

# Dynamic Sets

- Sets in computer science change over time  
⇒ Dynamic Sets
- May contain a unique object attribute called "key"
- Support common operations: insertion, deletion, membership / search

Search ( $s, k$ ) :  $k$ : key ,  $s$ : set

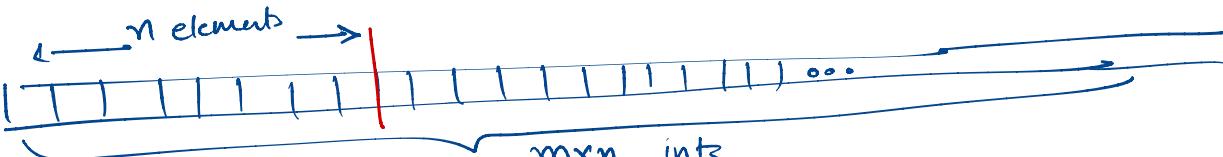
Insert ( $s, x$ ) :  $x$ : object (or pointer)

Delete ( $s, x$ )

Minimum ( $s$ ) , Maximum ( $s$ ) : returns object with min/max key.

successor ( $s, x$ ), predecessor ( $s, x$ ) : returns next/prev object in an ordered set  $s$

# Arrays

- Contiguous memory for storing elements of a set.
    - fixed length
    - Contiguous
    - same type of objects
  - Indexing an array
    - row major (a.k.a. Scan order)  
(Left → right) - (Top → bottom)
    - column major  
(top → bottom) - (Left → right)
  - Given starting address a, each element occupying b bytes in a s-based index ( $s=0, m$   
 $s=1$ )  
 $i^{\text{th}}$  element can be accessed at  
 $a + b(i-s)$  ← starting address
  - 2D arrays
    - allocate an  $m \times n$  array (of ints) in C :  
 $\text{int *mat} = (\text{int *}) \text{malloc}(\text{no. of elements} * \text{size of (elem)})$   
 $= (\text{int *}) \text{malloc}(m * n * \text{size of (int)})$
    - Access an element at  $(i, j)$  location  
 $\text{mat}[i * n + j]$  or  $*(\text{mat} + i * n + j)$
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# Stacks

- A dynamic set implementation where:

→ Implements a LIFO policy

↳ Last-in, first-out

→ Insert is called Push

→ Delete is called Pop

- Array-based implementation

→ a stack of at most size  $n$  is implemented by an array  $S[1..n]$

→ Attributes:  $s.\text{top}$ : index of the most recently inserted element

→  $S[1]$  is the bottom of stack

→ When  $s.\text{top} = 0$ , stack is empty "underflow"

→ When  $s.\text{top} = s.\text{size}$ , stack is full "overflow"

## Stack-Empty(s)

If  $s.\text{top} == 0$   
return true

else return false  $O(1)$

## Pop(s)

if stack-empty(s)  
error "underflow"

else  $s.\text{top} = s.\text{top} - 1$   
return  $s[s.\text{top} + 1]$

## Push(s, x)

If  $s.\text{top} == s.\text{size}$   
error "overflow"

else  $s.\text{top} = s.\text{top} + 1$   
 $s[s.\text{top}] = x$   $O(1)$

$O(1)$

# Queues

- In a queue, elements are deleted at one end and inserted at the other end
  - ↳ Implements FIFO policy
    - ↳ First-in, first-out
  - Insert op. is called Enqueue
  - Delete op. is called Dequeue
  - Two attributes Head (for dequeue)  
Tail (for enqueue)
- Array based implementation
  - ↳ A queue of at most  $(n-1)$  elements is implemented by an array  $Q[1..n]$
  - ↳  $Q.\text{head}$  → index of head
  - ↳  $Q.\text{tail}$  → index of next enqueue location
  - ↳ Elements reside in  $Q.\text{head}, Q.\text{head}+1, \dots, Q.\text{tail}-1$
  - ↳ Implement a "Circular indexing"
    - 1 follows  $n$
- Queue is empty when  $Q.\text{head} == Q.\text{tail}$   
(initially  $Q.\text{head} = Q.\text{tail} = 1$ )
  - ↳ Deque happens at head
  - ↳ Enqueue happens at tail.
  - ↳ ...
- Queue is full when  $Q.\text{head} = Q.\text{tail} + 1$   
OR (both  $Q.\text{head} = 1$  AND  $Q.\text{tail} = Q.\text{size}$ )

Assume  $n = Q.size$

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Enqueue (Q, x)

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$Q[Q.tail] = x$

if  $Q.tail == Q.size$

$Q.tail = 1$

else  $Q.tail = Q.tail + 1$

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→ check for "overflow"

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Dequeue (Q)

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$x = Q[Q.head]$

if  $Q.head == Q.size$

$Q.head = 1$

else  $Q.head = Q.head + 1$

return  $x$

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→ check for "underflow"