IN3140 - Assignment 1

Task 1

$$R_{z,90^\circ} = \begin{bmatrix} \cos 90 & -\sin 90 & 0 \\ \sin 90 & \cos 90 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

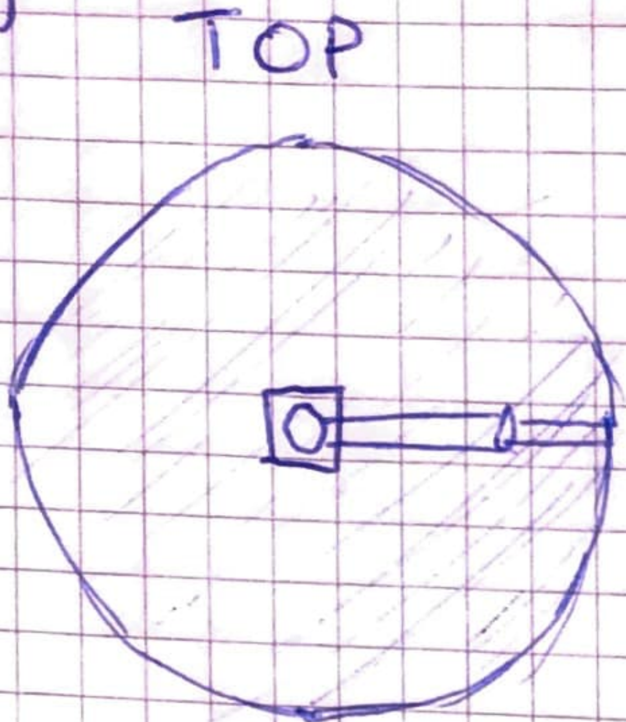
$$O_B - O_T = [-750, 250, 100]$$

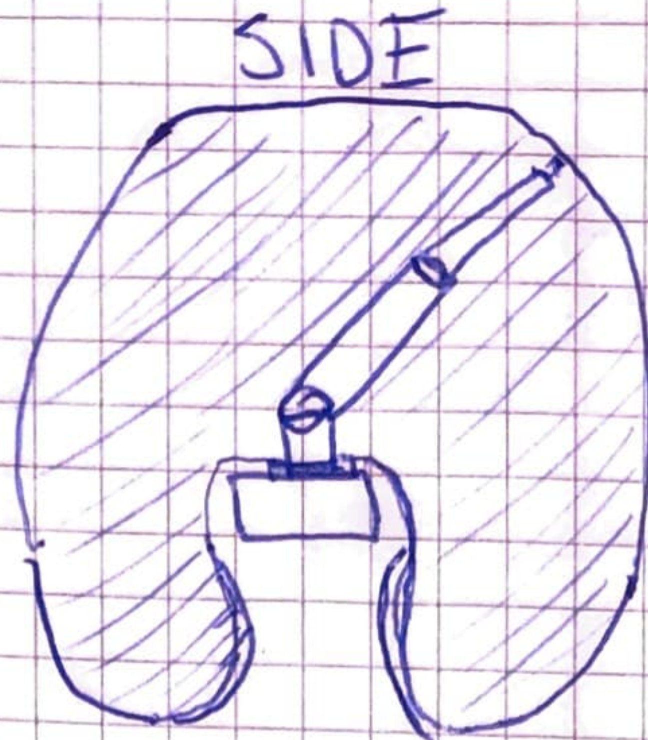
$$R_z \cdot [-750, 250, 100] = [250, -750, 100] = t$$

$$T_T^B = \begin{bmatrix} R_z & t \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 250 \\ 1 & 0 & 0 & -750 \\ 0 & 0 & 1 & 100 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

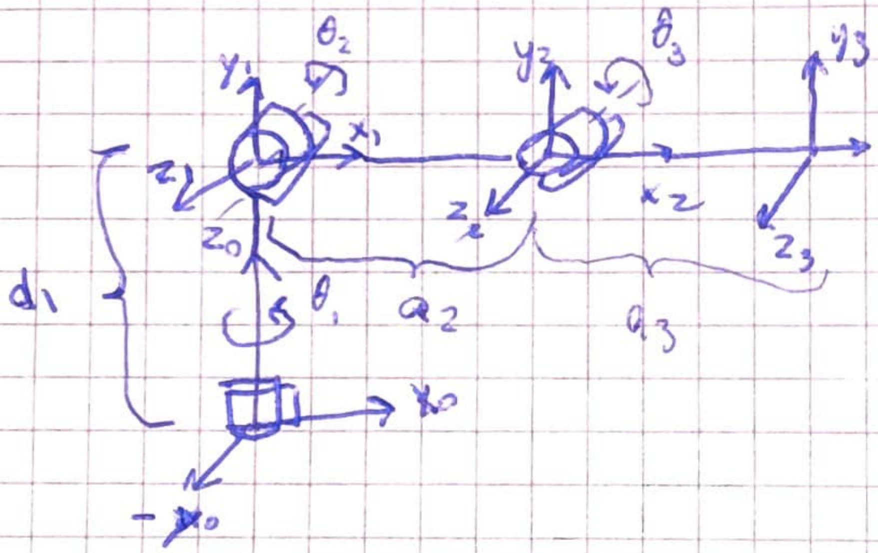
Task 2

A)





B)



Link	a_i	α_i	d_i	θ_i
1	0	90°	d_1	θ_1^*
2	a_2	0°	0	θ_2^*
3	a_3	0°	0	θ_3^*

$$A_i = Rot_{z\,\theta_i}\; Trans_{z\,d_i}\; Trans_{x\,a_i}\; Rot_{\alpha_i}$$

$$A_1^0 = \begin{bmatrix} \cos(\theta_1) & -\sin(\theta_1)\cos(\alpha_1) & \sin(\theta_1)\sin(\alpha_1) & a_1\cos(\theta_1) \\ \sin(\theta_1) & \cos(\theta_1)\cos(\alpha_1) & -\cos(\theta_1)\sin(\alpha_1) & a_1\sin(\theta_1) \\ 0 & \sin(\alpha_1) & \cos(\alpha_1) & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} \cos(\theta_2) & -\sin(\theta_2)\cos(\alpha_2) & \sin(\theta_2)\sin(\alpha_2) & a_2\cos(\theta_2) \\ \sin(\theta_2) & \cos(\theta_2)\cos(\alpha_2) & -\cos(\theta_2)\sin(\alpha_2) & a_2\sin(\theta_2) \\ 0 & \sin(\alpha_2) & \cos(\alpha_2) & d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} \cos(\theta_3) & -\sin(\theta_3)\cos(\alpha_3) & \sin(\theta_3)\sin(\alpha_3) & a_3\cos(\theta_3) \\ \sin(\theta_3) & \cos(\theta_3)\cos(\alpha_3) & -\cos(\theta_3)\sin(\alpha_3) & a_3\cos(\theta_3) \\ 0 & \sin(\alpha_3) & \cos(\alpha_3) & d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^0 = A_1^0 A_2^1 A_3^2 = \begin{bmatrix} C\theta_1 & 0 & S\theta_1 & 0 \\ S\theta_1 & 0 & -C\theta_1 & 0 \\ 0 & 1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_2 & -S\theta_2 & 0 & a_2C\theta_2 \\ S\theta_2 & C\theta_2 & 0 & a_2S\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} C\theta_3 & -S\theta_3 & 0 & a_3C\theta_3 \\ S\theta_3 & C\theta_3 & 0 & a_3S\theta_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} C\theta_1C\theta_2C\theta_3 - C\theta_1S\theta_2S\theta_3 & -C\theta_1C\theta_2S\theta_3 - S\theta_2C\theta_2C\theta_3 & S\theta_1 & A_3C\theta_1C\theta_2C\theta_3 - A_3C\theta_1S\theta_2\theta_3 + A_2C\theta_1C\theta_2 \\ S\theta_1C\theta_2C\theta_3 - S\theta_1S\theta_2S\theta_3 & -S\theta_2C\theta_2S\theta_3 - S\theta_1S\theta_2C\theta_3 & -C\theta_1 & A_3S\theta_1C\theta_2C\theta_3 - A_3S\theta_1S\theta_2S\theta_3 + A_2S\theta_1C\theta_2 \\ S\theta_2C\theta_3 + C\theta_2S\theta_3 & -S\theta_2S\theta_3 + C\theta_2C\theta_3 & 0 & A_3S\theta_2C\theta_3 + A_3C\theta_2S\theta_3 + A_2S\theta_2 + d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Task 3

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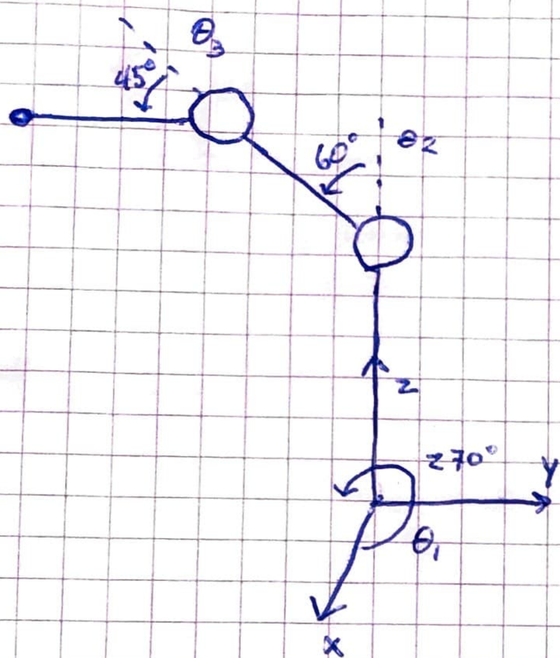
Model



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DH-parameters from Task 2c

Link	a_i	α_i	d_i	θ_i
1	0	90°	d_1	θ_1^*
2	a_2	0°	0	θ_2^*
3	a_3	0°	0	θ_3^*

This gives me A_3^0.

T_T^B from Task 1

$L1 = d_1 \quad L2 = a_2 \quad L3 = a_3$

$\theta_1 = 270° \quad \theta_2 = 30° \quad \theta_3 = -45°$

Coordinate column from A_3^0 w/ inserted variables:

$$\begin{bmatrix} a_3 C_1 C_2 C_3 - a_2 C_1 S_2 S_3 + a_2 C_1 C_2 \\ a_3 S_1 C_2 C_3 - a_3 S_1 S_2 S_3 + a_2 S_1 C_2 \\ a_3 S_2 C_3 + a_3 C_2 S_3 + a_2 S_2 + d_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ -323.9 \\ 176.69 \\ 1 \end{bmatrix} = g$$

Got to transform coordinate to task-coordinate:

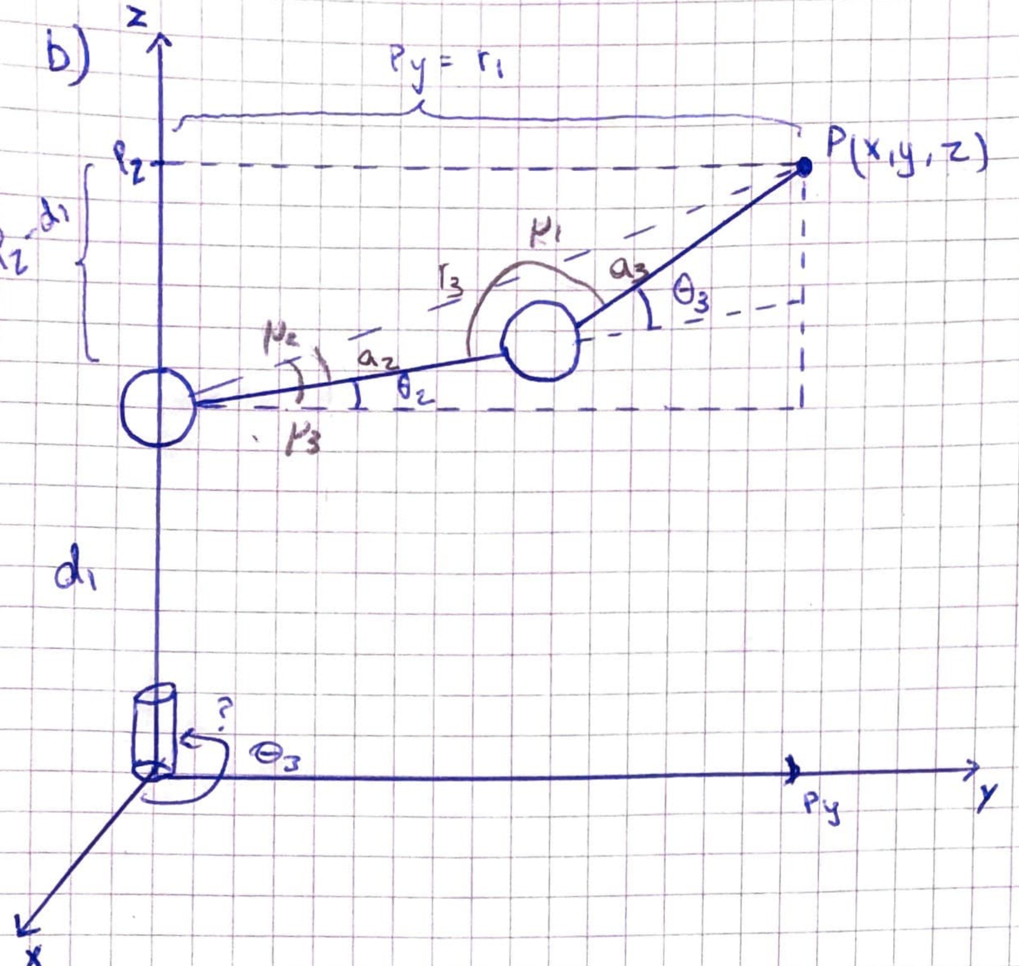
Inverse of T_T^B multiplied g: $\quad p^T = T_B^T p^B$

$$\begin{bmatrix} 0 & 1 & 0 & 750 \\ -1 & 0 & 0 & 250 \\ 0 & 0 & 1 & -100 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ -323.9 \\ 176.69 \\ 1 \end{bmatrix} = \begin{bmatrix} 426.1 \\ 250 \\ 76.69 \\ 1 \end{bmatrix} \Rightarrow p^t = (426.1, 250, 76.69)$$

Task 4

a) The two most common ways of deriving inverse kinematics is geometric and analytical. For the geometric way you solve the joint angles using trigonometry and geometry, while for the analytical way you solve a set of equations given the forward kinematics.

b)



$$r_3^2 = r_1^2 + r_2^2 \Rightarrow r_3 = \sqrt{r_1^2 + r_2^2}$$

$$r_3^2 = a_2^2 + a_3^2 - 2a_2a_3\cos(\mu_1)$$

$$\frac{r_3^2 - a_2^2 - a_3^2}{2a_2a_3} = \cos(\mu_1) \Rightarrow \mu_1 = \cos^{-1}\left(\frac{r_3^2 - a_2^2 - a_3^2}{2a_2a_3}\right)$$

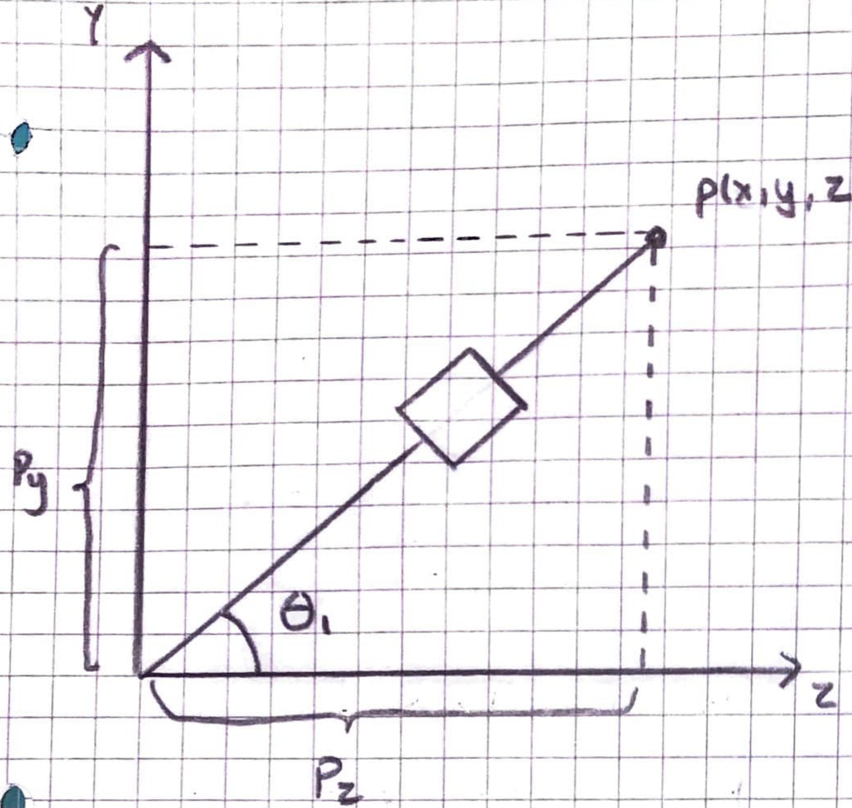
$$\underline{\theta_3 = 180° - \mu_1 = \pi - \mu_1}$$

$$a_3^2 = r_3^2 + a_2^2 - 2r_3a_2\cos(\mu_2)$$

$$\frac{a_3^2 - r_3^2 - a_2^2}{2r_3a_2} = \cos(\mu_2) \Rightarrow \mu_2 = \cos^{-1}\left(\frac{a_3^2 - r_3^2 - a_2^2}{2r_3a_2}\right)$$

$$\mu_3 = \operatorname{atan2}\left(\frac{r_2}{r_1}\right).$$

$$\underline{\theta_2 = \mu_3 - \mu_2}$$



$$\theta_1 = \operatorname{atan2}\left(\frac{P_y}{P_z}\right)$$

c) There will be four solutions. A elbow up and a elbow down solution. But since you can rotate the arm around, the coordinate systems for joint 2 and 3 will change which applies two more solutions to the elbow up/down. Therefore 4 solutions.