

Name:

SID:

GSI and section time:

Write down the names of the students around you as they appear on their SID.

Name of student on your left:

Name of student on your right:

Name of student behind you:

Name of student in front of you:

Note: For each question part, 20% of the points will be given to any blank answer, or to any *clearly* crossed-out answer.

Note: If you finish in the last 15 minutes, please remain seated and not leave early, to not distract your fellow classmates.

Instructions: You may consult one handwritten, two-sided sheet of notes. You may not consult other notes, textbooks, etc. Cell phones and other electronic devices are not permitted.

There are 4 questions. The last question is on page 9.

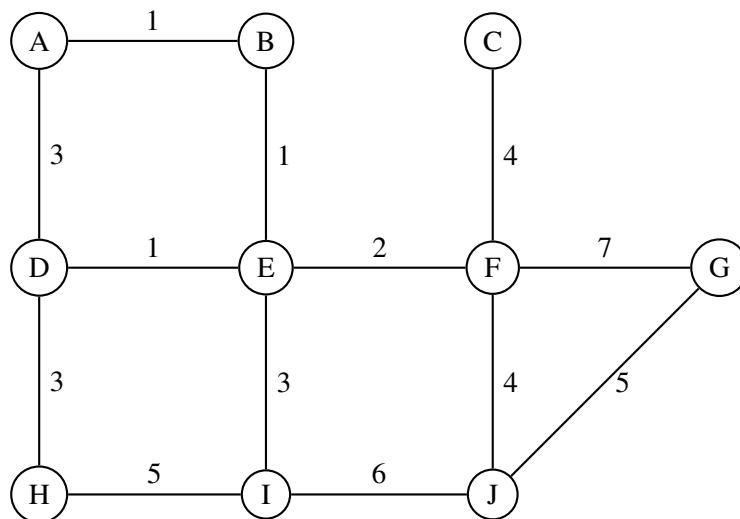
Answer all questions. On questions asking for an algorithm, make sure to respond in the format we request. Be precise and concise. Write in the space provided. Good luck!

Do not turn this page until an instructor tells you to do so.

1. (40 pts.) Short Answer

(a) (8 pts.) *Maximum Spanning Tree*

- (i) Find a **maximum** spanning tree of the graph below (indicate it by circling the weights of the edges included in the spanning tree).



- (ii) How many different maximum spanning trees does the above graph have?

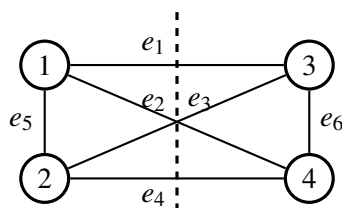
(b) (10 pts.) Linear Programming Fundamentals

$$\begin{aligned} \min \quad & 2x_1 + 4x_2 \\ \text{s.t.} \quad & x_1 + x_2 \geq 30 \\ & -x_1 + 2x_2 \geq 0 \\ & x_1, x_2 \geq 5 \end{aligned}$$

(i) Draw the feasible region of this linear program, and find the optimum of this LP.

(ii) Find the dual to the above linear program.

(c) (6 pts.) Cut Property



Consider the cut shown above that separates vertices 1 and 2 from 3 and 4 in the given graph. Suppose we are given the weights of all 4 edges across this cut: $w(e_1) = 1$, $w(e_2) = 2$, $w(e_3) = 2$, $w(e_4) = 3$. $w(e_5)$ and $w(e_6)$ are unknown. Circle all the answers that apply for the following questions:

(i) Which edge(s) **must be** part of every minimum spanning tree in G ?

☐ e_1 ☐ e_2 ☐ e_3 ☐ e_4

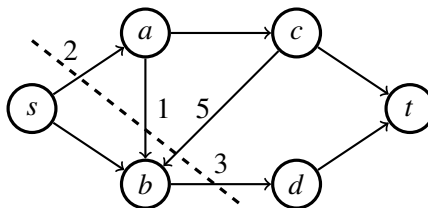
(ii) Which edge(s) **can be but aren't necessarily** part of a minimum spanning tree in G ?

☐ e_1 ☐ e_2 ☐ e_3 ☐ e_4

(iii) Which edge(s) **must not be** part of a minimum spanning tree in G ?

☐ e_1 ☐ e_2 ☐ e_3 ☐ e_4

(d) (4 pts.) Min Cut



In the graph G shown above, consider the cut that separates s and b from a, c, d, t . Assume that this is the min $s - t$ cut in G . In the max $s - t$ flow, what are the flow values through the following edges:

(i) $s \rightarrow a$: _____

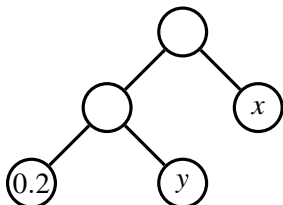
(ii) $a \rightarrow b$: _____

(iii) $c \rightarrow b$: _____

(iv) $b \rightarrow d$: _____

(e) (5 pts.) Huffman Encoding

Below is a Huffman encoding tree of 3 symbols, where leaf values 0.2, x and y indicate the frequencies of each symbol. Since they represent frequencies, $0.2 + x + y = 1$. Give the minimum and maximum values that x can take so that this tree still minimizes the expected length of the encoding.



(f) (7 pts.) Zero-Sum Games

		Column:	
Row:		c_1	c_2
	r_1	1	-1
	r_2	-1	1

In the zero-sum game shown above, Row seeks to maximize the value of the game, and Column seeks to minimize it. Assume that Row must announce her strategy (the probabilities r_1 and r_2) first, and Column determines his strategy (c_1 and c_2) afterwards.

For example, if Row announced the strategy “ $r_1 = 1, r_2 = 0$,” Column could respond with the strategy “ $c_1 = 0.25, c_2 = 0.75$,” and the value of the game would be -0.5 .

(i) What strategy should Row choose?

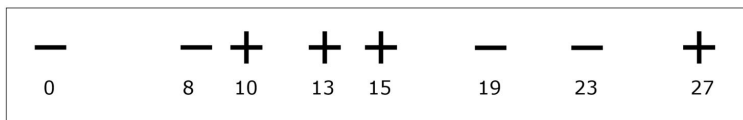
(ii) Assuming that Column responds optimally, what is the value of the game?

2. (20 pts.) Matching Terminals

There are n positive terminals and n negative terminals, arranged along the x -axis. The location of the t -th terminal (moving from left to right) is $x(t)$, and its sign is $s(t)$ (+ or -).

You must connect each positive terminal to a distinct negative terminal, using a total of n pieces of wire. You wish to choose the pairing of the terminals to minimize the total length of wire used.

Design a greedy algorithm to solve this problem. Use an exchange argument to prove that your algorithm finds the optimal solution. What is the running time of your algorithm?



The above picture shows a possible layout of terminals for $n = 4$. The location value $x(t)$ for each terminal t is written underneath the terminal.

3. (20 pts.) Knapsack with Weighty and Voluminous Objects

A thief walks into a house and notices n objects with values v_1, \dots, v_n , weights w_1, \dots, w_n , and volumes u_1, \dots, u_n . The thief has brought with him a bag that has a volume capacity of U , and can hold a total weight of W . He wishes to choose a subset of items whose total volume is at most U and weight at most W , and whose total value is as large as possible, but he must decide quickly and make his getaway. Give an efficient dynamic programming algorithm to help him choose. Clearly specify the subproblems, recurrence, the order in which you would solve the subproblems, and the running time.

4. (20 pts.) Breadth requirements

UC Berkeley's newly formed College of Computer Science has several categories of breadth requirements. Each class is offered by a single department, and may match multiple categories. Students must take at least one class from each category, and at most k classes from each department. If a class matches multiple categories, you must choose which category you're using it to fulfill: no class can be used to fulfill more than one category.

Formally, there are m classes $c_1 \dots c_m$, T departments $D_1 \dots D_T$, and n categories of breadth requirements $G_1 \dots G_n$. For some class c_i , let $H(c_i)$ be its department and $F(c_i)$ be the set of categories it can be used to fulfill. For example, you could have $H(c_{10}) = D_3$ and $F(c_{10}) = \{G_3, G_4, G_9\}$.

Given $G_1 \dots G_n$, $D_1 \dots D_T$, F , and H , and a subset of classes $c_1 \dots c_p$, you want to determine whether the p classes satisfy all the breadth requirements subject to the above constraints.

Alternatively, for 2/3 credit, you can ignore the departmental constraints (so the input is just $c_1 \dots c_p$, $G_1 \dots G_n$, and F).

- (a) Formulate the above problem as a linear program. Make sure to clearly identify and define what your variables are.

Continued on next page...

- (b) Now formulate the above problem as a network flow problem. Show how to determine from the max-flow whether the requirements are satisfied, and (if all requirements are satisfied) a mapping of which class contributes to which requirement. Prove the correctness of your reduction.

Main idea of reduction, together with sketch of max-flow network:

Proof:

Extra page

Use this page for scratch work, or for more space to continue your solution to a problem. If you want this page to be graded, please refer the graders here from the original problem page.

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