

**ECSE 403 lab assignment**  
**Fall 2018 , assignment 5**  
**Instructor: Prof. P. E. Caines**  
**Due 8<sup>th</sup> November 2018**

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## 1 Objective

The main goal of this assignment is to design and implement LEAD and LAG compensators for cart's position system.

## 2 Your responsibility

Your responsibility is to answer all questions which have been asked throughout this assignment and submit all your answers in addition to Matlab codes and Simulink results.

## 3 Questions

1. Using the coefficients you have found in **Lab3-Q4** (estimated coefficients of the system), define a transfer function in Matlab and find Bode diagram of the system (consider position as the output of the system). Using the bode diagram find the phase margin and gain margin of the system.
2. Use the value of  $K_p$ , in **Lab4-Q3**, plot the open-loop bode diagram of the system. Design a LEAD Controller, which can achieve phase margin of  $60^\circ$ .

**Hint:** If you do not remember design method for LEAD compensator you can follow these steps:

- (a) Suppose LEAD compensator is of the form:

$$C_{lead}(s) = K_{lead} \frac{\alpha Ts + 1}{Ts + 1}$$

- (b) Set  $K_{lead} = 1$ , since there is no requirement on steady-state error.  
 (c) Find the current open-loop phase margin of the system.  
 (d) Calculate required phase for reaching  $60^\circ$ , phase margin.(also include a small safety factor)  
 (e) Find coefficient  $\alpha$ , using following formula:

$$\phi_{max} = \sin^{-1} \left( \frac{\alpha - 1}{\alpha + 1} \right)$$

- (f) Find the desired  $w_m$  by looking at the Bode plot of the system and find the frequency at which gain equals:

$$-20 \log(K\sqrt{\alpha}) \quad dB$$

- (g) Find  $T$  using following equation:

$$T = \frac{1}{w_m \sqrt{\alpha}}$$

3. Using transfer function blocks in simlulink, simulate the step response of your estimated system in presence of proportional feedback controller, with and without the compensator. (In your Simulink model instead of physical system insert the estimated transfer function)
4. Using the cart system (physical system), find the step response of the system to proportional feedback controller with and without compensator(plot both step responses in one figure). Compare the results with simulations.
5. Describe the effect of LEAD compensator on rise-time, steady state error, and bandwidth of the closed loop system.
6. Add a LAG controller to the LEAD you just designed to get a LEAD-LAG system. Your LAG controller should reduce the steady state error to a ramp input by a factor of at least 10. Your combined system should have the specified phase margin of  $60^\circ$ .  
**Hint:** If you do not remember design method for LEAD compensator you can follow these steps:

(a) Suppose the the compensator is of the form :

$$c_{lag}(s) = K_{lag} \frac{Ts + 1}{\alpha.Ts + 1}$$

(b) calculate  $K_V = \lim_{s \rightarrow 0} s.H(s).K_{lag}.\beta$ . Where  $H(s)$  is your transfer function.

(c) using the fact that  $e_{ss} = K_V^{-1}$ , find proper  $K_{lag}$

(d) Determine the frequency  $w$  at which the phase equals  $-180^\circ + \text{desired phase margin} + 10^\circ$

(e) Calculate the amount of  $\alpha$ , using the desired dropped gain

$$\alpha = 10^{\frac{dB_{drop}}{20}}$$

(f) Fix the place of zero by setting  $T = \frac{10}{w}$ , to find  $T$ .

7. Using simulation environment you used in Question-3, plot the step response of proportional feedback controller with LEAD and LEAD-LAG compensators in one figure. plot the ramp response of proportional feedback controller with LEAD and LEAD-LAG compensators in one figure.
8. Using the cart system (physical system), find the step response of your estimated system in presence of proportional feedback controller, with LEAD-LAG compensator.
9. Compare step response of the your LEAD-LAG compensator and PID you tuned in **LAB4**