Physics 112 - Intro to Statistical and Thermal Physics - Spring 2023
Spoiler Set 04

Problem 4.2 - Diffusion

(a) Instead of the setup where we have temperature T on the left of the boundary and slightly higher temperature $T + dT$ on the right, use a setup where the temperatures are equal but we have a concentration n on the left and slightly higher concentration $n + dn$ on the right.

(e) You should find to within an order of magnitude that $D \sim 10^{-5}\text{m}^2/\text{s}$.

(f) This is an order-of-magnitude estimation problem involving a differential equation. A good way to approach this is to replace the partials with scales for the quantities. That is, we can replace something like $\frac{\partial^2 n}{\partial x^2}$ with N/x^2, where N is a typical number scale for the number of particles involved (sub-spoiler: this will wind up canceling out!) and x is the typical length scale for distances involved. Our goal is to find a time-scale.

Some chemistry! The "fruity" smell of perfume is from the compound ethyl octanoate, $C_{10}H_{20}O_2$. You can use this to estimate the relative mass and size of the perfume molecules as compared to air.

Problem 4.3 - Biased Random Walks and Binary State Systems

(a) **Extra Part** (*Not for Credit*) In the $N = 4$ case there are 16 microstates with macrostates and multiplicities $X = \pm 4\ell$ with $\Omega = 1$; $X = \pm 2\ell$ with $\Omega = 4$; $X = 0$ with $\Omega = 6$.

(d) Remember that the expectation value/average of some function $f(X)$ of our variable X is given by $\langle f(X) \rangle = \sum_i f(X_i) P(X_i)$.

(e) Since the Δx_i are all independent, $\langle \Delta x_i \rangle$ will be the same for all values of i.

(f) You can express $\langle \Delta x_i \Delta x_j \rangle$ using the Kronecker delta as $\delta_{ij}(\text{answer when } i = j) + (1 - \delta_{ij})(\text{answer when } i \neq j)$.

You should find $\langle \Delta x_i \Delta x_j \rangle = \ell^2 \left(\delta_{ij} + (1 - \delta_{ij})(p_R - p_L)^2 \right)$.

(h) The X that maximizes $P(X)$ will also be the X that maximizes $\ln P(X)$. The logarithm will be easier to work with!

You should find that the maximum occurs when $n = pN$.

Problem 4.4 - DataHub - Biased Random Walks

For part (a), you can generate the random walk by first generating an array of N random numbers chosen uniformly between 0 and 1. Numpy has a function that does this nicely. Once you have the array of numbers, you can convert that to a microstate by, for example, saying if a given number is less than θ then we will call that "spin up" and otherwise call it "spin down." Think about why that works!

♦