MEMORIAL UNIVERSITY OF NEWFOUNDLAND
DEPARTMENT OF MATHEMATICS AND STATISTICS

A1 **MATH 6201** DUE: JAN. 30, 2017

Hand in your solutions at the beginning of class on Monday Jan. 30, 2017.

1. Use the Lagrange interpolation formula to find a polynomial that passes through the points $(0, 1)$, $(2, 3)$ and $(3, 0)$. Use the Newton's divided difference form of the interpolating polynomial through the same points and verify that it coincides with the Lagrange form.

2. Derive the interpolation polynomial of degree two which interpolates $f(x) = \cos x$ at $x_0 = -\pi/2$, $x_1 = 0$ and $x_2 = \pi/2$. Use this polynomial to approximate $\cos(\pi/4)$. Find an upper bound for the error $|f(\pi/4) - P_2(\pi/4)|$ (using the error estimate presented in class) and compute the actual error in this representation.

3. Show that the so-called *θ-method*

$$u^{n+1} = u^n + \Delta t[\theta f(u^n) + (1 - \theta)f(u^{n+1})]$$

is of order 2 if $\theta = 1/2$ and otherwise of order 1. Prove that the method is convergent for every $\theta \in [0, 1]$.

4. (a) Re-derive the two-step Adams–Bashforth scheme using Taylor series expansions.

   (b) Derive the two-step Adams–Moulton method using polynomial interpolation.

5. Suppose the truncation error in a finite difference method is of order p and that the global error at some particular physical time t^n satisfies $|E| = \alpha(\Delta t)^p$, where α is a constant. A series of numerical simulations are conducted in which Δt is repeatedly halved, and the error $E(\Delta t)$ is plotted as a function of Δt on a log–log scale (this procedure is called a *convergence study*).

   (a) Show that the result will be a straight line of slope p.

   (b) Carry out a convergence study for (i) the forward Euler and (ii) the two-step Adams–Bashforth method for the test equation

$$u' = -u, \quad u(0) = 1,$$

   integrated up to $t = 1$, and compare the obtained slopes with lines with slopes of 1 and 2.