Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 2.17**

Problem 2.17 In this problem we explore some of the more useful theorems (stated without proof) involving Hermite polynomials.

(a) The **Rodrigues formula** says that

$$H_n(\xi) = (-1)^n e^{\xi^2} \left(\frac{d}{d\xi}\right)^n e^{-\xi^2}. \qquad [2.86]$$

Use it to derive H_3 and H_4.

(b) The following recursion relation gives you H_{n+1} in terms of the two preceding Hermite polynomials:

$$H_{n+1}(\xi) = 2\xi H_n(\xi) - 2n H_{n-1}(\xi). \qquad [2.87]$$

Use it, together with your answer in (a), to obtain H_5 and H_6.

2. **The Harmonic Oscillator**
In this problem, we will consider a particle in the simple harmonic oscillator potential, which is the potential for a mass on a spring:

$$V(x) = \frac{1}{2}m\omega^2.$$

Later, you will learn how to obtain the energy eigenfunctions and energy eigenvalues. But for now, we will give them to you. The (normalized) energy eigenfunctions are

$$\psi_n(x) = \frac{1}{\sqrt{2^n n!}}\left(\frac{m\omega}{\pi\hbar}\right)^{1/4} H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)\exp\left(-\frac{m\omega x^2}{2\hbar}\right), \qquad n = 0, 1, 2...$$

where $H_n(z)$ are the Hermite polynomials, which are defined in general by

$$H_n(z) = (-1)^n e^{z^2}\frac{d^n}{dz^n}(e^{-z^2}),$$

where $z = \sqrt{\frac{m\omega}{\hbar}}x$ in our case. For your convenience, the first few Hermite polynomials are

$$H_0(z) = 1$$
$$H_1(z) = 2z$$
$$H_2(z) = 4z^2 - 2$$
$$H_3(z) = 8z^3 - 12z$$
$$H_4(z) = 16z^4 - 48z^2 + 12.$$

(a) Write down $\psi_0(x)$, $\psi_1(x)$, and $\psi_2(x)$, and show that they are solutions to the time-independent Schödinger equation with energies $E_0 = \frac{1}{2}\hbar\omega$, $E_1 = \frac{3}{2}\hbar\omega$, and $E_2 = \frac{5}{2}\hbar\omega$, respectively.

(b) A particle begins at $t = 0$ with the (normalized) wavefunction

$$\Psi(x, t = 0) = \frac{2}{\sqrt{3}\pi^{1/4}} \left(\frac{m\omega}{\hbar}\right)^{5/4} x^2 \exp\left(-\frac{m\omega x^2}{2\hbar}\right)$$

.Write this state in terms of the energy eigenfunctions $\psi_n(x)$.

(c) What is the wavefunction of the particle at a later time t? Keep your answer in terms of the energy eigenfunctions$\psi_n(x)$.

(d) What is the expectation value of energy of the state $\Psi(x, t)$ as a function of time? [Hint: How does the Hamiltonian operator appear in the time-independent Schrödinger equation? There is no need to write out the explicit form of $\psi_n(x)$.]

3. **Griffiths 2.42**

Problem 2.42 Find the allowed energies of the *half* harmonic oscillator

$$V(x) = \begin{cases} (1/2)m\omega^2 x^2, & \text{for } x > 0, \\ \infty, & \text{for } x < 0. \end{cases}$$

(This represents, for example, a spring that can be stretched, but not compressed.) *Hint:* This requires some careful thought, but very little actual computation.

4. **Griffiths 3.15**

Problem 3.15 Show that two noncommuting operators cannot have a complete set of common eigenfunctions. *Hint:* Show that if $\hat{P}$ and $\hat{Q}$ have a complete set of common eigenfunctions, then $[\hat{P}, \hat{Q}]f = 0$ for any function in Hilbert space.

5. **Griffiths 3.22**

Problem 3.22 Consider a three-dimensional vector space spanned by an orthonormal basis $|1\rangle$, $|2\rangle$, $|3\rangle$. Kets $|\alpha\rangle$ and $|\beta\rangle$ are given by

$$|\alpha\rangle = i|1\rangle - 2|2\rangle - i|3\rangle, \quad |\beta\rangle = i|1\rangle + 2|3\rangle.$$

(a) Construct $\langle\alpha|$ and $\langle\beta|$ (in terms of the dual basis $\langle 1|$, $\langle 2|$, $\langle 3|$).

(b) Find $\langle\alpha|\beta\rangle$ and $\langle\beta|\alpha\rangle$, and confirm that $\langle\beta|\alpha\rangle = \langle\alpha|\beta\rangle^*$.

(c) Find all nine matrix elements of the operator $\hat{A} \equiv |\alpha\rangle\langle\beta|$, in this basis, and construct the matrix $\mathbf{A}$. Is it hermitian?