Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 3.27**

Problem 3.27 Sequential measurements. An operator $\hat{A}$, representing observable A, has two normalized eigenstates ψ_1 and ψ_2, with eigenvalues a_1 and a_2, respectively. Operator $\hat{B}$, representing observable B, has two normalized eigenstates ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2. The eigenstates are related by

$$\psi_1 = (3\phi_1 + 4\phi_2)/5, \quad \psi_2 = (4\phi_1 - 3\phi_2)/5.$$

(a) Observable A is measured, and the value a_1 is obtained. What is the state of the system (immediately) after this measurement?

(b) If B is now measured, what are the possible results, and what are their probabilities?

(c) Right after the measurement of B, A is measured again. What is the probability of getting a_1? (Note that the answer would be quite different if I had told you the outcome of the B measurement.)

2. **Griffiths 4.1**

***Problem 4.1**

(a) Work out all of the **canonical commutation relations** for components of the operators **r** and **p**: $[x, y]$, $[x, p_y]$, $[x, p_x]$, $[p_y, p_z]$, and so on. *Answer:*

$$[r_i, p_j] = -[p_i, r_j] = i\hbar\delta_{ij}, \quad [r_i, r_j] = [p_i, p_j] = 0, \qquad [4.10]$$

where the indices stand for x, y, or z, and $r_x = x$, $r_y = y$, and $r_z = z$.

(b) Confirm Ehrenfest's theorem for 3-dimensions:

$$\frac{d}{dt}\langle \mathbf{r} \rangle = \frac{1}{m}\langle \mathbf{p} \rangle, \quad \text{and} \quad \frac{d}{dt}\langle \mathbf{p} \rangle = \langle -\nabla V \rangle. \qquad [4.11]$$

(Each of these, of course, stands for *three* equations—one for each component.) *Hint:* First check that Equation 3.71 is valid in three dimensions.

(c) Formulate Heisenberg's uncertainty principle in three dimensions. *Answer:*

$$\sigma_x \sigma_{p_x} \geq \hbar/2, \quad \sigma_y \sigma_{p_y} \geq \hbar/2, \quad \sigma_z \sigma_{p_z} \geq \hbar/2, \qquad [4.12]$$

but there is no restriction on, say, $\sigma_x \sigma_{p_y}$.

3. **Griffiths 4.18**

***Problem 4.18** The raising and lowering operators change the value of m by one unit:

$$L_\pm f_l^m = (A_l^m) f_l^{m\pm 1}, \qquad [4.120]$$

where A_l^m is some constant. *Question:* What *is* A_l^m, if the eigenfunctions are to be *normalized*? *Hint:* First show that $L_\mp$ is the hermitian conjugate of $L_\pm$ (since L_x and L_y are *observables*, you may assume they are hermitian ... but *prove* it if you like); then use Equation 4.112. *Answer:*

$$A_l^m = \hbar\sqrt{l(l+1) - m(m \pm 1)} = \hbar\sqrt{(l \mp m)(l \pm m + 1)}. \qquad [4.121]$$

Note what happens at the top and bottom of the ladder (i.e., when you apply L_+ to f_l^l or L_- to f_l^{-l}).

4. **Griffiths 4.19**

∗Problem 4.19

(a) Starting with the canonical commutation relations for position and momentum (Equation 4.10), work out the following commutators:

$$
\begin{aligned}
&[L_z, x] = i\hbar y, \quad [L_z, y] = -i\hbar x, \quad [L_z, z] = 0, \\
&[L_z, p_x] = i\hbar p_y, \quad [L_z, p_y] = -i\hbar p_x, \quad [L_z, p_z] = 0.
\end{aligned}
\qquad [4.122]
$$

(b) Use these results to obtain $[L_z, L_x] = i\hbar L_y$ directly from Equation 4.96.

(c) Evaluate the commutators $[L_z, r^2]$ and $[L_z, p^2]$ (where, of course, $r^2 = x^2 + y^2 + z^2$ and $p^2 = p_x^2 + p_y^2 + p_z^2$).

(d) Show that the Hamiltonian $H = (p^2/2m) + V$ commutes with all three components of $\mathbf{L}$, provided that V depends only on r. (Thus H, L^2, and L_z are mutually compatible observables.)

5. B&J 6.12

6.12 Let $\hat{\mathbf{n}}$ be a unit vector in a direction specified by the polar angles (θ, ϕ). Show that the component of the angular momentum in the direction $\hat{\mathbf{n}}$ is

$$
\begin{aligned}
L_n &= \sin\theta\cos\phi L_x + \sin\theta\sin\phi L_y + \cos\theta L_z \\
&= \tfrac{1}{2}\sin\theta(\mathrm{e}^{-\mathrm{i}\phi}L_+ + \mathrm{e}^{\mathrm{i}\phi}L_-) + \cos\theta L_z.
\end{aligned}
$$

If the system is in simultaneous eigenstates of $\mathbf{L}^2$ and L_z belonging to the eigenvalues $l(l+1)\hbar^2$ and $m\hbar$,

(a) what are the possible results of a measurement of L_n?

(b) what are the expectation values of L_n and L_n^2?