Control Lab
ECSE 403 Fall 2018
Lab assignment1
Instructore: Prof. Caines
Due 28^{th} September 2018

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1 Objective

The main goal of this assignment is to review some concepts from the linear control course(ECSE 307), and to become familiar with some useful tools in matlab which helps us in designing and implementing controllers.

2 Your duty

Your duty is to answer all questions which have been asked throughout this assignment and submit all your answers in addition to matlab codes in mycourses website.

3 Model Description

The equation of motion of a DC motor can be described by:

$$J_m\ddot{\theta} + (b + \frac{K_t K_e}{R_a})\dot{\theta} = \frac{K_t}{R_a}v_a$$

where θ is the shaft angle (in radians) of the motor and v_a is the applied voltage.
System's parameters are as following:

- $J_m = 0.01kgm^2$ be the inertia of the rotor and the shaft.
- $b = 0.001Nmsec$ be the viscous friction coefficient
- $K_e = 0.02Vsec$ be the back emf constant
- $K_t = 0.002Nm/A$ be the motor torque constant.
- $R_a = 10\Omega$ be the armature resistance

Note that using SI units $K_e = K_t$.

4 Questions

1. Find the transfer function between the input voltage and the speed of the motor shaft($\frac{w(s)}{v_a(s)}$, where $w(s) = \dot{\theta}(s)$).(Plug in the coefficients above).[10 marks]
2. Using transfer function derived above and matlab, find the steady-state and time constant of the response of the motor to a step function.[10 marks]
   Hint: You can use following matlab commands for defining a transfer function: (random coefficient)
   ```
   s = tf('s')
   G = s/(s^2 + 4*s+1)
   ```
   Or instead, you can just use polynomial coefficients.
   ```
   G = tf([1],[1 4 1])
   ```
   and then use step command:
   ```
   stepplot(G)
   ```
3. Using previous step response find rise time and settling time of the system.[5 marks]
4. Using Final Value Theorem, calculate the steady state speed of the motor to step response theoretically. Compare theoretical value and values you found in matlab.[5 marks]
5. Find the transfer function between the shaft's angel and input voltage($\frac{\theta(s)}{v_a(s)}$).(Identify the order of the system with respect to new definition of input-output signals)[10 marks]
6. Consider the transfer function in question 1. Apply a unity feedback loop to the system. Draw block diagram of the system[5 marks], find the closed loop transfer function[5 marks].

7. Suppose a proportional controller is added to the system such that open-loop transfer function has changed from $G(s)$ to $K.G(s)$. Plot step response of the system choosing $K = \{0.1, 1, 10, 100\}$, in one figure [5 marks]. Describe the effect of proportional gain on step response's behaviour [5 marks].
   Hint: One way to plot different graphs on one figure is as following:
   ```
   figure(1);
   stepplot(h1);
   hold on;
   stepplot(h2);
   .
   .
   hold off;
   ```
8. Consider the transfer function in question 5, repeat steps of question 7 for this system[5 marks.] Describe the effect of proportional gain on step response's behaviour [5 marks].(In this case you should explain the effect of proportional gain on the overshoot, rise-time, and settling time)
9. Analytically find the maximum value of K that can be used if an overshoot of at most 20% is desired?[10 marks] *Hint*: You can refer to formulas in chapter 3 of [1]
10. Ignoring previous constraint, find the value for K, which provides a rise time of 4 seconds.[10 marks] *Hint*: You can refer to formulas in chapter 3 of [1]
11. For the transfer function which you derived in question 5, plot the Bode diagram of the system[5 marks]. From the Bode plot, find Gain-Margin and Phase-Margin of the system[5 marks].
    Write the definition of Gain-Margin and Phase-Margin[5 marks]. Describe the importance of these two numbers[5 marks].
    Hint : In order to plot Bode diagram you can use following command
    ```
    bodeplot(h1)
    ```
12. For the transfer function which you derived in question 5, plot the root locus of the system[5 marks]. Explain application of root locus diagrams[5 marks]. Using root locus diagram, find the gain in which system becomes unstable[5 marks].
13. Consider the transfer function you derived in question 5. We add a controller block with following transfer function instead of proportional controller.
    $$\frac{s + 0.35}{s + 0.75}$$

Plot root lucos of the system considering this additional block[5 marks]. Using the root locus diagram, Is there a gain for which system becomes unstable?[5 marks]

References

[1] G. F. Franklin, D. J. Powell, and A. Emami-Naeini, *Feedback Control of Dynamic Systems*, 4th ed. Upper Saddle River, NJ, USA: Prentice Hall PTR, 2001.