Please note, all Griffiths problems come from our class text, the second edition.

1. **Griffiths 4.28**

∗Problem 4.28 For the most general normalized spinor χ (Equation 4.139), compute $\langle S_x \rangle$, $\langle S_y \rangle$, $\langle S_z \rangle$, $\langle S_x^2 \rangle$, $\langle S_y^2 \rangle$, and $\langle S_z^2 \rangle$. Check that $\langle S_x^2 \rangle + \langle S_y^2 \rangle + \langle S_z^2 \rangle = \langle S^2 \rangle$.

2. **Griffiths 4.29**

∗Problem 4.29

(a) Find the eigenvalues and eigenspinors of $\mathbf{S}_y$.

(b) If you measured S_y on a particle in the general state χ (Equation 4.139), what values might you get, and what is the probability of each? Check that the probabilities add up to 1. *Note: a* and *b* need not be real!

(c) If you measured S_y^2, what values might you get, and with what probabilities?

3. **Griffiths 4.34**

∗Problem 4.34

(a) Apply S_- to $|1\,0\rangle$ (Equation 4.177), and confirm that you get $\sqrt{2}\hbar|1\,-1\rangle$.

(b) Apply $S_\pm$ to $|0\,0\rangle$ (Equation 4.178), and confirm that you get zero.

(c) Show that $|1\,1\rangle$ and $|1\,-1\rangle$ (Equation 4.177) are eigenstates of S^2, with the appropriate eigenvalue.

4. **Matrix Representation**

2 Matrix Representation for $j = 1$

In this problem, we would like to compute probability distribution of measurements of J_x, J_y, and J_z for particles with $j = 1$.

(a) We will do this problem in the $\{|j=1, m_z=1\rangle, |j=1, m_z=0\rangle, |j=1, m_z=-1\rangle\}$ basis. How does the $\hat{J}_z$ operator look in this basis? Note that we will have a 3×3 matrix. (Hint: Think about which eigenvalues the matrix should have.)

(b) Recall that $\hat{J}_\pm = \hat{J}_x \pm i\hat{J}_y$. Show that $\hat{J}_+^\dagger = \hat{J}_-$.

(c) Show that $\hat{J}_\mp \hat{J}_\pm = \hat{\mathbf{J}}^2 - \hat{J}_z^2 \mp \hbar \hat{J}_z$.

(d) Determine the form of the raising and lowering operators in the $\{|j=1, m_z\rangle\}$ basis. (Hint: First determine which elements are non-zero by recalling how the raising and lowering operators act on the basis states. Then use the above facts to determine exactly what the non-zero elements are.)

(e) Use the raising and lowering operators to construct the representations of $\hat{J}_x$ and $\hat{J}_y$ in the $\{|j=1, m_z\rangle\}$ basis.

(f) Use these matrices to find the representations of the eigenstates of both $\hat{J}_x$ and $\hat{J}_y$ in the $\{|j=1, m_z\rangle\}$ basis and their corresponding eigenvalues.

(g) A particle is prepared in the state $|j=1, m_z=1\rangle$ and then J_x is measured. What are the possible J_x measurement results, i.e. states, and their respective probabilities? What is the expectation value of the angular momentum in the x-direction of $|j=1, m_z=1\rangle$?

(h) If we measure $J_x = \hbar$ and then we measure J_y, what is the expectation value of the angular momentum in the y-direction?

(i) If we instead measured J_z again after measuring $J_x = \hbar$, what is the probability that we get the original state $|j=1, m_z=1\rangle$? You should find that simply making the measurement of J_x changes the state; you can have the value of J_z change just by measuring J_x!

5. **Measuring Spin**

Imagine you have a beam of spin 1/2 particles moving in the y-direction. We can set up an inhomogeneous magnetic field to interact with the particles, separating them according to their spin component in the direction of the magnetic field, $\mathbf{B}\cdot\hat{\mathbf{S}}$. This is the Stern-Gerlach experiment, depicted in Fig. 1.

(a) You set up a magnetic field in the z-direction. As the beam of particles passes through it, it splits in two equal beams: one goes up, corresponding to the spin-up particles (those whose $\hat{S}_z$ eigenvalue was $+\frac{\hbar}{2}$), and the other goes down, corresponding to the spin-down particles. Now, you take the beam that went up and pass it through another magnetic field in the z-direction. Does the beam split? If so, what fraction of the particles go to each side?

(b) Instead, you pass the beam through a z-field, take the beam that went up, and pass it through a magnetic field in the x-direction. Does the beam split? If so, what fraction of the particles go to each side?

(c) You select one of the beams from part b above, and pass it through another magnetic field in the z-direction. Does the beam split? If so, what fraction of the particles go to each side? Compare with part a and explain.

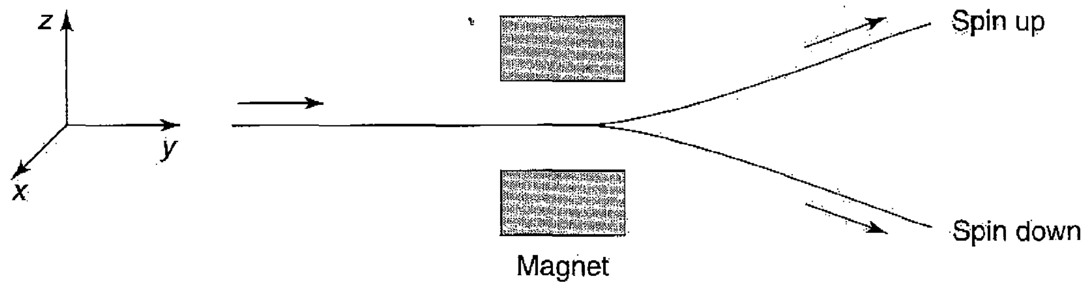

FIGURE 4.11: The Stern-Gerlach apparatus.

Figure 1: The Stern-Gerlach experiment

(d) Suppose we start with N particles. We first pass them through a magnetic field in the z-direction, and block the beam that goes down. After this process, you find that only $\frac{N}{2}$ particles remain. They then go through a magnetic field in the x-z plane, an angle θ from the z-axis, and the beam that goes against the direction of the field is blocked. Then you have a magnetic field in the z-direction again, and block the beam that goes up this time. How many particles come out? Compare with the case without the middle magnetic field.