Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 2.4**

∗Problem 2.4 Calculate $\langle x \rangle$, $\langle x^2 \rangle$, $\langle p \rangle$, $\langle p^2 \rangle$, σ_x, and σ_p, for the nth stationary state of the infinite square well. Check that the uncertainty principle is satisfied. Which state comes closest to the uncertainty limit?

2. **Infinite square well** (based on Griffiths 2.8, 2.38, 2.39)
   (a) Suppose you have a particle "at rest", equally likely to be found anywhere in the well, at $t = 0$. What should its mean momentum be? What is its wavefunction?
   (b) If you measured the energy of the particle, what possible values could you obtain and what are their probabilities?
   (c) What is the expected value of the energy?
   (d) Write down the wavefunction at some later time t. (Leave it as an infinite sum.)
   (e) Show that at time $t = 4ma^2/\pi\hbar$, the wavefunction returns to its initial state.
   (f) Suppose the well was somehow expanded to double the length, keeping the centre unchanged, without perturbing the wavefunction of the particle. Now, if you measured the energy, what possible values could you obtain and with what probabilities?

3. **Griffiths 2.31**

Problem 2.31 The Dirac delta function can be thought of as the limiting case of a rectangle of area 1, as the height goes to infinity and the width goes to zero. Show that the delta-function well (Equation 2.114) is a "weak" potential (even though it is infinitely deep), in the sense that $z_0 \to 0$. Determine the bound state energy for the delta-function potential, by treating it as the limit of a finite square well. Check that your answer is consistent with Equation 2.129. Also show that Equation 2.169 reduces to Equation 2.141 in the appropriate limit.

4. **Delta function well scattering**
   Instead of sending a particle from infinity with well-defined momentum p, suppose we had a Gaussian wavepacket, with some small momentum uncertainty σ_p (i.e. large position uncertainty σ_x), with mean momentum p and mean position $-d$ (with $d \gg \sigma_x$, so it's on the left of the well).
   (a) What is the probability that after a very long time, the particle remains near $x = 0$? (You may leave your answer in terms of an integral.)
   (Hint: which energy eigenstates are localised?)
   (b) If this probability is not zero, comment on the apparent contradiction between $R + T = 1$ and your result.
   (Hint: what happens to this probability as we take $d \to \infty$ or $\sigma_p \to 0$?)

5. **Periodic potentials** (see B&J§4.8)
   Suppose we have a potential which is periodic with period L, i.e. $V(x + L) = V(x)$

(a) Show that if $\psi(x)$ is a stationary state, then $\psi(x+nL)$ is also a stationary state with the same energy, for any integer n.

(b) Given any stationary state $\psi(x)$, construct a set of stationary states $\phi_k(x)$ with the same energy, with the property that $\phi_k(x+L) = e^{ikL}\phi_k(x)$. Check that you can recover ψ from the ϕ_ks.
(Hint: use a linear combination $\phi_k(x) = \sum_{n=-\infty}^{\infty} c_n \psi(x+nL)$ with appropriately chosen coefficients c_n.)

(c) Define $u_k(x) = e^{-ikx}\phi_k(x)$ and show that u_k is periodic, i.e. $u_k(x+L) = u_k(x)$. Hence we may choose an energy eigenbasis in which the stationary states are $\phi_k(x) = e^{ikx}u_k(x)$. This is known as *Bloch's theorem*.