Please note, all Griffiths problems come from our class text, the second edition.

1. **Bloch Sphere**
A particle with magnetic moment $\hat{\mu}$ is placed in a magnetic field $\mathbf{B}$. As a result, it has energy

$$\hat{H} = -\hat{\mu} \cdot \mathbf{B}.$$

The magnetic moment of an electron (which is spin $\frac{1}{2}$) is related to its spin by its gyromagnetic ratio γ (which has units of charge over mass):

$$\hat{\mu} = \gamma \hat{\mathbf{S}}.$$

We will perform this problem in the S_z eigenbasis: $|+z\rangle$ has eigenvalue $+\hbar/2$ and $|-z\rangle$ has eigenvalue $-\hbar/2$.

(a) If we orient the magnetic field such that $\mathbf{B} = B_0\hat{\mathbf{z}}$, (here $\hat{\mathbf{z}}$ is a unit vector not an operator) what are the energy eigenstates and eigenvalues?

(b) If we orient the magnetic field in another direction, but we do not change its magnitude, argue why the energy eigenvalues are the same.

(c) Let us orient the magnetic field such that $\mathbf{B} = B_0\hat{\mathbf{n}}$, where

$$\hat{\mathbf{n}} = \sin\theta\cos\phi\hat{\mathbf{x}} + \sin\theta\sin\phi\hat{\mathbf{y}} + \cos\theta\hat{\mathbf{z}}.$$

Again, $\hat{\mathbf{n}}$, $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ are unit vectors and not operators. Show that the energy eigenstates are given as

$$|+n\rangle = \cos(\theta/2)|+z\rangle + e^{i\phi}\sin(\theta/2)|-z\rangle$$

,

$$|-n\rangle = \sin(\theta/2)|+z\rangle - e^{i\phi}\cos(\theta/2)|-z\rangle\,.$$

(d) The state $|+n\rangle$ gives us a way to map from the surface of a sphere, parametrised by (θ, ϕ), to the space of physical states of a two-level quantum mechanical system,

$$|\psi(\theta,\phi)\rangle = \cos(\theta/2)|+z\rangle + e^{i\phi}\sin(\theta/2)|-z\rangle\,.$$

Show that this is a bijection between the sphere and space of physical states. That is, show that any normalized state of a two-level system can be uniquely determined by a point on a sphere with coordinates (θ, ϕ). [Hint: Begin by showing that any two-level state is equivalent to a $|+n\rangle$. Then show that all coordinate pairs (θ, ϕ) that refer to the same point on a sphere also refer to the same quantum state]. We call the sphere made of these points the Bloch sphere.

(e) Draw the Bloch sphere and place points indicating where the eigenstates of S_x, S_y, and S_z are mapped to. [Hint: the state

$$\cos\frac{\theta}{2}|+z\rangle + e^{i\phi}\sin\frac{\theta}{2}|-z\rangle$$

is at the coordinate (θ, ϕ).]

(f) Calculate $\langle S_x\rangle$, $\langle S_y\rangle$, and $\langle S_z\rangle$ for the state $|\psi(\theta,\phi)\rangle$.

2. **Rotation Matrices**
   Do this problem after problem 1. It will make more sense if you do it that way
   (a) If we rotate the $|+z\rangle$ state around the $\hat{\mathbf{x}}$-axis by angle φ, what will it become? What about $|-z\rangle$? [Hint: Make use of the physical connection from the previous problem. If a state was in the $|+z\rangle$ state, it would have a specific energy when a magnetic field is pointing in the $\hat{\mathbf{z}}$ direction. If we rotate out (unphysical) coordinates so that the magnetic field points in the $\hat{\mathbf{n}}$ direction, the state must still have the same energy, and the state that has that energy is $|+n\rangle$. So if we rotate $\hat{\mathbf{z}}$ to $\hat{\mathbf{n}}$, then $|+z\rangle$ becomes $|+n\rangle$.]
   (b) Use the above result to find the matrix representation (in the S_z eigenbasis) of the operator $\hat{R}(\varphi\hat{\mathbf{x}})$ that rotates a spin state around the $\hat{\mathbf{x}}$-axis by an angle φ.

3. **Griffiths 4.2**

∗Problem 4.2 Use separation of variables in *cartesian* coordinates to solve the infinite *cubical* well (or "particle in a box"):

$$V(x, y, z) = \begin{cases} 0, & \text{if } x, y, z \text{ are all between 0 and } a; \\ \infty, & \text{otherwise.} \end{cases}$$

(a) Find the stationary states, and the corresponding energies.

(b) Call the distinct energies $E_1, E_2, E_3, \ldots$, in order of increasing energy. Find E_1, E_2, E_3, E_4, E_5, and E_6. Determine their degeneracies (that is, the number of different states that share the same energy). *Comment:* In *one* dimension degenerate bound states do not occur (see Problem 2.45), but in three dimensions they are very common.

(c) What is the degeneracy of E_{14}, and why is this case interesting?

4. **Griffiths 4.38**

∗Problem 4.38 Consider the **three-dimensional harmonic oscillator**, for which the potential is

$$V(r) = \frac{1}{2}m\omega^2 r^2. \qquad [4.188]$$

(a) Show that separation of variables in cartesian coordinates turns this into three one-dimensional oscillators, and exploit your knowledge of the latter to determine the allowed energies. *Answer:*

$$E_n = (n + 3/2)\hbar\omega. \qquad [4.189]$$

(b) Determine the degeneracy $d(n)$ of E_n.