of the velocity fields at the solar surface do not show this trend [12] (Fig. 1c). The motions of surface granules and the slightly deeper supergranules are clearly present, but the "giant cells" which are theorized to be driven at the base of the 14-density-scale-height-deep convection zone are not observed.

This paints a troubling picture. Years of simulation results – and the Mixing Length Theory (MLT) of convection that they inform – seem to be wrong. Recent work by Brandenburg (2016) [**?**] has sought to incorporate nonlocal effects in convection into MLT, but recent simulations [16] have shown that regions which carry a great amount of flux convectively do not necessary locally drive the convection. It is the tendency of simulators in the field to ensuring that the physics in our simulations are correct, but this often comes at the price of having testable hypotheses which can be meaningfully explored. When running complex, 3D simulations, computational cost often limits the number of simulations which can be run, and parameter space studies are often out of the question.

Drawing on the knowledge and expertise of those in the physics community who study incompressible, unstratified Rayleigh-Bénard convection, we have recently examined hydrodynamic, compressible convection in simple stratified domains [13]. We discovered that these somewhat complicated systems *transport heat in the same way as incompressible systems*, and that the Mach number of convection can be easily specified *a priori*. By setting up a simple experiment, we were able to learn something concrete about the fundamental nature of convection.

Here we propose a trio of similar, small experiments. In each experiment, we test the effects of one new piece of physics in order to gain an understanding of how they influence the convective motions and in order to gain a frame of reference for how to understand these effects in simulations with more complicated, "more realistic" physics.

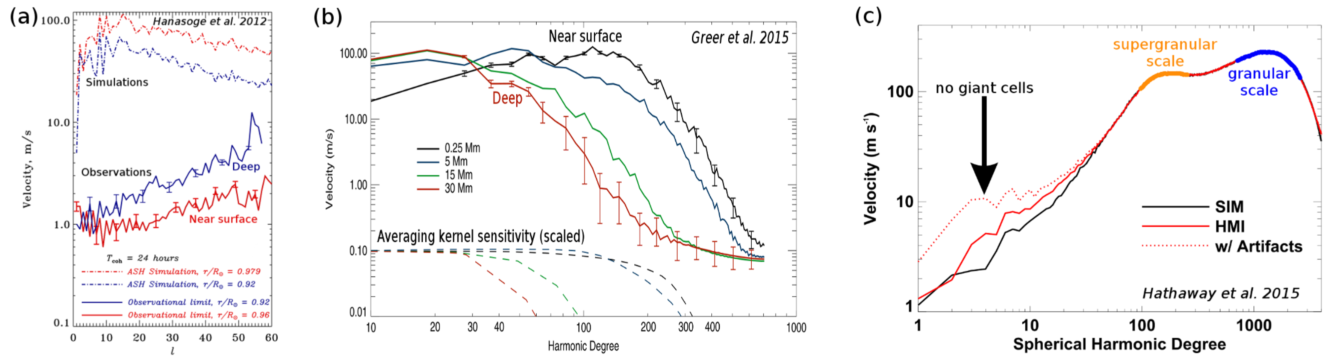

FIG. 1. (a) Power spectra of solar convective velocities are shown for both observations and simulations, and both near the surface and deeper [9]. Observations are obtained using time-distance helioseismology, and show velocities roughly two orders of magnitude lower than those predicted by simulations, and a decrease in power approaching larger length scales rather than the opposite. (b) Further observations of solar velocity power using ring-diagram helioseismology. Here, velocity magnitudes are roughly in line with those predicted by simulations, but show decreasing power as larger scales are approached, unlike what is expected from simulations [10]. A simple spectrum of horizontal velocities at the solar surface, obtained using line-of-sight Doppler velocities [12]. The length scales of surface granules and deeper supergranules appear as distinct features, but the hypothesized giant cells are not observed at low wavenumber.

II. PROPOSED PROJECT

I propose a series of small projects which will probe and quantify the effects of key elements of solar convection on the dynamics of the evolved flows. Each of these pieces of physics is often included in the more complex global simulations or local area models.

The first two projects involve understanding mechanisms of driving convection that are not frequently used in simple simulations. The third project involves a careful study of a more complicated piece of physics which is often included, but whose effects have not been studied in careful detail.

A. Mini Project 1: Internally heated convection

Our first experiment involves the study of atmospheres where a constant internal heating term is included in the energy equation. This is the simplest form of internal heating, and is similar to that which has been studied in Rayleigh-Bénard convection, just with compressibility and stratification included (e.g., [15]). We have found that these systems can be constructed so as to naturally include stable layers underlying unstable, convection regions. These systems are important to study, because the changing opacities in the interior of the solar convective zone effectively act like an internal heating term, depositing energy and driving convection.

We find that these systems essentially have the same input control parameters as simple polytropic convection [13], which include the Rayleigh number (Ra, the strength of convective driving and amount of turbulence), the Prandtl number (Pr, the ratio of the thermal to viscous diffusivity), the characteristic superadiabaticity of the atmosphere (which sets the Mach number of the flows), and the depth of the atmosphere. In the internally heated systems, we find that the magnitude of the internal heating directly relates to the superadiabaticity of the atmosphere, and thus Ma, but we must split up the depth into two parameters: the depth of the *unstable* region, and the depth of the new *stable* region. In [13], we explored the effects of the magnitude of the superadiabaticity on the resulting convection, and found that – except where the Mach number is very high – it has almost no effect on the other properties of the convective flows.

In this project, we will focus on understanding how the depth of the atmosphere, and the amount of stratification felt by the flows, affects the resultant convection. We will set the depth of the convection zone (to 1, 3, and then 5 density scale heights), and for each of these depths we will run simulations for shallow and deep radiative zones. We will examine the evolved stratification of these atmospheres in order to determine where convection is driven and where it is not. Preliminary results show that the so-called "Deardorff zones" present in the simulations of [16] are also present in these simple simulations of internally heated convection and do not require complex forms of the conductivity to be achieved. We expect that the extent of these subadiabatic zones in which convection carries the flux can be determined *a priori* in these simple simulations, but further work must be done.

We're looking for the motions at the surface and also the stratification.

B. Mini Project 2: Hydrogen recombination

Most studies of convection are driven either by the choice of fixed boundary conditions (hot at bottom, cold at top) or through the volumetric depoisition of energy throughout the domain (internal heating). Here we propose a project in which we utilize a nonlinear equation of state to represent the ionization / reionization of hydrogen near the surface of the Sun. We examine the effects that this has on the driving boundary layer region near the top of the simulation, and we do blahblahblah.

In order to carefully study the effects of the ionization of hydrogen, or other similar phase changes, we must first understand the reference state for such an atmosphere. It will no longer simply be polytropic, as the nonlinear EOS rules out the simple polytropic assumption. Once the appropriate state which is in hydrostatic and thermal equilibrium despite this phase change is discovered, we will then carefully study the effects of the temperature of ionization (which will in turn determine its depth), and also the ionization energy of the transition. In standard simulations of convection, the interactions of the flows with a hard boundary forms a thin boundary layer, which scales downwards as the diffusivities shrink, and this boundary generally drives convection. We suspect that we can create atmospheres with larger or smaller boundary layers than the natural thermal boundary length scale, which will nominally drive convection on different length scales than the natural one. We are interested in seeing how these "too large" convective flows interact with their surroundings, and in seeing how the atmosphere naturally evolves in the presence of these flows.

We are particularly interested in determining the effects of *where* in the atmosphere the transition from ionized to neutral Hydrogen occurs, and also *how much energy* is involved in the ionization process. These are two simple controls which can be examined in full detail through a suite of simulations. We hypothesize that, for sufficiently energetic ionization processes (such as that of neutral hydrogen), a natural boundary layer will form between an overlying stable layer and underlying convecting region in the atmosphere. We are further interested in determining how the ionization energy determines the length scale of the boundary layer.

Using these two control knobs, we are very interested in determining the startification of the evolved solution. This will tell us how large of a region is driving convection (is it a small layer near the reionization, or does it extend to a great depth below that? We are also very interested in the filling factor of convection compared to simple boundary-driven convection.

III. MINI PROJECT 3: KRAMER'S OPACITY

While it is convenient to use a simple form of the radiative flux of the form of Fourier's law of conduction (flux $\propto -\kappa\nabla T$), as such a formulation makes it very simple to understand what regions in the atmosphere are stable and which are unstable, and exactly *how* unstable they are, things are not so simple in nature. While the general form is correct, κ is, in general, not a constant value through the depth of the atmosphere, but rather takes the form of a Kramer's opacity, such as $\kappa \propto T^3/\rho$. Building upon our previous work, we will determine the proper adiabatic gradient for a system with this form of radiative flux, understand how to quantify heat transport and determine how to set the Mach number of the experiment. Then, we will run experiments at low and high Mach number. We will compare a κ which is allowed to change with time and a κ that changes with height, but not time. We hypothesize that, for low Mach number flows, the time variance of κ, which makes it a fully nonlinear term, is unimportant, as low Ma flows are by definition very small compared to the background. Once we understand which regimes it is appropriate to use a fluctuating verses non-fluctuating κ, we will as further questions.

We are curious to see if this more realistic form of the conductivity significantly changes the transport properties of the overall convection: does it affect the heat transport (Nu)? How does the change in the adiabatic temperature gradient under this formulation change our intuition? We can likely naturally have heavily driven regions and lightly driven regions of naturally-occuring, internally heated convection. Our studies from project 1 will help inform how to interpret the divergence of this flux term as an internal heating source.

IV. NUMERICAL TOOLS AND FEASIBILITY

I will use the open-source Dedalus[1] pseudospectral framework [14] to carry out my simulations. Dedalus is a flexible solver of general partial differential equations, making it extremely easy to study diverse sets of equations under many

[1] http://dedalus-project.org/

different atmospheric constraints. I have already published one paper using this tool [13], have submitted another paper, and am now adept at using it to create suites of simulations in short timeframes.

I will primarily study 2D convective solutions in plane-parallel atmospheres in order to gain intuition about the mean behavior of vertical profiles within the atmosphere. Once I have a grasp on how my measurements vary in 2D across parameter space, I will run select 3D simulations to verify whether or not that behavior holds in 3D, as I did in my previous paper [13]. In cases where 2D and 3D diverge, I will quantify how and why they do so, but most questions I am asking are quite basic, and most of the systems I propose to study here have not been studied in the compressible context, at least not recently. By primarily studying in 2D, and by carefully selecting my 3D runs once I know which parameters I must examine more carefully, I can complete a full suite of simulations, such as those in my previous paper [13], using roughly 3 million CPU-hours. Through my advisor, I have access to an allocation on NASA Pleiades of roughly 20 million CPU-hours/year, so one- or two- of the following projects of the scope I am proposing can be completed each year.

Furthermore, my recent work (Anders, Brown, & Oishi 2018, submitted to PRFluids) has shown that properly constructed boundary value problems, coupled with initial value problems, can fast-forward the slow thermal evolution of these convective simulations. This work was done in Boussinesq, Rayleigh-Bénard convection but can be easily extended to stratified convection, and will greatly extend both the number of simulations we are able to complete and the level of turbulent driving (while attaining converged atmospheres) that we are able to solve.

A. Timeline of proposed work

I need to expand this.

a. Year 1:

- (Fall '18) Finalize Internally Heated convection work, submit a paper to ApJL.
- (Spring-Summer '19) Run simulations of nonlinear EOS, write paper, submit to ApJ. Begin opacity project.

b. Year 2:

- (Fall '19 - Spring '20) Finalize opacity project, submit ApJ. Write and finalize thesis by end of Spring '20.

V. RELEVANCE TO NASA

I need to make this 2017 strategic plan. The proposed work fits with NASA's 2014 Strategic Plan objective 1.4: "Understand the Sun and its interactions with Earth and the solar system, including space weather." This work also fits in with one of the three overarching science goals of the Heliophysics section of NASA's 2014 Science Plan: "Develop the knowledge and capability to detect and predict extreme conditions in space to protect life and society and to safeguard human and robotic explorers beyond earth." Furthermore, simulated observables created by this work will be directly comparable to data retrieved by the currently operational Solar Dynamics Observatory and the future NASA-ESA Solar Orbiter mission.

VI. SUMMARY

Here I summarize all the stuff I said above.

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