Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 2.12**

***Problem 2.12** Find $\langle x \rangle$, $\langle p \rangle$, $\langle x^2 \rangle$, $\langle p^2 \rangle$, and $\langle T \rangle$, for the nth stationary state of the harmonic oscillator, using the method of Example 2.5. Check that the uncertainty principle is satisfied.

2. **Griffiths 2.13**

Problem 2.13 A particle in the harmonic oscillator potential starts out in the state

$$\Psi(x, 0) = A[3\psi_0(x) + 4\psi_1(x)].$$

(a) Find A.

(b) Construct $\Psi(x, t)$ and $|\Psi(x, t)|^2$.

(c) Find $\langle x \rangle$ and $\langle p \rangle$. Don't get too excited if they oscillate at the classical frequency; what would it have been had I specified $\psi_2(x)$, instead of $\psi_1(x)$? Check that Ehrenfest's theorem (Equation 1.38) holds for this wave function.

(d) If you measured the energy of this particle, what values might you get, and with what probabilities?

3. **Griffiths 3.35**

$*{*}*$**Problem 3.35 Coherent states of the harmonic oscillator.** Among the stationary states of the harmonic oscillator ($|n\rangle = \psi_n(x)$, Equation 2.67) only $n = 0$ hits the uncertainty limit ($\sigma_x \sigma_p = \hbar/2$); in general, $\sigma_x \sigma_p = (2n+1)\hbar/2$, as you found in Problem 2.12. But certain *linear combinations* (known as **coherent states**) also minimize the uncertainty product. They are (as it turns out) *eigenfunctions of the lowering operator*:[32]

$$a_-|\alpha\rangle = \alpha|\alpha\rangle$$

(the eigenvalue α can be any complex number).

(a) Calculate $\langle x\rangle$, $\langle x^2\rangle$, $\langle p\rangle$, $\langle p^2\rangle$ in the state $|\alpha\rangle$. *Hint:* Use the technique in Example 2.5, and remember that a_+ is the hermitian conjugate of a_-. Do *not* assume α is real.

(b) Find σ_x and σ_p; show that $\sigma_x \sigma_p = \hbar/2$.

(c) Like any other wave function, a coherent state can be expanded in terms of energy eigenstates:

$$|\alpha\rangle = \sum_{n=0}^{\infty} c_n |n\rangle.$$

Show that the expansion coefficients are

$$c_n = \frac{\alpha^n}{\sqrt{n!}} c_0.$$

(d) Determine c_0 by normalizing $|\alpha\rangle$. *Answer:* $\exp(-|\alpha|^2/2)$.

(e) Now put in the time dependence:

$$|n\rangle \to e^{-iE_n t/\hbar}|n\rangle,$$

and show that $|\alpha(t)\rangle$ remains an eigenstate of a_-, but the eigen*value* evolves in time:

$$\alpha(t) = e^{-i\omega t}\alpha.$$

So a coherent state *stays* coherent, and continues to minimize the uncertainty product.

(f) Is the ground state ($|n = 0\rangle$) itself a coherent state? If so, what is the eigenvalue?

(g) Are the coherent states orthogonal? Compute $\langle\alpha|\beta\rangle$.

4. **Griffiths 3.39**

∗∗Problem 3.39

(a) For a function $f(x)$ that can be expanded in a Taylor series, show that

$$f(x+x_0) = e^{i\hat{p}x_0/\hbar} f(x)$$

(where x_0 is any constant distance). For this reason, $\hat{p}/\hbar$ is called the **generator of translations in space.** *Note:* The exponential of an *operator* is defined by the power series expansion: $e^{\hat{Q}} \equiv 1 + \hat{Q} + (1/2)\hat{Q}^2 + (1/3!)\hat{Q}^3 + \ldots$.

(b) If $\Psi(x,t)$ satisfies the (time-dependent) Schrödinger equation, show that

$$\Psi(x, t+t_0) = e^{-i\hat{H}t_0/\hbar}\Psi(x,t)$$

(where t_0 is any constant time); $-\hat{H}/\hbar$ is called the **generator of translations in time.**

(c) Show that the expectation value of a dynamical variable $Q(x, p, t)$, at time $t + t_0$, can be written[34]

$$\langle Q\rangle_{t+t_0} = \langle \Psi(x,t)| e^{i\hat{H}t_0/\hbar}\hat{Q}(\hat{x}, \hat{p}, t+t_0) e^{-i\hat{H}t_0/\hbar}|\Psi(x,t)\rangle.$$

Use this to recover Equation 3.71. *Hint:* Let $t_0 = dt$, and expand to first order in dt.