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Controlling the rotational constraint in stratified, compressible convection

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ABSTRACT

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1. INTRODUCTION

1. There's lots of research on rotating convection. Stratified and boussinesq. Laboratory experiments and numerics. Cylindrical, spherical, and cartesian.

2. Often in astrophysics, global rotating convection simulations aim to gain insight into the solar dynamo.

3. Recent observations, the convective conundrum, have called into question our understanding of deep convective motions in the Sun. Recent work by Featherstone & Hindman (2016) show that if the convection there is rotationally constrained, it could suppress deep convections and line up with observations better.

4. We find no explanation in the literature for how to set the rotational constraint of convection (Ro) while changing the Reynolds number to study more rotationally constrained convection. If rotational constraints are primary drivers of the nature of convection in the natural object of interest (e.g, Sun), then it will be very important to fix the Ro of convective motions while varying other parameters.

5. Much work has focused on how Nu scales with Ra at fixed Ta. In this work, we show how to study convection at a fixed rossby number in the laminar and turbulent regimes, much as we learned how to study low and high Mach number convection in AB17.

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Recent work by Featherstone & Hindman (2016) needs to be talked about. Rotating dynamo stuff Busse (2002); Brown et al. (2008, 2010, 2011) Papers on rotating RB convection that might matter: Hathaway & Somerville (1983); Julien et al. (1996); Zhong et al. (2009); Julien et al. (2012); Stellmach et al. (2014). Papers on rotating stratified f-planes that might matter: Brummell et al. (1996, 1998); Calkins et al. (2015). Honestly, most stratified, rotating sims seem to avoid simplicities (like the f-plane) as though they were the plague.

2. EXPERIMENT

We study fully compressible, stratified convection as we previously did in Anders & Brown (2017), with the inclusion of rotation. We study an ideal gas whose equation of state is $P = R\rho T$ and with an adiabatic index $\gamma = 5/3$. We nondimensionalize the atmosphere such that $R = 1$ and $P = \rho = T = 1$ at the top of the domain. The initial stratification is polytropic, such that

$$\begin{aligned} \rho_0(z) &= (1 + L_z - z)^m, \\ T_0(z) &= (1 + L_z - z), \end{aligned} \tag{1}$$

where m is the polytropic index, z increases upwards in the range $z = [0, L_z]$, and $L_z \equiv e^{n_\rho/m} - 1$ is the depth of the atmosphere, where n_ρ specifies the number of density scale heights that the atmosphere spans. We specify the instability of the atmosphere through the superadiabatic excess, $\epsilon = m - m_{ad}$, where $m_{ad} = (\gamma - 1)^{-1}$ is the adiabatic polytropic index, and ϵ controls the Mach number of the flows (Anders & Brown 2017). The domain is a 3D cartesian box whose horizontal extent is in the range $x, y = [0, AL_z]$, where A is the aspect ratio of the domain. As has been studied previously by e.g., Julien et al. (1996); Brummell et al. (1996), we study a domain in which the gravity and rotational vector are antiparallel, $\boldsymbol{g} = -g\hat{z}$, and $\boldsymbol{\Omega} = \Omega\hat{z}$.

We evolve the velocity ($\boldsymbol{u}$), temperature, and log density according to the Fully Compressible Navier-Stokes equations,

$$\frac{\partial \ln \rho}{\partial t} + \nabla \cdot \boldsymbol{u} = -\boldsymbol{u} \cdot \nabla \ln \rho, \tag{2}$$

$$\begin{aligned}\frac{\partial \boldsymbol{u}}{\partial t} + 2\boldsymbol{\Omega} \times \boldsymbol{u} + \nabla T - \nu \nabla \cdot \bar{\bar{\boldsymbol{\sigma}}} - \bar{\bar{\boldsymbol{\sigma}}} \cdot \nabla \nu = \\ - \boldsymbol{u} \cdot \nabla \boldsymbol{u} - T \nabla \ln \rho + \boldsymbol{g} + \nu \bar{\bar{\boldsymbol{\sigma}}} \cdot \nabla \ln \rho,\end{aligned} \tag{3}$$

$$\begin{aligned}\frac{\partial T}{\partial t} - \frac{1}{c_V} \left(\chi \nabla^2 T + \nabla T \cdot \nabla \chi \right) = \\ - \boldsymbol{u} \cdot \nabla T - (\gamma - 1) T \nabla \cdot \boldsymbol{u} \\ + \frac{1}{c_V} \left(\chi \nabla T \cdot \nabla \ln \rho + \nu \left[\bar{\bar{\boldsymbol{\sigma}}} \cdot \nabla \right] \cdot \boldsymbol{u} \right),\end{aligned} \tag{4}$$

with the viscous stress tensor given by

$$\sigma_{ij} \equiv \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} \delta_{ij} \nabla \cdot \boldsymbol{u} \right). \tag{5}$$

The kinematic viscosity, ν, thermal diffusivity, χ, and strength of rotation Ω are set at the top of the domain by the Rayleigh number (Ra), Prandtl number (Pr), and Taylor number (Ta),

$$\mathrm{Ra} = \frac{g L_z^3 \Delta S / c_P}{\nu \chi}, \;\; \mathrm{Pr} = \frac{\nu}{\chi}, \;\; \mathrm{Ta} = \left(\frac{2 \Omega L_z^2}{\nu} \right)^2, \tag{6}$$

where $\Delta S = \epsilon n_\rho / m$ is the specific entropy difference between $z = 0$ and $z = L_z$, and the specific heat at constant pressure is c_P.

We measure the resulting Rossby number, Nusselt number, and Reynolds number of all flows in order to understand the various regimes of convection which are open to us.

Figure 1. (a) The critical Rayleigh number, as a function of the Taylor number, is plotted as a solid black line. Paths of constant Convective Rossby Number (red dashed line), constant supercriticality (orange dashed line), and COPRIME (blue solid line) are shown through parameter space. (b) Evolved Rossby number is plotted vs. Taylor number along multiple constant COPRIME paths, such as the solid blue line in (a). After a sharp increase at low Ta, the evolved Rossby number flattens out and stays nearly constant across orders of magnitude of Ta.

3. RESULTS & DISCUSSION

This is where figures go and other important things that we like to talk about.

Figure 2. (a) Evolved Nusselt number vs. Rayleigh number / Ra_crit along constant COPRIME paths. A classic scaling law of $Nu \propto Ra^{2/7}$ is observed. (b) Evolved Reynolds number vs. Rayleigh number / Ra_crit along constant COPRIME paths. A classic scaling law of $Re \propto Ra^{1/2}$ is observed. These laws are reminiscent of standard scaling laws of Re and Nu in non-rotational convection (SOURCES SOURCES SOURCES). This suggests that at fixed Rossby number on a constant COPRIME path (Fig. 1), varying the Rayleigh number affects the evolved dynamics in a manner similar to a nonrotating fluid.

Figure 3. Here's some pretty plots, we'll figure out what we plot later.

Figure 4. (a) Horizontally-averaged profiles of the Rossby number are shown vs. z for a constant COPRIME = X. (b) Horizontally-averaged profiles of the entropy gradient are shown vs. z for a constant COPRIME = X. (c) Vorticity boundary layer thickness normalized by entropy boundary layer thickness as a function of Ta/Ta_crit for multiple COPRIME paths. When this measure is $\gg 1$, we expect the flows to be buoyancy dominated, when it is $\ll 1$, we expect the flows to be rotationally dominated, and when it is ~ 1, we anticipate that both effects are very important.

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