Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 2.18**

Problem 2.18 Show that $[Ae^{ikx} + Be^{-ikx}]$ and $[C\cos kx + D\sin kx]$ are equivalent ways of writing the same function of x, and determine the constants C and D in terms of A and B, and vice versa. *Comment:* In quantum mechanics, when $V = 0$, the exponentials represent *traveling* waves, and are most convenient in discussing the free particle, whereas sines and cosines correspond to *standing* waves, which arise naturally in the case of the infinite square well.

2. **Gaussians** (based on G1.3)
   A Gaussian distribution, parametrised by μ, σ, is given by

$$\rho(x) = A\exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right). \tag{1}$$

   (a) Define

$$I = \int_{-\infty}^{\infty} e^{-x^2}\,\mathrm{d}x\,. \tag{2}$$

   Show that $I = \sqrt{\pi}$.
   (Hint: find I^2 by transforming to polar coordinates and then changing variables)

   (b) Hence find the normalisation constant A.

   (c) Find $\langle x\rangle$, $\langle x^2\rangle$, $\sigma(x)$ of the given Gaussian.
   (Hint: $\int x^2 e^{-ax^2}\,\mathrm{d}x = -\frac{\partial}{\partial a}\int e^{-ax^2}\,\mathrm{d}x$)

   (d) Sketch the Gaussian. Indicate μ and σ on your sketch or describe the effect of changing them.

3. **G2.20**

∗∗**Problem 2.20** This problem is designed to guide you through a "proof" of Plancherel's theorem, by starting with the theory of ordinary Fourier series on a *finite* interval, and allowing that interval to expand to infinity.

(a) Dirichlet's theorem says that "any" function $f(x)$ on the interval $[-a, +a]$ can be expanded as a Fourier series:

$$f(x) = \sum_{n=0}^{\infty} [a_n \sin(n\pi x/a) + b_n \cos(n\pi x/a)].$$

Show that this can be written equivalently as

$$f(x) = \sum_{n=-\infty}^{\infty} c_n e^{in\pi x/a}.$$

What is c_n, in terms of a_n and b_n?

(b) Show (by appropriate modification of Fourier's trick) that

$$c_n = \frac{1}{2a} \int_{-a}^{+a} f(x) e^{-in\pi x/a}\, dx.$$

(c) Eliminate n and c_n in favor of the new variables $k = (n\pi/a)$ and $F(k) = \sqrt{2/\pi}\, a c_n$. Show that (a) and (b) now become

$$f(x) = \frac{1}{\sqrt{2\pi}} \sum_{n=-\infty}^{\infty} F(k) e^{ikx} \Delta k; \quad F(k) = \frac{1}{\sqrt{2\pi}} \int_{-a}^{+a} f(x) e^{-ikx}\, dx,$$

where Δk is the increment in k from one n to the next.

(d) Take the limit $a \to \infty$ to obtain Plancherel's theorem. *Comment:* In view of their quite different origins, it is surprising (and delightful) that the two formulas—one for $F(k)$ in terms of $f(x)$, the other for $f(x)$ in terms of $F(k)$—have such a similar structure in the limit $a \to \infty$.

4. **G2.21**

Problem 2.21 A free particle has the initial wave function

$$\Psi(x, 0) = Ae^{-a|x|},$$

where A and a are positive real constants.

(a) Normalize $\Psi(x, 0)$.

(b) Find $\phi(k)$.

(c) Construct $\Psi(x, t)$, in the form of an integral.

(d) Discuss the limiting cases (a very large, and a very small).