Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 1.7**

$*$**Problem 1.7** Calculate $d\langle p\rangle/dt$. *Answer*:

$$\frac{d\langle p\rangle}{dt} = \left\langle -\frac{\partial V}{\partial x}\right\rangle. \qquad [1.38]$$

Equations 1.32 (or the first part of 1.33) and 1.38 are instances of **Ehrenfest's theorem**, which tells us that *expectation values obey classical laws.*

2. **Griffiths 1.15**

$**$**Problem 1.15** Suppose you wanted to describe an **unstable particle**, that spontaneously disintegrates with a "lifetime" τ. In that case the total probability of finding the particle somewhere should *not* be constant, but should decrease at (say) an exponential rate:

$$P(t) \equiv \int_{-\infty}^{+\infty} |\Psi(x,t)|^2\, dx = e^{-t/\tau}.$$

A crude way of achieving this result is as follows. In Equation 1.24 we tacitly assumed that V (the potential energy) is *real*. That is certainly reasonable, but it leads to the "conservation of probability" enshrined in Equation 1.27. What if we assign to V an imaginary part:

$$V = V_0 - i\Gamma,$$

where V_0 is the true potential energy and Γ is a positive real constant?

(a) Show that (in place of Equation 1.27) we now get

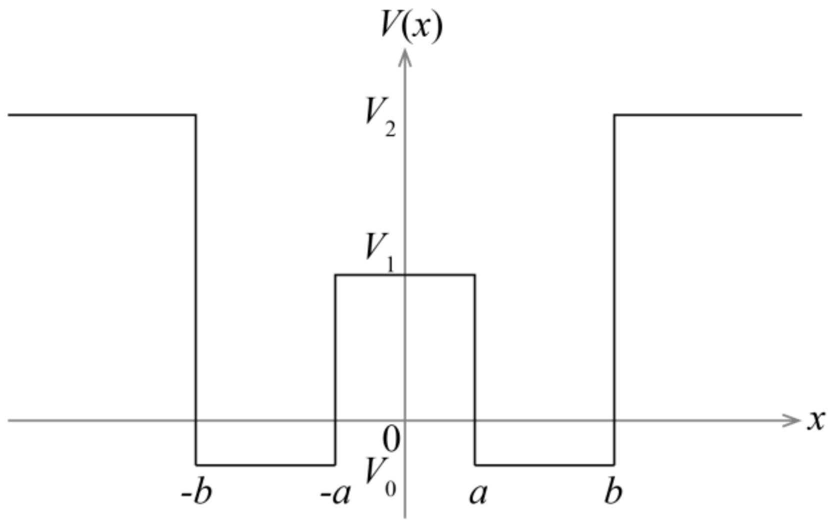
$$\frac{dP}{dt} = -\frac{2\Gamma}{\hbar}P.$$

(b) Solve for $P(t)$, and find the lifetime of the particle in terms of Γ.

3. **Calculate the following commutators**
(Recall that $[\hat{A}, \hat{B}] = \hat{A}\hat{B} - \hat{B}\hat{A} = -[\hat{B}, \hat{A}]$)

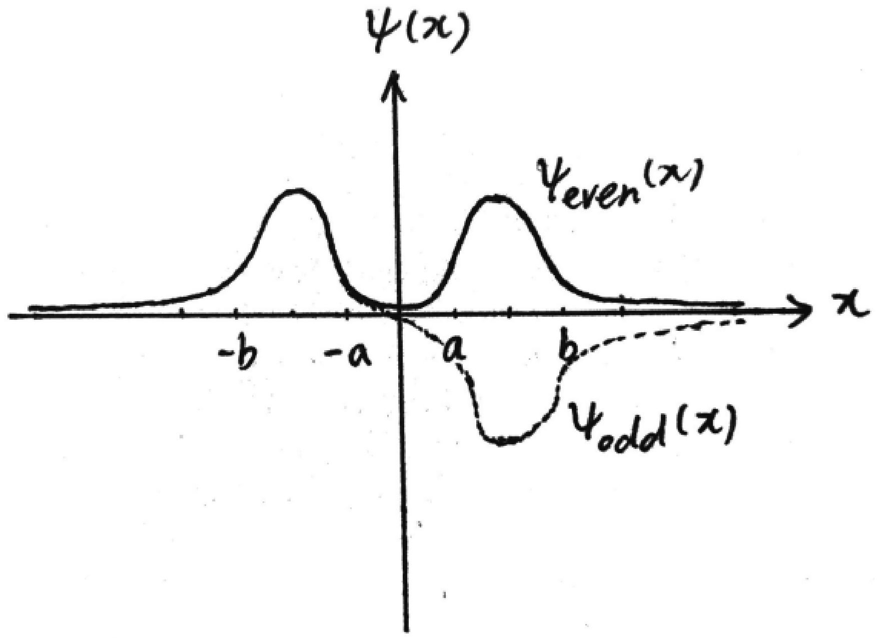
   (a) $[\hat{x}, \hat{x}]$
   (b) $[\hat{x}, \hat{p}]$
   (c) $[\hat{x}^2, \hat{p}]$
   (d) $[\hat{x}, f(\hat{x})]$
   (e) $[\hat{p}, f(\hat{x})]$
   (f) $[\hat{x}, \hat{H}]$ for general potential $V(x)$

4. **Qualitative Sketches of Wavefunctions**
Consider the following potential $V(x)$:



We will be looking for solutions to the time-independent Schrodinger equation.

   (a) For what range of values E
      i. are we certain there does not exist a solution?
      ii. do we have solutions which are bound states?
      iii. do we have solutions which are unbound states?
   (b) Consider the case $V_0 < E < V_1$. Where are the classically allowed and disallowed regions? In other words, if a classical particle has total energy E, where is it possible or impossible for the particle to be?
   (c) The two lowest energy solutions (*i.e.* the ground state and first excited state) are sketched below:



Which wavefunction, ψ_{even} or ψ_{odd}, has a lower energy? Explain why qualitatively.

(d) Sketch the third and fourth energy levels (*i.e.* the second and third excited states), assuming their energies still satisfy $V_0 < E < V_1$. Clearly label the two wavefunctions, as well as the locations of $\pm a$ and $\pm b$.

5. **B&J 3.12**

3.12 Let E_n denote the bound-state energy eigenvalues of a one-dimensional system and let $\psi_n(x)$ denote the corresponding energy eigenfunctions. Let $\Psi(x,t)$ be the wave function of the system, normalised to unity, and suppose that at $t = 0$ it is given by

$$\Psi(x, t = 0) = \frac{1}{\sqrt{2}} e^{i\alpha_1} \psi_1(x) + \frac{1}{\sqrt{3}} e^{i\alpha_2} \psi_2(x) + \frac{1}{\sqrt{6}} e^{i\alpha_3} \psi_3(x)$$

where the α_i are real constants.

(a) Write down the wave function $\Psi(x,t)$ at time t.

(b) Find the probability that at time t a measurement of the energy of the system gives the value E_2.

(c) Does $\langle x \rangle$ vary with time? Does $\langle p_x \rangle$ vary with time? Does $E = \langle H \rangle$ vary with time?