Physics 137A Lectures 1 & 2

The origins of quantum theory

- Successes of classical physics
  - → Newtonian mechanics
  - → Maxwellian electromagnetism
  - → Boltzmann statistical mechanics

⟹ But, catastrophic breakdown when applied to blackbody radiation + atomic spectra

Blackbody Radiation

- 1792: Wedgwood remarks that all objects heated to the same temperature glow the same color

- 1800s: Improvements in spectroscopy show that solids emit continuous spectra; gases emit discrete lines

- 1859: Kirchoff proposes model of thermal radiation emission

  ↓ $R(\lambda, T)$ emissive power per unit area

Blackbody:

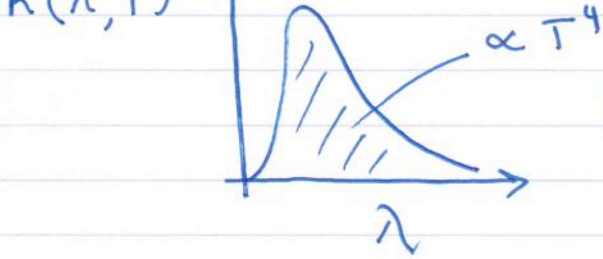
eg. cavity where light bounces many times off the walls

- Perfect absorber / Perfect emitter
- Equilibrium established between walls at temperature T and light field

Classical Observations:

A. Stefan's Law (1879)

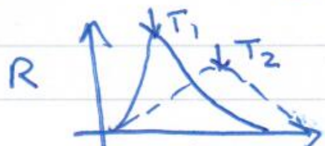
→ experimentally finds that total radiation emitted from a glowing solid $\propto T^4$



$$R(T) = \int_0^\infty R(\lambda,T)\, d\lambda = \underbrace{\sigma}_{5.67\times10^{-8}\ \frac{W}{m^2K^4}} T^4$$

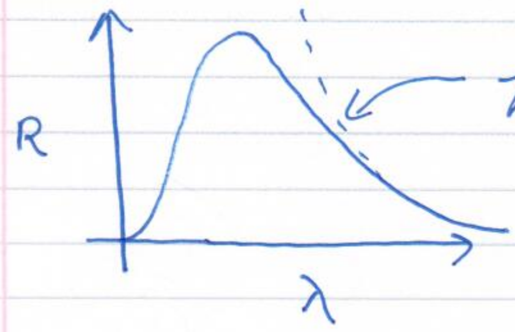
B. Wien's Law

→ From spectroscopic data $\lambda_{max} T = 2.898 \times 10^{-3}\ m\cdot K$



$T_1 > T_2$

c. Rayleigh - Jeans Law



fit a functional form to this part of the spectrum

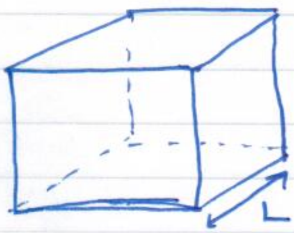
$$\propto \frac{8\pi k_B T}{\lambda^4}$$

BUT.... $\int R\, d\lambda$ would diverge!! Infinite power emitted!

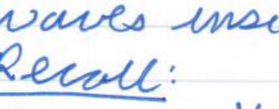
UV CATASTROPHE

DERIVATION OF CLASSICAL & QUANTUM BLACKBODY RADIATION FORMULAE

- Consider cubic cavity of side L



Need to calculate standing waves inside cavity. Recall:



modes of a fixed string

Must Satisfy 3-D wave equation

$$\nabla^2 \psi(\vec{r},t) = \frac{1}{c^2}\frac{\partial}{\partial t^2}\psi(\vec{r},t)$$

wave amplitude

$\nabla^2 = \frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$

c speed of light

Boundary conditions are that
$\psi = 0$ at cavity walls

ie. $\psi(x=0, y, z, t) = \psi(x=L, y, z, t) = \ldots = 0$

$\rightarrow$ solution $\psi(\vec{r}, t) = A(t) \sin(k_x x) \sin(k_y y) \cdot \sin(k_z z)$

Boundary conditions require $k_i = \frac{n_i \pi}{L}$

$i = x, y, k$ n is an integer

Can express as standing wave with a time varying amplitude:

$$A(t)\, B(x, y, z)$$

Substitute into wave equation

$$-(n_1^2 + n_2^2 + n_3^2) \frac{\pi^2}{L^3} A(t) B(x, y, z) = \frac{1}{c^2} B(x, y, z) \frac{\partial^2}{\partial t^2} A(t)$$

$\rightarrow$ guess solution for $A(t)$

$$A(t) = A_0 \cos(\omega t) + \varphi$$

$$\omega^2 = \frac{c^2 \pi^2}{L^2} (n_x^2 + n_y^2 + n_z^2)$$

Now: We need to count how many different ways (modes) a given ω can be distributed over n_x, n_y, n_z

Let $g(\omega) = \frac{dN(\omega)}{d\omega}$ # of modes

# of modes per unit frequency

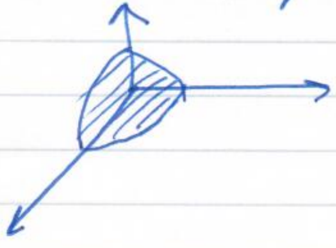
Thus:

$$N(\omega) = \int_0^{\omega} g(\omega)\, d\omega$$

For a given ω, the possible values of n_x, n_y, n_z are bound and must obey

$$n_x^2 + n_y^2 + n_z^2 \leq \frac{\omega^2 L^2}{c^2 \pi^2}$$

- Picture a sphere of radius $\frac{\omega L}{c\pi}$
- All the points inside represent allowed modes



Consider 1/8 of the whole sphere to account for all positive values of n_x, n_y, n_z

$$\Rightarrow N(\omega) = \frac{1}{8}\left(\frac{4}{3}\pi \frac{\omega^3 L^3}{c^3 \pi^3}\right)$$

$$= \frac{\omega^3 V}{6 c^3 \pi^2}$$ volume $= L^3$

Can convert to linear frequency

$$\omega = 2\pi f$$

$$N(f) = \frac{8\pi^3 f^3 V}{6 c^3 \pi^2}$$

$$g(f) = \frac{dN(f)}{df} = \frac{4\pi f^2 V}{c^3}$$

Each mode has 2 polarization directions

$$g(f) = \frac{8\pi f^2}{c^3} V$$

** Up to this point, we have not invoked any assumptions from <u>classical</u> or <u>quantum</u> physics. The derivation is correct up to this point. Now, we will see differences.

CLASSICAL CALCULATION

- Each mode has energy $k_B T$ (equipartition)

→ Thus, the energy density between f and $f+df$ is $g(f) k_B T \, df = \frac{8\pi}{c^3} f^2 V k_B T \, df$

We get the energy density by dividing by V

$$\rho(f) = \frac{8\pi}{c^3} f^2 k_B T$$

To convert to λ units, use $f = c/\lambda$

$$|df| = \frac{c\,d\lambda}{\lambda^2}$$

<u>Note</u>: (−) sign in derivative changes direction of $\int$

$$\rho(\lambda) = \frac{8\pi}{c^3}\frac{c^2}{\lambda^2}\frac{c}{\lambda^2} k_B T = \boxed{\frac{8\pi k_B T}{\lambda^4}}$$

* This is precisely the Rayleigh-Jeans result!

The formula works fine for $k_B T \gg \omega$

but as $f \to \infty$

$\rho \to \infty$

UV Catastrophe

QUANTUM
CALCULATION

* Planck postulates that each oscillator cannot take on all values of energy

- Energy is gained/lost in units of hf

Planck's constant

$$E_n = \overset{\text{\# of quanta}}{n}hf$$

$$\overline{E} = \sum_{n=0}^{\infty} nhf \underbrace{\frac{e^{-nhf/k_BT}}{\sum_{n=0}^{\infty} e^{-nhf/k_BT}}}_{\text{normalized probability of occupation}}$$

let $x = e^{-hf/k_BT}$

$$\overline{E} = hf \sum_{n=0}^{\infty} n \frac{x^n}{\sum_{n=0}^{\infty} x^n} = hf \frac{x}{1-x}$$

$$= hf \frac{e^{-hf/k_BT}}{1 - e^{-hf/k_BT}} = hf \frac{1}{e^{hf/k_BT} - 1}$$

Thus: $\rho(f,T) = \frac{g(f)}{V}\overline{E} = \frac{8\pi h f^3}{c^3} \frac{1}{e^{hf/k_BT} - 1}$

Convert to λ $\quad f\lambda = c \quad df = -\frac{c\,d\lambda}{\lambda^2}$

$$\boxed{\rho(\lambda,T) = \frac{8\pi hc}{\lambda^5}\frac{1}{e^{\frac{hc}{\lambda k_B T}}-1}}$$

This is the correct formula and matches $R(\lambda,T)$ perfectly!!