

Physics 112 - Intro to Statistical and Thermal Physics - Spring 2023
Spoiler Set 11

Problem 11.1 - Debye-bye-bye

(a) The number of one-phonon states in a given energy range is found by integrating the density of states over that energy range.

(b) You might want to use the substitution $x = E/k_B T$. Since there is a maximum phonon energy, make sure your integral over E only ranges from 0 to E_D . Be sure to change this upper limit when you do an integral substitution. $\langle N_{\text{phon}} \rangle = 9N \left(\frac{T}{T_D} \right)^3 \int_0^{T_D/T} \frac{x^2}{e^x - 1} dx$.

(f) Note that we are in the regime $T \ll T_D$.

Problem 11.2 - The Degenerate Fermi Gas

(d) Recall that $(1 + \epsilon)^p \approx 1 + p\epsilon + \frac{1}{2}p(p-1)\epsilon^2$ out to second order.

(e) Knowing $\langle U \rangle$ we can find the heat capacity C_V and from there... well, we did something similar in the previous problem set!

Problem 11.3 - White Dwarf Stars and Neutron Stars

(a) The number of electrons to use is $N = M/2m_p$ (since M/m_p is the total number of nucleons in the star, the number of protons is half the number of nucleons, and the number of electrons is equal to the number of protons).

(d) In terms of the number of electrons N and the volume V you should find $E_F = \hbar c(3\pi^2 N/V)^{1/3}$ and $\langle U_F \rangle = \frac{3}{4}NE_F$.

