Table of Contents:

We're going to **break up into groups of 2-6 people** and work on the problems in this document. After about an hour, we will jump to section 4 to make sure everyone gets a chance to talk as a big group about exam strategies, then we will go back to working on problems. You must leave the room by 6 pm.

If you can't come to the review session, feel free to email with questions, and/or work with other friends that can't make it.

Quantum Trivia

1. Are the following operators Hermitian and/or linear?

$$\hat{A}: \quad \hat{A}\Psi(x) = \Psi^*(x)\,\Psi(x)$$

$$\hat{B} = -i\hat{P}^2 + i\hat{X}^2$$

$$\hat{C}_a: \quad \hat{C}_a\Psi(x) = \Psi(x+a) \quad \text{for some } a \in \mathbb{R}$$

$$\hat{D} = \hat{P}^2\hat{X}^2 + \hat{X}^2\hat{P}^2$$

2. Can you think of something to add to B or D to make them Hermitian? (or maybe they already are)

3. Calculate the following commutators:

$$[\hat{X}, \hat{P}^2] =$$

$$[\hat{X}, \hat{P}^3] =$$

$$[\hat{X}, \hat{P}^q] =$$

$$[\hat{X}, e^{i\hat{P}a}] =$$

$$[\hat{P}, \hat{X}^q] =$$

4. Under what conditions is this true?

$$[\hat{P}, \hat{H}] = 0$$

Offer some physical interpretation. (Hint, what does it mean if something commutes with the Hamiltonian? The Hamiltonian generates time translation. What does it mean if something commutes with momentum? Momentum generates spatial translation)

5. Under what conditions is this true?

$$[\hat{X}, \hat{H}] = 0$$

What would it mean if it were true?

6. Under what conditions is this true?

$$\langle [\hat{x}, \hat{H}] \rangle = 0$$

What would it mean if it were true?

7. Under what conditions is this true?

$$[\langle \hat{x} \rangle, \hat{H}] = 0$$

What would it mean if it were true?

Normalize the following wavefunctions defined on x = (0, a):

$$\psi(x) = A\left(\frac{x}{a}\right)^2$$

$$\psi(x) = B\cos\left(\frac{x\pi}{a}\right)$$

$$\psi(x) = C\ln\left(\frac{x}{a}\right)$$

$$\psi(x) = De^{-x/a}$$

List all the types of functions that can be normalized on the range x = all real numbers.

Before moving to the next section: Say something nice to someone in your study group.

Well, then

What are the energy eigenstates of a particle in the infinite square well? (You should be able to do this on the fly, make up whatever constants you need to define)

With how you defined the boundaries of the well, what is the expectation value of x ($\langle x \rangle$) in any of the eigenstates?

If $\langle x \rangle$ is the same in each eigenstate, is it possible to make a linear combination of eigenstates such that $\langle x \rangle$ is different? How?

In the first and second energy eigenstate, compare $\langle x \rangle$ with where you'd actually expect to find the particle.

If you put air in a box, it distributes completely evenly. But energy eigenstates of the infinite square well have specific humps. Explain the inconsistency.

Come up with a wavefunction that can live in the box but is not a stationary state. Calculate the overlap of this wavefunction with the ground state.

If you wanted to keep a particle in the box in the lowest possible energy state, would you put it in a small or big box? Do you have any intuition for why this is the right answer?

If you have a particle in the ground state in a box, and you expand the walls of the box, what happens to the particle's energy? Is this a violation of the first law of thermodynamics? (Energy is conserved) What if you contract the walls of the box?

Scattering Problems

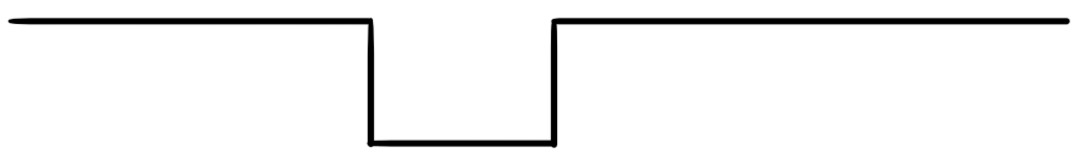
(Choose 1, come back if there's time)

Find the chance of a particle coming in from the right making it to the left side.

$$V(x) = \begin{cases} -V_0, & 0 \le x \le a \\ 0, & \text{else} \end{cases}$$

Assume

$0 < V_0 < E$



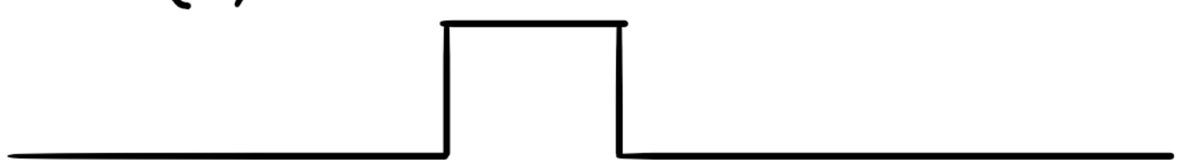
Compare this to the classical case. If you throw a frisbee over a ditch, is there a chance it will be reflected? Does your answer make sense?

Find the chance of a particle coming in from the right making it to the left side.

$$V(x) = \begin{cases} V_0, & 0 \leq x \leq a \\ 0, & \text{else} \end{cases}$$

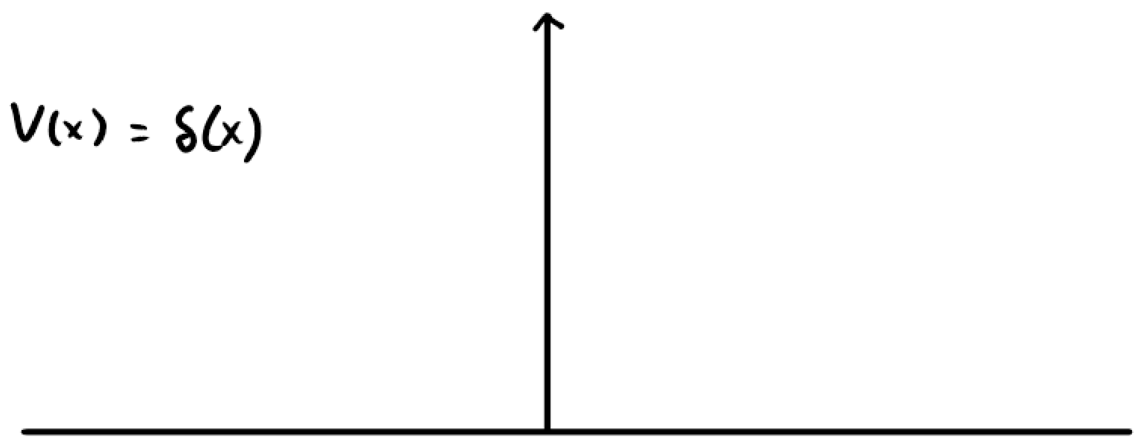
Assume

$0 < V_0 < E$



Compare this to the classical case. If you throw a frisbee over a bump on the ground, is there a chance it will be reflected? Does your answer make sense?

Find the chance of a particle coming in from the right making it to the left side.



Big Brain thinking

List all possible scattering problems you could see on the exam:

List all possible other questions you could see on the exam:

- What kinds of things do you do on the day before an exam, and the day of the exam, to prep?

- What mental state works best for exams? How can you get into that mental state?

- What does your performance on this exam (good or bad) say about your self-worth? (Hint: It's actually a quantifiable number, that happens to be my favorite number)

- What are some unconventional exam strategies that you have?

- Partial Credit: Your second-best friend (Ben is your best friend)

- Why do exams always stress me out?