Heat transport in stratified convection across mach number blahdy blah

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This is where an abstract defining what we're doing with stratified convection and Nusselt numbers will go.

INTRODUCTION

Compressible convection is ubiquitous among natural systems such as stellar envelopes and planetary atmospheres. The widely-studied Rayleigh-Bénard convection in which up- and downflows are symmetric and the Rayleigh number (Ra, the ratio of buoyant driving to diffusive damping) serves as the primary control parameter lacks a number of the key features which make studies of stratified convection difficult to master. Early studies of stratified convection in two [1–4] and three [5–7] dimensions utilized polytropically stratified atmospheres, in which the temperature gradient, thermal diffusivity, and gravity are constant throughout the depth of the atmosphere and the atmosphere is in hydrostatic equilibrium. While perhaps not perfectly realistic and often abandoned for atmospheres constructed using more realistic radiative transfer properties (cite some papers), the polytrope is a particularly useful reference state for stratified convection studies.

A canonical value of the "polytropic index" used in the study of convection of monatomic ideal gases is $m = 1$, such that the density and temperature profile are both linear. Recently, [8] argued that as the polytropic index approaches the adiabatic value of $m_{ad} = 1.5$, the convective flux of the system becomes negligible compared to the background flux gradient, and the Nusselt number of the system approaches zero. They argue that, as a result, values of $m \to -1$ are ideal polytropes to study, as the radiative flux approaches zero and the convective flux carries the whole of the system flux in equilibrium.

Here, we argue for The definition of a new Nusselt number, where the radiative flux is defined relative to the *adiabatic* state rather than to a linear temperature profile. We demonstrate that values of m close to adiabatic show a Nusselt number (similar?) to [9] and that such values of m are useful in allowing us to probe low-Mach number convection, such as that deep within the Sun's convection zone (cite a paper for that?)

MODEL & EQUATIONS

We study a fluid whose equation of state is that of an ideal gas, $P = R^*\rho T$ and whose initial stratification is polytropic, where

$$\begin{aligned} \rho(z) &= \rho_{00}(z_0 - z)^m \\ T(z) &= T_{00}(z_0 - z)^m \end{aligned} \tag{1}$$

and z increases upwards. We define the height of the atmosphere such that n_ρ density scale heights fit in the atmosphere, or such that $\ln[\rho(L_z)/\rho(0)] = n_\rho$. We nondimensionalize our equations such that $P = \rho = T = 1$ at the top of the atmosphere, requiring $z_0 \equiv L_z + 1$ and $R^* = T_{00} = \rho_{00} = 1$. We specify the entropy gradient at the top of the atmosphere as $\nabla S(L_z) = -\epsilon$ such that the polytropic index of the atmosphere is $m = (\gamma - 1)^{-1} - \epsilon = m_{ad} - \epsilon$.

We evolve the Fully Compressible Navier-Stokes equations with an energy-conserving energy equation, which take the form:

$$\frac{D\ln\rho}{Dt} + \nabla\cdot(\boldsymbol{u}) = 0 \tag{2}$$

$$\rho\frac{D\boldsymbol{u}}{Dt} = -\nabla P + \rho\boldsymbol{g} - \nabla\cdot\bar{\bar{\boldsymbol{\Pi}}} \tag{3}$$

$$\begin{aligned} \rho c_V\left(\frac{DT}{Dt} + (\gamma-1)T\nabla\cdot(\boldsymbol{u})\right) + \nabla\cdot(-\kappa\nabla T) = \\ -\left(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla\right)\cdot\boldsymbol{u} \end{aligned} \tag{4}$$

where $D/Dt \equiv \partial_t + \boldsymbol{u}\cdot\nabla$ and the viscous stress tensor is defined as

$$\Pi_{ij} \equiv -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot(\boldsymbol{u})\right), \tag{5}$$

where μ is the *dynamic viscosity* (in units of [mass $\cdot$ length^{-1} $\cdot$ time^{-1}]) and δ_{ij} is the kronecker delta function. We also define the *kinematic viscosity*, which has units of a classic diffusion coefficient, as $\nu \equiv \mu/\rho$. In a similar fashion, we define the thermal diffusivity, $\chi \equiv \kappa/\rho$ [note: Kundu has a c_P on the bottom, here].

Dotting Eq. 3 with $\boldsymbol{u}$ and adding it to Eq. 4, we retrieve the full energy equation in conservation form,

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\rho\left[\frac{|\boldsymbol{u}|^2}{2} + c_V T + \phi\right]\right) + \\ &\nabla\cdot\left(\rho\boldsymbol{u}\left[\frac{|\boldsymbol{u}|^2}{2} + h + \phi\right] + \boldsymbol{u}\cdot\bar{\bar{\boldsymbol{\Pi}}} - \rho\chi\nabla T\right) = 0, \end{aligned} \tag{6}$$

where $h \equiv c_V T + P/\rho$ is the system enthalpy and $\phi = -gz$ is the gravitational potential. All of the

We use initial conditions of randomized temperature perturbations on the order of 10^{-6} below the background temperature field (EVAN – TRY TO DO 1E-6*EPSILON). We impose stress free, impenetrable, constant temperature boundary conditions.

The efficiency of convection is defined by the Nusselt number. While the Nusselt number is well-defined in Rayleigh-Bénard convection as the amount of total convective and radiative flux divided by the steady-state background flux [9, 10], a well-defined Nusselt number is more elusive in stratified convection. We suggest that a more general definition of the Nusselt number is similar to that defined by Rayleigh-Bénard convection, but where both the numerator and denominator have the radiative flux of the adiabatic temperature gradient of the atmosphere removed. Such a definition was first used by [3] who wrote it as

$$N \equiv \frac{F_T - F_A}{F - F_A}, \tag{7}$$

where F_T is the sum of fluxes in the system, $F_A = \kappa g/c_P$ is the radiative flux of the corresponding adiabatic profile of the atmosphere, and $F = \Delta T/Lz$ is the radiative flux carried by a linear temperature profile across the domain. Under the specific case of constant temperature boundary conditions, ΔT is constant and the denominator of the Nusselt number reduces to $F - F_A = -\kappa\epsilon\nabla T_0(\gamma-1)/\gamma$. Thus, the denominator of the Nusselt number is directly proportional to the level of superadiabaticity of the background atmosphere, as is the time-dependent value of $F_T - F_A$. Under the Boussinesq approximation of Rayleigh-Bénard convection, the lack of density variation means that the temperature field is directly linked to entropy, and the adiabatic temperature gradient is definitionally zero. In the case of [8], the case of $\epsilon \to (m_{ad}+1)$ is examined, but this also corresponds to the case where $g \to 0$ and the corresponding adiabatic atmospheric temperature gradient is isothermal.

Control Parameters

In addition to the explicitly scaling the level of superadiabaticity of the background entropy gradient via ϵ, we utilize the nondimensional Prandtl (Pr) and Rayleigh (Ra) numbers to determine our atmospheric diffusivities. Pr is the ratio of kinematic viscosity to thermal diffusivity, which we set equal to one. Ra is the ratiof buoyant driving to diffusivity of the form

$$\mathrm{Ra} \equiv \frac{g L_z^3 (\Delta S/c_P)}{\nu\chi}, \tag{8}$$

where $\Delta S/c_P$ is the entropy difference between the top and bottom boundaries of the reference state. Note that $\nu \propto \chi \propto \rho^{-1}$ such that Ra increases with depth as the square of the density. In this work we specify Ra at the top of the domain.

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