Draft version September 14, 2018
Typeset using LaTeX **twocolumn** style in AASTeX62

Controlling the rotational constraint in stratified, compressible convection

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(Received September 14, 2018; Revised September 14, 2018; Accepted September 14, 2018)

Submitted to ApJ

ABSTRACT

Keywords: convection — happy caterpillars

1. INTRODUCTION

Convection under the influence of rotation has been studied in great detail in recent decades. In the incompressible boussinesq case, numerous authors have studied rotating convection in both laboratory and numerical settings. The behavior of the heat transport in these systems has been the focus of many of these studies, particularly the difference of behavior of heat transport in rotationally-constrained and unconstrained regimes (King et al. 2009; Zhong et al. 2009; Stevens et al. 2009; Julien et al. 2012). Studies of both laboratory (cite, cite) and numerical (cite, cite) experiments coincide well in these regimes.

More complicated experiments of rotational convection are often conducted using numerical tools in spherical geometries. Often these studies aim to gain insight into the solar dynamo (Glatzmaier & Gilman 1982; Busse 2002; Brown et al. 2008, 2010, 2011; Augustson et al. 2012; Guerrero et al. 2013; Käpylä et al. 2014). Numerous interesting phenomenological discoveries have been made in such studies, but they often employ very different degrees of rotational constraint, but their basic system behavior often differs significantly from the sun. For example, dynamo simulations often produce antisolar differential rotation profiles at nominally solar values of rotational constraint.

In recent years, helioseismic imaging of flows in the Sun have suggested that power in large-scale convective motions is much lower than convective simulations and mixing length theory predict (Hanasoge et al. 2012; Greer et al. 2015). This suggests that either convection is not driven deep in the solar convection zone or its motions are masked before they reach the surface by some process. A number of possible mechanisms have been suggested as culprits for this behavior, including entropy rain (Brandenburg 2016). One other possibility is that the interior convection zone is rotationally constrained, and that this behavior is reducing low wavenumber power at the surface. The recent work of Featherstone & Hindman (2016) suggests that this is a strong possibility, and the observations of Greer et al. (2016) suggest that flows in the deep solar convection zone are rotationally constrained. The degree of rotational constraint under which convective flows occur can greatly change the character of the resultant situation, and so in the case of astrophysical contexts (solar studies such as those mentioned above and also planetary studies such as e.g., Soderlund et al. (2015)), it is important to study convection in the same rotational regime as the astrophysical object of interest in order to meaningfully understand results.

One difficulty in studying rotational convection is that it is often unclear from input parameters whether or not the resulting convective state will be rotationally constrained. In Anders & Brown (2017) (hereafter AB17), we studied non-rotating, hydrodynamic, compressible convection. We showed how the evolved Reynolds number, Peclet number (Re and Pe, two measures of turbulence in the evolved state), and Mach number (Ma, the ratio of flow speed to the local sound speed) of the convective solution can be specified through a properly constructed reference atmosphere. Upon the inclusion of rotation, a fourth dynamical measure of the solution becomes meaningful: the Rossby number (Ro, the ratio of advective dynamics to rotational constraint). Low Ro flows are rotationally constrained, while high Ro flows are not. While the literature contains a wealth of infor-

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mation regarding how the magnitude of rotation affects the heat transport characteristics of convection, we find no work which simply links the rotational constraint of evolved solutions with input parameters.

In this work, we extend the study of AB17 to rotationally-influenced, f-plane polytropic atmospheres, as have been previously studied by e.g., Brummell et al. (1996, 1998); Calkins et al. (2015). Our goal is to determine how the input parameters which we studied previously couple with a new input parameter, the Taylor number (Ta, Julien et al. (1996)), which sets the magnitude of the rotational vector. In section 2, we describe our atmosphere, numerical experiment, and paths through parameter space. In section 3, we present the results of our experiments and we offer concluding remarks.

2. EXPERIMENT

We study fully compressible, stratified convection as we previously did in Anders & Brown (2017), with the inclusion of rotation. We study an ideal gas whose equation of state is $P = R\rho T$ and with an adiabatic index $\gamma = 5/3$. We nondimensionalize the atmosphere such that $R = 1$ and $P = \rho = T = 1$ at the top of the domain. The initial stratification is polytropic, such that

$$\begin{aligned} \rho_0(z) &= (1 + L_z - z)^m, \\ T_0(z) &= (1 + L_z - z), \end{aligned} \tag{1}$$

where m is the polytropic index, z increases upwards in the range $z = [0, L_z]$, and $L_z \equiv e^{n_\rho/m} - 1$ is the depth of the atmosphere, where n_ρ specifies the number of density scale heights that the atmosphere spans. We specify the instability of the atmosphere through the superadiabatic excess, $\epsilon = m - m_{ad}$, where $m_{ad} = (\gamma - 1)^{-1}$ is the adiabatic polytropic index, and ϵ controls the Mach number of the flows (Anders & Brown 2017). The domain is a 3D cartesian box whose horizontal extent is in the range $x, y = [0, AL_z]$, where A is the aspect ratio of the domain. As has been studied previously by e.g., Julien et al. (1996); Brummell et al. (1996), we study a domain in which the gravity and rotational vector are antiparallel, $\boldsymbol{g} = -g\hat{z}$, and $\boldsymbol{\Omega} = \Omega\hat{z}$.

We evolve the velocity ($\boldsymbol{u}$), temperature, and log density according to the Fully Compressible Navier-Stokes equations,

$$\frac{\partial \ln\rho}{\partial t} + \nabla\cdot\boldsymbol{u} = -\boldsymbol{u}\cdot\nabla\ln\rho, \tag{2}$$

$$\begin{aligned} \frac{\partial \boldsymbol{u}}{\partial t} + 2\boldsymbol{\Omega}\times\boldsymbol{u} + \nabla T - \nu\nabla\cdot\bar{\bar{\boldsymbol{\sigma}}} - \bar{\bar{\boldsymbol{\sigma}}}\cdot\nabla\nu = \\ -\boldsymbol{u}\cdot\nabla\boldsymbol{u} - T\nabla\ln\rho + \boldsymbol{g} + \nu\bar{\bar{\boldsymbol{\sigma}}}\cdot\nabla\ln\rho, \end{aligned} \tag{3}$$

$$\begin{aligned} \frac{\partial T}{\partial t} - \frac{1}{c_V}\left(\chi\,\nabla^2 T + \nabla T\cdot\nabla\chi\right) = \\ -\boldsymbol{u}\cdot\nabla T - (\gamma-1)T\nabla\cdot\boldsymbol{u} \\ + \frac{1}{c_V}\left(\chi\nabla T\cdot\nabla\ln\rho + \nu\left[\bar{\bar{\boldsymbol{\sigma}}}\cdot\nabla\right]\cdot\boldsymbol{u}\right), \end{aligned} \tag{4}$$

with the viscous stress tensor given by

$$\sigma_{ij} \equiv \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot\boldsymbol{u}\right). \tag{5}$$

The kinematic viscosity, ν, thermal diffusivity, χ, and strength of rotation Ω are set at the top of the domain by the Rayleigh number (Ra), Prandtl number (Pr), and Taylor number (Ta),

$$\mathrm{Ra} = \frac{gL_z^3\Delta S/c_P}{\nu\chi},\;\; \mathrm{Pr} = \frac{\nu}{\chi},\;\; \mathrm{Ta} = \left(\frac{2\Omega L_z^2}{\nu}\right)^2, \tag{6}$$

where $\Delta S = \epsilon n_\rho/m$ is the specific entropy difference between $z = 0$ and $z = L_z$, and the specific heat at constant pressure is c_P.

We measure the resulting Rossby number, Nusselt number, and Reynolds number of all flows in order to understand the various regimes of convection which are open to us.

Figure 1. (a) The critical Rayleigh number, as a function of the Taylor number, is plotted as a solid black line. Paths of constant Convective Rossby Number (red dashed line), constant supercriticality (orange dashed line), and COPRIME (blue solid line) are shown through parameter space. (b) Evolved Rossby number is plotted vs. Taylor number along multiple constant COPRIME paths, such as the solid blue line in (a). After a sharp increase at low Ta, the evolved Rossby number flattens out and stays nearly constant across orders of magnitude of Ta.

3. RESULTS & DISCUSSION

This is where figures go and other important things that we like to talk about.

3.1. *acknowledgements*

EHA acknowledges the support of the University of Colorado's George Ellery Hale Graduate Student Fellowship. This work was additionally supported by NASA

Figure 2. (a) Evolved Nusselt number vs. Rayleigh number / Ra_crit along constant COPRIME paths. A classic scaling law of $Nu \propto Ra^{2/7}$ is observed. (b) Evolved Reynolds number vs. Rayleigh number / Ra_crit along constant COPRIME paths. A classic scaling law of $Re \propto Ra^{1/2}$ is observed. These laws are reminiscent of standard scaling laws of Re and Nu in non-rotational convection (SOURCES SOURCES SOURCES). This suggests that at fixed Rossby number on a constant COPRIME path (Fig. 1), varying the Rayleigh number affects the evolved dynamics in a manner similar to a nonrotating fluid.

Figure 3. Here's some pretty plots, we'll figure out what we plot later.

Figure 4. (a) Horizontally-averaged profiles of the Rossby number are shown vs. z for a constant COPRIME = X. (b) Horizontally-averaged profiles of the entropy gradient are shown vs. z for a constant COPRIME = X. (c) Vorticity boundary layer thickness normalized by entropy boundary layer thickness as a function of Ta/Ta_crit for multiple COPRIME paths. When this measure is $\gg 1$, we expect the flows to be buoyancy dominated, when it is $\ll 1$, we expect the flows to be rotationally dominated, and when it is ~ 1, we anticipate that both effects are very important.

LWS grant number NNX16AC92G. Computations were conducted with support by the NASA High End Computing (HEC) Program through the NASA Advanced Supercomputing (NAS) Division at Ames Research Center on Pleiades with allocations GID s1647. We thank Jeff Oishi for many useful discussions.

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