Heat transport in stratified convection across mach number blahdy blah

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This is where an abstract defining what we're doing with stratified convection and Nusselt numbers will go.

INTRODUCTION

Blah Blah stuff.

MODEL & EQUATIONS

We study a fluid whose equation of state is that of an ideal gas, $P = R^*\rho T$ and whose initial stratification is polytropic, where

$$\begin{aligned} \rho(z) &= \rho_{00}(z_0 - z)^m \\ T(z) &= T_{00}(z_0 - z)^m \end{aligned} \tag{1}$$

and z increases upwards. We define the height of the atmosphere such that n_ρ density scale heights fit in the atmosphere, or such that $\ln[\rho(L_z)/\rho(0)] = n_\rho$. We nondimensionalize our equations such that $P = \rho = T = 1$ at the top of the atmosphere, requiring $z_0 \equiv L_z + 1$ and $R^* = T_{00} = \rho_{00} = 1$. We specify the entropy gradient at the top of the atmosphere as $\nabla S(L_z) = -\epsilon$ such that the polytropic index of the atmosphere is $m = (\gamma - 1)^{-1} - \epsilon = m_{ad} - \epsilon$.

We evolve the Fully Compressible Navier-Stokes equations with an energy-conserving energy equation, which take the form:

$$\frac{D\ln\rho}{Dt} + \nabla\cdot(\boldsymbol{u}) = 0 \tag{2}$$

$$\rho\frac{D\boldsymbol{u}}{Dt} = -\nabla P + \rho\boldsymbol{g} - \nabla\cdot\bar{\bar{\boldsymbol{\Pi}}} \tag{3}$$

$$\begin{aligned} \rho c_V\left(\frac{DT}{Dt} + (\gamma-1)T\nabla\cdot(\boldsymbol{u})\right) + \nabla\cdot(-\kappa\nabla T) = \\ -\left(\bar{\bar{\boldsymbol{\Pi}}}\cdot\nabla\right)\cdot\boldsymbol{u} \end{aligned} \tag{4}$$

where $D/Dt \equiv \partial_t + \boldsymbol{u}\cdot\nabla$ and the viscous stress tensor is defined as

$$\Pi_{ij} \equiv -\mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3}\delta_{ij}\nabla\cdot(\boldsymbol{u})\right), \tag{5}$$

where μ is the *dynamic viscosity* (in units of [mass $\cdot$ length^{-1} $\cdot$ time^{-1}]) and δ_{ij} is the kronecker delta function. We also define the *kinematic viscosity*, which has units of a classic diffusion coefficient, as $\nu \equiv \mu/\rho$. In a similar fashion, we define the thermal diffusivity, $\chi \equiv \kappa/\rho$ [note: Kundu has a c_P on the bottom, here].

Dotting Eq. 3 with $\boldsymbol{u}$ and adding it to Eq. 4, we retrieve the full energy equation in conservation form,

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\rho\left[\frac{|\boldsymbol{u}|^2}{2} + c_V T + \phi\right]\right) + \\ &\quad\nabla\cdot\left(\rho\boldsymbol{u}\left[\frac{|\boldsymbol{u}|^2}{2} + h + \phi\right] + \boldsymbol{u}\cdot\bar{\bar{\boldsymbol{\Pi}}} - \rho\chi\nabla T\right) = 0, \end{aligned} \tag{6}$$

where $h \equiv c_V T + P/\rho$ is the system enthalpy and $\phi = -gz$ is the gravitational potential. All of the

We use initial conditions of randomized temperature perturbations on the order of 10^{-6} below the background temperature field (EVAN – TRY TO DO 1E-6*EPSILON).

Control Parameters

In addition to the explicitly scaling the level of superadiabaticity of the background entropy gradient via ϵ, we utilize the nondimensional Prandtl (Pr) and Rayleigh (Ra) numbers to determine our atmospheric diffusivities. Pr is the ratio of kinematic viscosity to thermal diffusivity, which we set equal to one. Ra is the ratiof buoyant driving to diffusivity of the form

$$\mathrm{Ra} \equiv \frac{gL_z^3(\Delta S/c_P)}{\nu\chi}, \tag{7}$$

where $\Delta S/c_P$ is the entropy difference between the top and bottom boundaries of the reference state. Note that $\nu \propto \chi \propto \rho^{-1}$ such that Ra increases with depth as the square of the density. In this work we specify Ra at the top of the domain.

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